



# An improved intuitionistic fuzzy interval two-stage stochastic programming for resources planning management integrating recourse penalty from resources scarcity and surplus

Shanshan Guo <sup>a</sup>, Fan Zhang <sup>a</sup>, Chenglong Zhang <sup>a</sup>, Youzhi Wang <sup>a</sup>, Ping Guo <sup>a, b, \*</sup>

<sup>a</sup> Center for Agricultural Water Research in China, China Agricultural University, Beijing, 100083, China

<sup>b</sup> Wuwei Experimental Station for Efficient Water Use in Agriculture, Ministry of Agriculture and Rural Affairs, Wuwei, 733000, China

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## ABSTRACT

Two-stage programming (TSP) is popular in resources planning management, especially for limited and precious resources. Remarkable study has been done to improve the model performance. However, one of the biggest obstacle is lack of objectivity when it comes to penalty quantification derived from recourse behavior. Besides, much attention has been paid in the resources deficiency penalty but little in resources residual, which may lead to wasting. In order to clarify the physical meaning of mathematical equation for recourse penalty from both resources scarcity and surplus, the production frontier was estimated and the technical efficiency and shadow prices of resources were introduced into TSP to characterize the resources deficiency and residual penalty, respectively. Then, an intuitionist fuzzy interval two-stage stochastic programming (IFITSP) was generated integrating the uncertainty of fuzzy membership and traditional TSP. An integrated solving approach was proposed coupling several previous uncertain programming methods and an improved robust interval TSP method. A case study was conducted in an arid area of northwest China to schedule agricultural cultivation scale based on limited water resources. The inefficiencies were [0.26, 0.49], [0.14, 0.37] and [0, 0.03] for GZ, LZ, and GT. The shadow prices of GZ, LZ, and GT in 2015 were 12.94, 2.61, 2.67 Yuan/m<sup>3</sup> respectively, indicating the severe water crisis of GZ. The relatively unbiased and abundant decision could be generated by the developed IFITSP to help decision makers with various preferences make tradeoff between benefits and basic crop production requirement as well as balance resources deficiency and surplus. The results also show that the developed model could unveil the uncertainty influence of model inputs on decision strategies and trigger managers to deeply analyze subjective effect and associated risk. By comparison, the proposed methodology can not only clarify the physical meaning of penalty but deal with more complex uncertainty than previous methods. Therefore, the established model can provide reliable and scientific support for resources planning with recourse.

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## 1. Introduction

Resources scarcity is threatening human sustainable development around the world. Climate change, population growth, environmental contamination and industrial expansion have worsened the resources crisis which urges managers to develop high-efficient

resources management approaches (Singh, 2012). In response to mitigating the resources crisis, the reasonable resources planning is among the most popular resources management and has been widely discussed worldwide.

Remarkable contributions have been done surrounding the optimal resources planning and one of the achievements is two-stage programming (TSP), which is capable of modifying the pre-determined targets (called first-stage decision) based on the overall influence of uncertain events and generating corresponding decision (called second-stage decision) after the uncertain events happen. Resources planning optimization research on TSP covers all walks of life, ranging from agriculture (Fu et al., 2018; Zhang et al.,

\* Corresponding author. Center for Agricultural Water Research in China, China Agricultural University, Tsinghuadong Street No.17, Beijing, 100083, China.

E-mail addresses: [hbpngxss@cau.edu.cn](mailto:hbpngxss@cau.edu.cn) (S. Guo), [Zhagf@cau.edu.cn](mailto:Zhagf@cau.edu.cn) (F. Zhang), [zhangcl1992@cau.edu.cn](mailto:zhangcl1992@cau.edu.cn) (C. Zhang), [2587804769@qq.com](mailto:2587804769@qq.com) (Y. Wang), [guop@cau.edu.cn](mailto:guop@cau.edu.cn) (P. Guo).

2017), hydrology (Ding et al., 2017; Hu et al., 2016; Yu et al., 2016), environment (Han et al., 2012; Han et al., 2013; Han and LEE, 2011), energy (Yun et al., 2017), and transportation (Barbarosoglu and Arda, 2004) to medicine (Dillon et al., 2017), manufacture (Alfieri et al., 2012), and networks (Wu and Kucukyavuz, 2018). A highlighted part in TSP is penalty quantification caused by uncertain events, which is usually expressed as benefits loss from resources deficiency according to previous studies. However, the previous achievements share two problems. Quantification method is the first one as the loss from missing the targets is difficult to compute precisely. The conventional method which is widely adopted in previous studies is to amplify the profits that could have been obtained from the resources shortage by multiplying some coefficients offered by decision makers, which are subjective and unreliable if decision makers are unsophisticated and uninitiated. The second one is neglecting the potential economic value of residual resources. Once the allocation strategy is decided, the waste of precious resources from exceeding the targets should also be punished. The first problem of how to quantify the punishment will arise again when the second one is taken into account. Therefore, a systematic and objective approach needs to be generated to solve aforementioned problems.

Production efficiency analysis was proposed to examine how a particular production sector transform their inputs into quantities of outputs (Simar and Wilson, 2015). Production frontier is defined in the relevant input-output spaces as the locus of the maximal attainable level of output corresponding to a given level of inputs (Cazals et al., 2002). Many approaches have been generated for productive efficiency analysis and frontier estimation, such as the nonparametric data envelopment analysis (DEA) (Farrell, 1957) and the parametric stochastic frontier analysis (SFA) (Effiong, 2007; Katuwal et al., 2016). However, DEA ignores any stochastic noise while SFA demands hypothesis on functional form and parameters. For improvement, stochastic nonparametric envelopment of data (StoNED) was established by combining the DEA-type nonparametric frontier with the SFA-style stochastic homoscedastic composite error term (Kuosmanen, 2012; Kuosmanen and Johnson, 2017). By estimating the production frontier, the technical efficiency and shadow price of inputs can be obtained. In order to solve the above problems in TSP, the technical efficiency was introduced to quantify the penalty caused by resources deficiency and the shadow price of resources was adopted to quantify the penalty caused by resources residual in this study. Then the physical meaning of resources deficiency penalty would be the biggest potential benefits of lacking resources under ideal production condition and that of resources residual penalty would be the marginal benefits of surplus resources under present condition. Thus, the physical meaning of the resources deficiency and residual penalty can be clarified and becomes independent on subjective judgment of managers but relies on the statistic data.

There are various types of uncertainty associated with data quality, human subjective judgment and other factors within a resources planning problem. Besides stochastic programming driven by random events, the extension of TSP has been examined in combination with interval programming and fuzzy programming (Li and Guo, 2015; Zhang et al., 2018a). Among these improvements, fuzzy programming has advantage in less data requirement than stochastic programming and higher disposability than interval programming when encountering high uncertain parameters (Chen et al., 2017). However, most of the recent research focuses on the traditional fuzzy sets (FSs) with distinct and certain membership (Guo et al., 2010; Han et al., 2013; Xu et al., 2017). Since the introduction of FSs by Zadeh (1965), ambiguity has also been found in membership and several concepts have been defined to quantify the blurry property of membership, among which are intuitionistic

fuzzy set (IFS) (Atanassov, 1986) and hesitant fuzzy set (HFS) (Torra, 2010). IFS possesses the capacity of quantifying the uncertainty on the membership function of FS and permits some hesitant on the membership by restraining the membership within a scope. As a generalization of IFS, HFS can characterize higher-level uncertain feature in membership but is more complex and difficult to tackle than IFS. These improvements are more flexible and practical than traditional FS theory (Li et al., 2017). The collaboration of TSP and IFS has less been explored and needs more efforts.

Therefore, this study aims to improve previous TSP for resources planning from both physical meaning and programming approach. On one hand, the proposed TSP structure clarified the penalty from resources deficiency by introducing the technical inefficiency from production frontier evaluation. Meanwhile, considering the likely penalty caused by resources residual, the shadow price of resources was brought in, thus supplementing and enriching the physical meaning of two-stage planning. On the other hand, the IFS theory was conflated with traditional interval two-stage programming (ITSP) in order to solve more complex fuzzy uncertainty and an intuitionistic fuzzy interval two-stage stochastic programming (IFITSP) was proposed. Corresponding solving method was developed aimed at the new-developed model. Finally, the agricultural cultivation scale planning under limited water resources was demonstrated as case study to illuminate the proposed model (Li and Guo, 2015; Zhang et al., 2017). The detailed study framework is depicted in Fig. 1.

## 2. Methodology

### 2.1. Production frontier estimation and shadow price

Considering a production process with input  $xin$  and output  $yot$ , the frontier production function  $f$  indicates the maximum output that can be produced with the given inputs. Here, we describe the model for single-output multiple-input case, i.e.  $xin \in \mathfrak{R}_+^m$ ,  $yot \in \mathfrak{R}_+$  and  $f: \mathfrak{R}_+^m \rightarrow \mathfrak{R}_+$ . Each observed or actual output  $yot_i$  may differ from  $f(xin_i)$  due to the inefficiency and noise, which can be combined known as composite error. The popular multiplicative error structure was adopted in this study and the production function with a multiplicative error term can be expressed as:

$$yot_i = f(xin_i) \exp(\varepsilon_i) = f(xin_i) \exp(v_i - u_i) \quad \forall i = 1, \dots, n \quad (1)$$

where,  $f(xin_i)$  is the production frontier,  $\varepsilon_i$  is composite error term and  $\varepsilon_i = v_i - u_i \quad \forall i$ ,  $v_i$  denotes the random disturbance and  $u_i$  is inefficiency term, satisfying  $u_i \geq 0$ . We assume that: 1) function  $f$  belongs to the class of continuous, monotonic increasing and globally concave functions that can be differentiable; 2) terms  $v_i$  and  $u_i$  are statistically independent with each other as well as input  $xin_i$ , where the disturbance term  $v_i$  has a symmetric distribution with zero mean and a constant, finite variance  $\sigma_v^2$ , while the inefficiency term  $u_i$  has an asymmetric distribution with a positive expected value  $\mu$  and a finite variance  $\sigma_u^2$ . As the composite error violates the Causs-Markov properties due to  $E(\varepsilon_i) = -E(u_i) = -\mu < 0$ , a modification was made then after taking the logarithm of both sides, equation (1) can be rephrased as:

$$\begin{aligned} \ln yot_i &= \ln f(xin_i) + \varepsilon_i = [\ln f(xin_i) - \mu] \\ &+ [v_i - u_i + \mu] = g(xin_i) + \delta_i \quad \forall i = 1, \dots, n \end{aligned}$$

where  $\delta_i$  is the modified composite error term and function  $g$  is the average-practical production function. According to the StoNED (Kuosmanen, 2012; Kuosmanen and Kortelainen, 2007), the shape of frontier production function can be estimated using the following nonlinear programming:

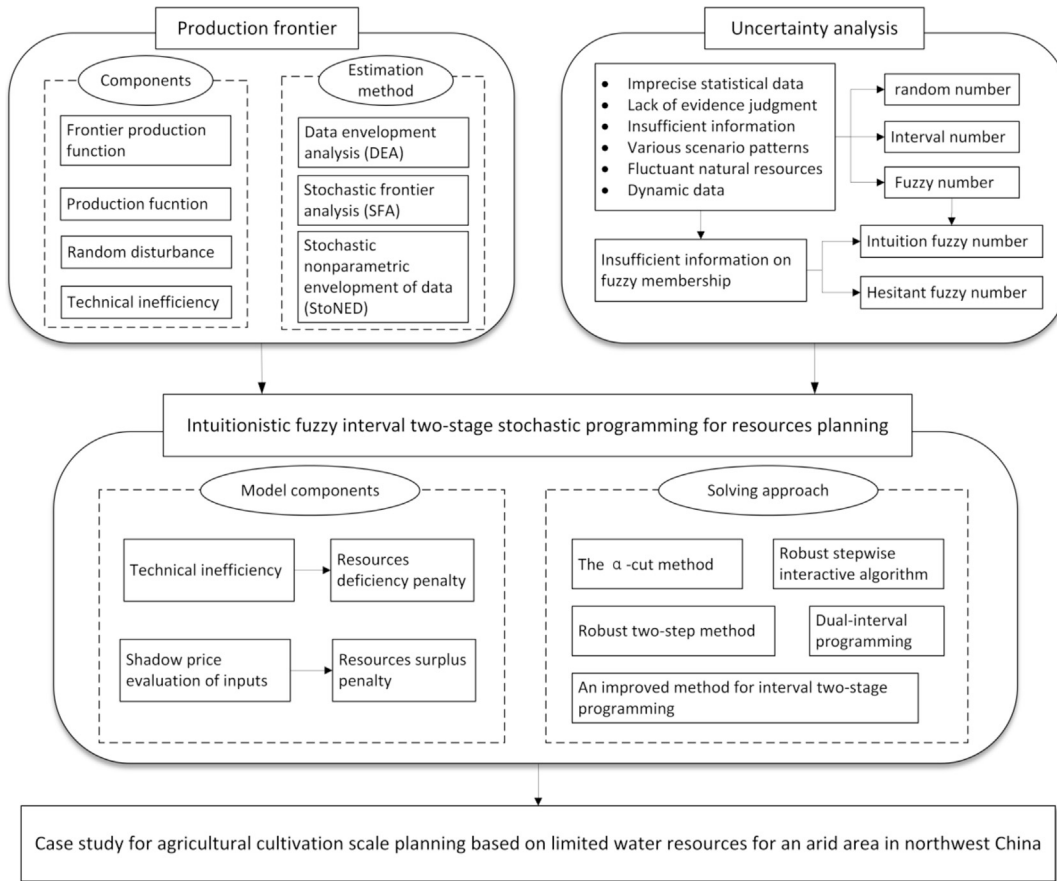


Fig. 1. Study framework of IFITSP considering recourse penalty from resources scarcity and surplus.

$$\begin{aligned}
 & \min_{\alpha, \beta} \sum_{i=1}^n \delta_i^2 \\
 & \text{s.t.} \\
 & \delta_i = \ln yot_i - \ln \hat{yot}_i \quad \forall i = 1, \dots, n \\
 & \hat{yot}_i = \alpha_i + \beta_i' xin_i \quad \forall i = 1, \dots, n \\
 & \alpha_i + \beta_i' xin_i \leq \alpha_h + \beta_h' xin_h \quad \forall i, h = 1, \dots, n \\
 & \beta_i \geq 0 \quad \forall i = 1, \dots, n
 \end{aligned} \tag{2}$$

where  $\hat{yot}_i$  is the estimator of  $yot_i$ , and  $\alpha_i, \beta_i$  are not parameters of the estimated function  $g$  but rather they characterize tangent hyperplanes to the unknown function  $g$  at point  $xin_i$ , thus they are specific to each observation  $i$ .

The modified composite error term  $\delta_i$  can be solved according to model (2). Then the method of moments can be used to estimate  $\sigma_v$  and  $\sigma_u$  based on the assumption that the technical inefficiency has a half normal distribution  $u_i \sim |N(0, \sigma_u^2)|$  and the random disturbance is normally distributed  $v_i \sim N(0, \sigma_v^2)$ . Then the variance can be:  $\hat{\sigma}_u =$

$$\sqrt[3]{\frac{\hat{M}_3}{\left(\frac{2}{\pi}\right)\left(1-\frac{3}{\pi}\right)}}; \quad \hat{\sigma}_v = \sqrt{\hat{M}_2 - \left(\frac{\pi-2}{\pi}\right)\hat{\sigma}_u^2}, \quad \text{where } \hat{M}_2 =$$

$\frac{1}{n} \sum_{i=1}^n (\hat{\delta}_i - E(\hat{\delta}_i))^2$ , and  $\hat{M}_3 = \frac{1}{n} \sum_{i=1}^n (\hat{\delta}_i - E(\hat{\delta}_i))^3$ . Thus the frontier production function can be obtained by  $f(xin_i) = \hat{yot}_i \exp(\hat{\sigma}_u \sqrt{2/\pi})$ . Meanwhile, the technical inefficiency of each  $i$  can be estimated as:

$$E(u_i | \hat{\varepsilon}_i) = -\frac{\hat{\varepsilon}_i \hat{\sigma}_u^2}{\hat{\sigma}_u^2 + \hat{\sigma}_v^2} + \frac{\hat{\sigma}_u^2 \hat{\sigma}_v^2}{\hat{\sigma}_u^2 + \hat{\sigma}_v^2} \left[ \frac{\phi(\hat{\varepsilon}_i / \hat{\sigma}_v)}{1 - \Phi(\hat{\varepsilon}_i / \hat{\sigma}_v)} \right]$$

where  $\hat{\varepsilon}_i = \hat{\delta}_i - \hat{\sigma}_u \sqrt{2/\pi}$  is the estimator of composite error term and  $\phi$  is the standard normal density function while  $\Phi$  is the standard normal cumulative distribution function.

When decision makers pursue to maximize the production profits, an optimization problem can be expressed as:

$$\max_{xin, yot} p_{yot}' yot - p_{xin}' xin \quad \text{s.t. } F(xin, yot) = 0$$

where  $p_{yot}$  and  $p_{xin}$  are prices of output and input,  $F(xin, yot)$  is a transformation function corresponding to the production function. Lagrange multiplier method can be applied to solve the above optimization problem and the shadow price of input can be derived as:

$$p_{xin_i} = p_{yot_i} \frac{\partial \hat{g}(xin_i)}{\partial xin_i} \exp(\hat{\mu}) = p_{yot_i} \hat{\beta}_i \exp(\hat{\mu}) \tag{3}$$

where  $\hat{\beta}$  is the solution of model (2) (Shen and Lin, 2017).

## 2.2. Interval two-stage stochastic programming

A generalized TSP model can be expressed as follows (Birge and

Louveau, 1988):

$$\begin{aligned} \max z &= cx + E_{\omega \in \Omega} [Q(x, \omega)] \\ &\text{s.t.} \\ &ax \leq b \\ &x \geq 0 \end{aligned}$$

$$\left\{ \begin{aligned} Q(x, \omega) &= \min q(y, \omega) \\ &\text{s.t.} \\ T(\omega)x + W(\omega)y &\leq H(\omega) \\ y &\geq 0 \end{aligned} \right.$$

where  $x$  is first-stage decision variable,  $y$  is second-stage decision variable,  $\omega$  is random variable defined in probability space  $(\Omega, F, Pr)$ ,  $E$  denotes expectation, and  $c$ ,  $a$  and  $b$  are known coefficient matrices,  $T(\omega)$ ,  $W(\omega)$ , and  $H(\omega)$  are functions of the random variable  $\omega$ .

Based on previous study (Huang and Loucks, 2000; Li et al., 2011; Xie et al., 2013; Zhang and Guo, 2018), the random variable can be discretized into a couple of certain values with associated probabilities and interval number can be adopted to reflect the uncertainty from parameters in the model. Then a simplified ITSP model can be derived from the generalized TSP above as:

$$\max z = c^\pm x^\pm + \sum_{h=1}^n p_h q(y_h^\pm, \omega_h^\pm) \tag{4a}$$

subject to:

$$a^\pm x^\pm \leq b^\pm \tag{4b}$$

$$T(\omega_h^\pm)x + W(\omega_h^\pm)y_h^\pm \leq H(\omega_h^\pm); \forall h \tag{4c}$$

$$x^\pm \geq 0 \tag{4d}$$

$$y_h^\pm \geq 0; \forall h \tag{4e}$$

where  $p_h$ ,  $h = 1, \dots, n$  is the discrete probabilities,  $\pm$  means the parameter is an interval with “-” for lower bound and “+” for upper bound. A popular solving method (Huang and Loucks, 2000) was proposed integrating the discreteness of random sets and the two-step method for interval programming (Huang et al., 1992).

### 2.3. Intuitionistic fuzzy set theory

Membership of FS can reflect the degree to which elements belong to the sets while the degree cannot always be defined precisely. Compared with FS, IFS can quantify the uncertainty existing in the membership by adopting membership and non-membership jointly denoting the belonging degree (Atanassov, 2000). Let  $S$  be fixed as a reference set, an IFS  $A$  on  $S$  is defined as  $A = \{ \langle s, \mu_A(s), \nu_A(s) \rangle | s \in S \}$ , where  $\mu_A(s)$  and  $\nu_A(s)$  denote the degrees of membership and non-membership of the element  $s$  to the set  $A$ , respectively, with the conditions of  $0 \leq \mu_A(s) \leq 1$ ,  $0 \leq \nu_A(s) \leq 1$ , and  $0 \leq \mu_A(s) + \nu_A(s) \leq 1$ . Besides,  $\pi_A(s) = 1 - \mu_A(s) - \nu_A(s)$  is called the degree of indeterminacy or hesitancy of  $s$  to  $A$  (Atanassov, 1986). Particularly, if  $\mu_A(s) + \nu_A(s) = 1$ , the IFS will be equal to a FS. Based on IFS, HFS was introduced to extend the IFS by covering all possible membership values. Similarly, let  $S$  be a fixed set, a HFS  $E$  on  $S$  is in terms of a function that when applied to  $S$  returns a subset of  $[0,1]$ , which can be expressed as  $E = \{ \langle s, h_E(s) \rangle | s \in S \}$ , where  $h_E(s)$  is a set of some values in  $[0,1]$ , denoting the possible membership degrees of  $s$  to the set  $E$  (Torra, 2010). This definition encompasses IFS as a particular case, which means the

IFS and HFS can transform to each other to some extent. Given an IFS  $A$ , the corresponding membership of HFS can be expressed as  $h_A(s) = [\mu_A(s), 1 - \nu_A(s)]$ , if  $\mu_A(s) \neq 1 - \nu_A(s)$ . Oppositely, given a HFS  $E$ , the corresponding IFS can be defined as the envelopment of  $h_E$ , i.e.  $\mu_E(s) = \min h_E(s)$ ;  $\nu_E(s) = 1 - \max h_E(s)$  (Xia and Xu, 2011).

However, too much uncertain information on the membership of a FS may bring difficulty in accurately quantifying it. Some accuracy functions or score functions have been proposed to help solve this problem, such as  $\xi_A(s) = [\mu_A(s) + 1 - \nu_A(s)]/2$  (Geng et al., 2013) or  $\xi_A(s) = (1 - \gamma)\mu_A(s) + \gamma[1 - \nu_A(s)]$  (Burillo and Bustince, 1996) for IFS and  $\xi_E(s) = \frac{1}{n} \sum_{i=1}^n h_{Ei}(s)$  for HFS (Farhadinia, 2013; Xia and Xu, 2011). However, these score functions lose a lot of uncertain information of their membership values. In order to improve the quantification method of IFS, the interval membership value was adopted in this study, i.e.  $\xi_A(s) = [\min(\mu_A(s), 1 - \nu_A(s)), \max(\mu_A(s), 1 - \nu_A(s))]$  and  $\xi_E(s) = [\min h_E(s), \max h_E(s)]$ . The membership interval is relatively reasonable as an accuracy method when there is few information about the relationship among different possible membership values.

When it comes to FS with continuous membership functions, defuzziness should first conducted before the fuzzy programming can be solved, wherein the  $\alpha$ -cut method has proved effective and recognized (Li and Hong, 2013; Lu et al., 2010; Sudha and Anitha, 2015). In this study, the triangular intuitionistic fuzzy number was discussed because of its simplicity and convenience and it was expressed as  $A = (a_1, a_2, a_3; a_1', a_2, a_3')$  shown in Fig. 2, where  $\mu_A = (a_1, a_2, a_3)$  and  $\nu_A = (a_1', a_2, a_3')$  with  $a_1' < a_1 < a_2 < a_3 < a_3'$ . Specially, if  $a_1' = a_1$  and  $a_3 = a_3'$ , the IFS will be a FS and if  $a_1' = a_1 = a_2 = a_3 = a_3'$ , the IFS will be a certain value. Based on the  $\alpha$ -cut method in fuzzy sets and the newly developed interval accuracy method, the IFS parameters with membership function can be defuzzified into a couple of dual intervals. Then the  $\alpha$ -cutted dual interval can be explicated as  $[A^{\alpha\pm}] = \left[ \left[ \underline{A}^{\alpha-}, \bar{A}^{\alpha-} \right], \left[ \underline{A}^{\alpha+}, \bar{A}^{\alpha+} \right] \right]$  as shown Fig. 2, which is a subset containing all the elements whose belonging degree  $\geq \alpha$ . The lower bound of the  $\alpha$ -cutted  $A$  is taken value between  $\underline{A}^{\alpha-}$  and  $\bar{A}^{\alpha-}$  while the upper bound is taken value between  $\underline{A}^{\alpha+}$  and  $\bar{A}^{\alpha+}$  without any information about the bound-value-taken rule. Therefore, the  $\alpha$ -cutted dual interval has two extremes: the minimum interval or the internal interval  $[\underline{A}^{\alpha-}, \underline{A}^{\alpha+}]$  and the maximum interval or the external interval  $[\bar{A}^{\alpha-}, \bar{A}^{\alpha+}]$ .

### 2.4. Solving method

Therefore, the intuitionist fuzzy two-stage stochastic programming (IFTSP) can be depicted based on the IFS mentioned above and TSP. Note that the model depicted in this paper belongs to a family of linear programming and in order to exhibit the method clearly, the objective here is exclusively set to maximize the benefits and minimize the benefits loss caused by recourse as an example. The developed IFTSP can be formulated as follows:

$$\max z = \tilde{c} \cdot \tilde{x} - \sum_{h=1}^n \tilde{p}_h \cdot \tilde{d}_h \cdot \tilde{y}_h \tag{5a}$$

subject to

$$\tilde{a} \cdot \tilde{x} \leq \tilde{b} \tag{5b}$$

$$\tilde{T}_h \cdot \tilde{x} + \tilde{W}_h \cdot \tilde{y}_h \leq \tilde{H}_h, \forall h \tag{5c}$$

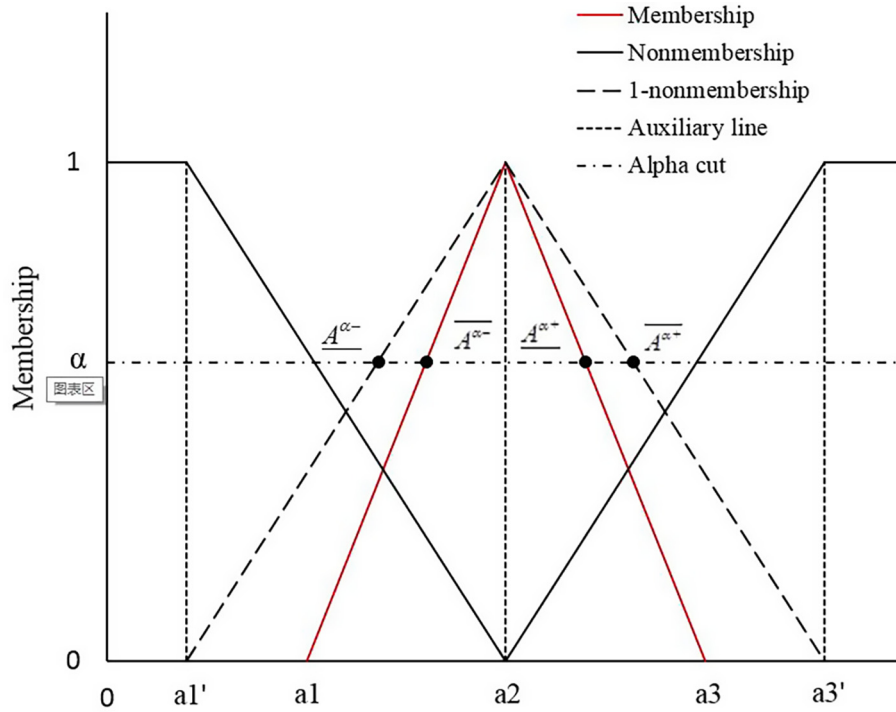


Fig. 2. A general view of the triangular IFS.

$$\tilde{x} \geq 0 \tag{5d}$$

$$\tilde{y}_h \geq 0, \forall h \tag{5e}$$

where  $\sim$  denotes the parameter is an IFS that can be expressed as  $(a_1, a_2, a_3; a_1', a_2, a_3')$ ;  $T, W$ , and  $H$  are linear parameters associated with random events; and constraint (5c) is a reshaping of constraint (4c). Set a couple of  $\alpha$  values from high to low, i.e.  $0 \leq \alpha_{i+1} < \alpha_i \leq 1$ ;  $i$  and  $i + 1 \in \{1, \dots, m\}$  and achieve corresponding  $\alpha$ -cut values of IFSs. The model (5) can be transformed by using  $\alpha$ -cut method and rephrased as a group of dual-interval two-stage stochastic programming (DITSP) submodels with associated  $\alpha$ , shown as model (6).

$$\max [z^{\alpha_i \pm}]^\pm = [c^{\alpha_i \pm}]^\pm \cdot [x^{\alpha_i \pm}]^\pm - \sum_{h=1}^n [p_h^{\alpha_i \pm}]^\pm \cdot [d_h^{\alpha_i \pm}]^\pm \cdot [y_h^{\alpha_i \pm}]^\pm \tag{6a}$$

subject to

$$[a^{\alpha_i \pm}]^\pm \cdot [x^{\alpha_i \pm}]^\pm \leq [b^{\alpha_i \pm}]^\pm \tag{6b}$$

$$[T_h^{\alpha_i \pm}]^\pm \cdot [x^{\alpha_i \pm}]^\pm + [W_h^{\alpha_i \pm}]^\pm \cdot [y_h^{\alpha_i \pm}]^\pm \leq [H_h^{\alpha_i \pm}]^\pm; \forall h \tag{6c}$$

$$[x^{\alpha_i \pm}]^\pm \geq 0 \tag{6d}$$

$$[y_h^{\alpha_i \pm}]^\pm \geq 0; \forall h \tag{6e}$$

where  $[\pm]^\pm$  means the dual-interval number. In order to achieve a relatively robust solution, the robust stepwise interactive algorithm (RSIA) was adopted when splitting the IFTSP into a group of DITSPs with a couple of additional interactive constraints (Fan et al., 2015).

Note that the law is unknown as mentioned above on how to

take bound value between external and internal bounds except for the two extreme bound values, which means there is little distributional information on bounds and thus the random interval method by Joslyn (Joslyn, 2003; Liu et al., 2009; Zhai et al., 2016) do not apply to the model (6). Therefore, two extreme scenarios were discussed in this paper as a simplification, i.e. the internal and external intervals. Then the DITSP can be converted into two conventional single-interval TSP submodels. In order to guarantee the reliability of solution, the internal interval submodel was solved first and additional interaction constraints were incorporated when the external interval submodel was dealt with. Additionally, the first-stage variables are predetermined inputs defined by decision makers and the two-stage programming needs to modify them according to the upcoming uncertain events. Therefore, auxiliary variables should be introduced as  $x^\pm = xl + t \cdot \Delta x$ , where  $xl$  denotes the lower bound of predefined first-stage decision variables (Huang and Loucks, 2000; Li et al., 2006).

There has been remarkable research on improved interval programming after the two-step method was generated (Fan and Huang, 2012; Huang and Cao, 2011; Huang and Moore, 1993; Zhou et al., 2008). Among these promotion, the robust two-step method is outstanding due to its good performance and convenience (Fan and Huang, 2012). However, when it comes to the ITSP, the method proposed by Huang and Loucks (2000) has been widely applied along with other compound ITSP coupled with other types of uncertainty (Li et al., 2011; Xie et al., 2013; Zhang et al., 2017). However, one of the weakness of ITSP in these research is that the first-stage decision variable solved in the first submodel will be directly used into the second submodel rather than being an interactive constraint. This method will obviously lead to over-optimistic first-stage decision when the best-case submodel is first calculated (the objective is maximum and the upper-bound model is first solved, vice versa) and to over-pessimistic first-stage decision when the worst-case submodel is first solved (the objective is maximum and the lower-bound model is first solved, vice versa). In

order to avoid this disadvantage and provide managers with a relatively flexible decision, the second submodel is computed under a supplemented interactive constraint based on the first submodel solution. Then instead of an exact first-stage modified decision with strong subjective color, a restrained interval decision could be offered. The detailed solving process of IFTSP can be expressed as follows in Fig. 3.

### 3. Case study

#### 3.1. Study area and problem overview

The study area is located in the middle reaches of Heihe River Basin (HRB), northwest China (97°37′~102°06′ E, 37°44′~42°40′ N), covering Ganzhou District (GZ), Linze County (LZ), and Gaotai County (GT) three administrative regions of Zhangye City, Gansu Province, seen in Fig. 4. As the second largest inland basin of China, HRB is faced with severe water crisis. The middle reaches of HRB is the main water consumption, where climate is characterized by high evaporation of about 1400 mm/year and low rainfall of 140 mm/year (Guo et al., 2019; Zhang et al., 2018b). Meanwhile, the middle reaches of HRB is one of the most important agricultural production base in northwestern China. Precious water resources grows deficit because of the comprehensive influence including social development, environment deteriorate and climate change. The agricultural sustainability encounters big challenge the history has ever seen, which urges managers to efficiently allocate the limited water resources (Zhang et al., 2019). The agricultural

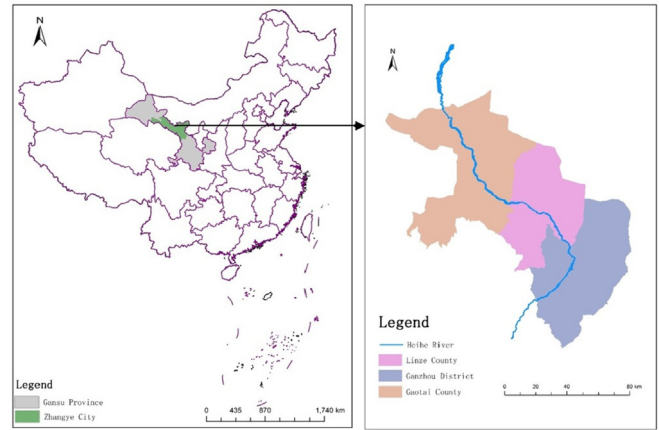


Fig. 4. Study area.

cultivation scale planning based on water allocation which regulates the agricultural irrigated area targets of different crops has been emphasized by local managers.

As a transformation of agricultural water resources planning, agricultural cultivation scale planning under limited water availability is also paid attention by researchers, which applies for arid area where the agricultural cultivation scale is highly dependent upon available water resources (Li and Guo, 2015; Zhang et al., 2017). The previous objectives of TSP in this area are mainly

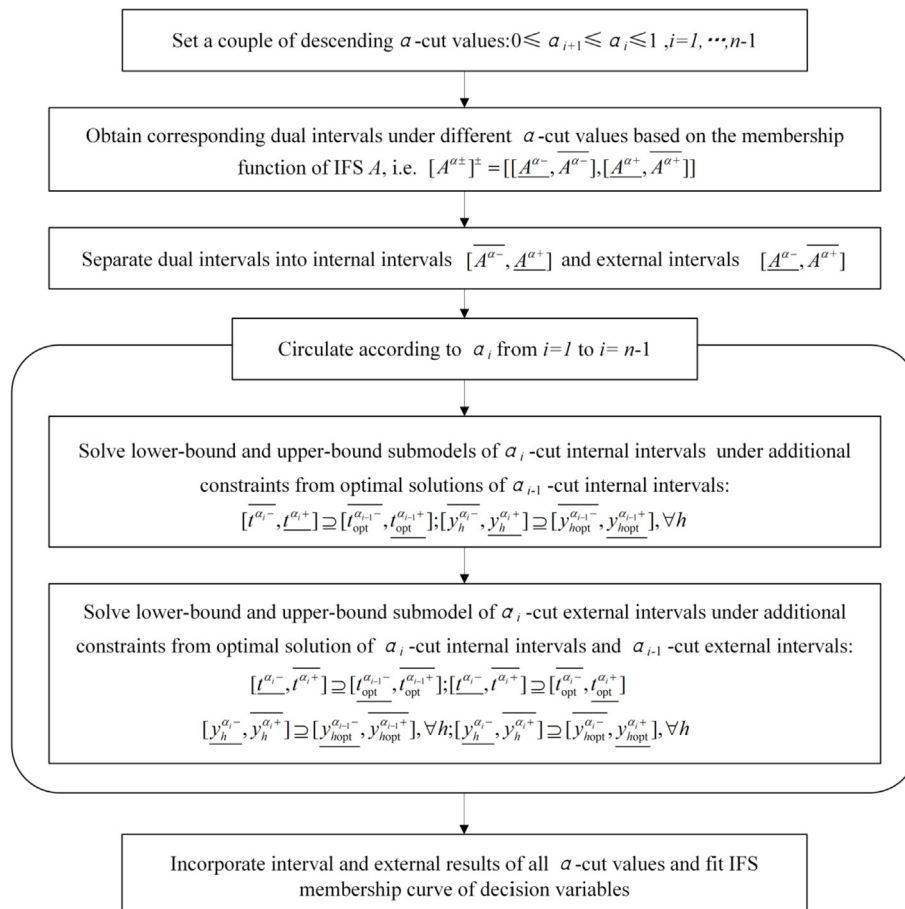


Fig. 3. The solving process of IFTSP.

targeted at the benefits of local farmers, which highlight the recourse penalty caused by water deficiency. Thus, the derived irrigation targets from conventional research relatively prefer to avoid the water shortage penalty while the water surplus is ignored. However, the water resources once allocated is difficult to transfer to other sectors timely in practice. Thus the water wasting is equally important from the perspective of the irrigation district managers, especially for the arid areas like the middle researches of HRB where the water resources is crucial for local development. Besides, the previous study is subjective when it comes to penalty quantification. Meanwhile, due to government intervention, the current price of water resources is too low to reflect its potential economic benefits. Therefore, a systematic and unbiased method is desired to solve these problems.

3.2. Modelling

Therefore, based on the methodology developed in this paper, the production frontier and shadow price of water resources were introduced into the conventional TSP. The objective is maximizing the agricultural benefits from the perspective of local managers and minimizing the penalty led by water scarcity and waste. Moreover, considering the widespread uncertainty, the intuitionist fuzzy feature of the model parameters were analyzed. Due to the data limitation, not all of the parameters can be qualified as IFS but single-interval number or certain value. Therefore, based on the available data, the IFITSP was developed and the definition of symbols can be found in Table 1.

$$\begin{aligned} \max f = & \sum_{i=1}^3 \sum_{j=1}^3 \tilde{B}_{ij} \cdot AT_{ij}^{\pm} - \sum_{i=1}^3 \sum_{j=1}^3 \sum_{h=1}^3 p_h \cdot \tilde{B}_{ij} \cdot \tilde{A}S_{ijh} \cdot \exp(\mu_i^{\pm}) \\ & - \sum_{i=1}^3 \sum_{h=1}^3 p_h \cdot WSP_i^{\pm} \cdot \tilde{W}res_{ih} \end{aligned} \tag{7a}$$

wherein  $\tilde{W}res_{ih} = \tilde{W}A_{ih} - \sum_{j=1}^3 (AT_{ij}^{\pm} - \tilde{A}S_{ijh}) \cdot M_{ij}^{\pm} \quad \forall i, h$  and  $AT_{ij}^{\pm} = ATL_{ij} + t_{ij} \cdot T_{\Delta A_{ij}}$  with  $\Delta AT_{ij} = ATR_{ij} - ATL_{ij}$ .

The constraints involve water availability, land resources availability, food security, fairness constraint, irrigation area constraint and nonnegative constraint.

(1) Water availability

$$\sum_{j=1}^3 (AT_{ij}^{\pm} - \tilde{A}S_{ijh}) \cdot M_{ij}^{\pm} \leq \tilde{W}A_{ih} \quad \forall i, h \tag{7b}$$

(2) Land resources availability

$$\sum_{j=1}^3 (AT_{ij}^{\pm} - \tilde{A}S_{ijh}) \leq A_i^{\pm} \quad \forall i, h \tag{7c}$$

(3) Food security

$$\sum_{j=1}^2 (AT_{ij}^{\pm} - \tilde{A}S_{ijh}) \cdot Y_{ij}^{\pm} \geq Pop_i \cdot FD_{min} \quad \forall i, h \tag{7d}$$

(4) Fairness constraint

$$\left| \frac{\sum_{i=1}^3 \sum_{j=1}^3 \tilde{A}S_{ijh}}{\sum_{i=1}^3 \sum_{j=1}^3 AT_{ij}^{\pm}} - \frac{\sum_{j=1}^3 \tilde{A}S_{ijh}}{\sum_{j=1}^3 AT_{ij}^{\pm}} \right| \leq \sigma \quad \forall i, h \tag{7e}$$

(5) Irrigation area constraint

$$A_{minij}^{\pm} \leq AT_{ij}^{\pm} - \tilde{A}S_{ijh} \leq A_{maxij}^{\pm} \quad \forall i, j, h \tag{7f}$$

(6) Nonnegative constraint

**Table 1**  
Definition and explanation of model symbols.

Indices	
<i>i</i>	Administrative district. <i>i</i> = 1, 2, 3 denotes GZ, LZ, and GT, respectively.
<i>j</i>	Crop type. <i>j</i> = 1, 2, 3 means spring wheat, maize, and economic crops.
<i>h</i>	Inflow level. <i>h</i> = 1, 2, 3 represents high, medium, and low hydrological years
Sign	
±	Single-interval number. '-' means lower bound and '+' upper bound.
~	IFS
Decision variables	
<i>AT<sub>ij</sub></i>	The first-stage decision variable (10 <sup>3</sup> ha)
<i>t<sub>ij</sub></i>	The ratio of irrigation target and the auxiliary first-stage decision variable
<i>AS<sub>ijh</sub></i>	The missed irrigation area due to water deficiency and the second-stage decision variable (10 <sup>3</sup> ha)
Coefficients	
<i>B<sub>ij</sub></i>	The benefits produced by unit irrigation area of <i>j</i> th crop in <i>i</i> th district (10 <sup>3</sup> Yuan/ha)
<i>ATL<sub>ij</sub></i> & <i>ATR<sub>ij</sub></i>	The lower bound and upper bound of predetermined first-stage targets from decision makers (10 <sup>3</sup> ha)
<i>p<sub>h</sub></i>	The probability of <i>h</i> th hydrological level year
<i>μ<sub>i</sub></i>	The expected technical inefficiency, and exp( <i>μ<sub>i</sub></i> ) denotes the true production inefficiency, i.e. Farrell inefficiency
<i>WSP<sub>i</sub></i>	The agricultural water shadow price (Yuan/m <sup>3</sup> )
<i>Wres<sub>ih</sub></i>	The water residual (10 <sup>6</sup> m <sup>3</sup> )
<i>WA<sub>ih</sub></i>	The available irrigation water resources (10 <sup>6</sup> m <sup>3</sup> )
<i>M<sub>ij</sub></i>	The irrigation quota per irrigation area of <i>j</i> th crop in <i>i</i> th district (10 <sup>3</sup> m <sup>3</sup> /ha)
<i>A<sub>i</sub></i>	The available cultivation area (10 <sup>6</sup> m <sup>3</sup> )
<i>Y<sub>ij</sub></i>	The yield per irrigation area of <i>j</i> th crop in <i>i</i> th district (kg/ha)
<i>Pop<sub>i</sub></i>	The present population in <i>i</i> th district (10 <sup>3</sup> capita)
<i>FD<sub>min</sub></i>	The minimum food demand per capita (kg per capita)
<i>σ</i>	Fairness coefficient
<i>A<sub>minij</sub></i> & <i>A<sub>maxij</sub></i>	The minimum and maximum irrigation area of <i>j</i> th crop in <i>i</i> th district (10 <sup>3</sup> ha)

$$\begin{aligned}
 AT_{ij}^{\pm} - \widetilde{AS}_{ijh} &\geq 0 \quad \forall i, j, h \\
 \widetilde{AS}_{ijh} &\geq 0 \quad \forall i, j, h
 \end{aligned}
 \tag{7g}$$

### 3.3. Data collection and processing

Data for the case study are mainly from the statistical yearbooks, irrigation management yearbook, government reports, field survey, and literature study. Firstly, the production frontier estimation was conducted by evaluating the relationship between gross agricultural value and several key inputs during agricultural production including irrigation area, agricultural electricity, dosage of agricultural fertilizer and agricultural water usage. The 2001–2015 data of GZ, LZ, and GT were used from the Zhangye statistical yearbooks (Zhangye-SY, 2016) and irrigation management yearbooks (Zhangye-IMY, 2016). The descriptive statistics of input and output variables is shown in Table 2. The price of the output in shadow price equation, i.e. Equation (3) was set to 1 since the agricultural value has been measured in monetary units. As the inflow is random and naturally follows a Pearson type III distribution according to the inflow statistical material from Yingluoxia hydrological station (Zhang et al., 2017). Based on the frequency analysis, three levels were divided representing the low, medium and high water availability with associated probabilities. Considering the data noise, it is bias to appoint a single certain value to express a corresponding level. Besides, the cognition of low, medium and high standards highly depends on subjective judgment. Together with the uncertainty of other minor water sources like groundwater, the IFS would be suitable to express the water availability and the data are shown in Table 3.

Table 4 shows the benefits from crop production, the predetermined irrigation targets, irrigation quota for each crop, and the yield per hectare of three main cultivated crops in the middle researches of HRB. The benefits per unit cultivated area was calculated with the yield per hectare multiplying the corresponding market price and was expressed as IFSs due to its relatively sufficient data size. The predetermined irrigation targets were decided by local managers. The irrigation quota is from Zhangye irrigation management yearbooks (Zhangye-IMY, 2016) and literature (Li et al., 2019; Wang et al., 2017). Based on the China Dietary Guidelines, the food demand per capita was chosen as 400 kg. The fairness coefficient has important significant in balancing the crop area and maintaining the crop diversity. The value of 0.4 was adopted in this study as a standard of relatively reasonable allocation (Yang et al., 2015).

## 4. Result analysis and discussion

Based on the method proposed in Section 2.1, the production frontier was estimated to obtain the technical efficiency and shadow price of irrigation water resources. Then these parameters were introduced into IFITSP model exhibited in Section 3.2 then the developed solving method developed in Section 2.4 was applied to solve the IFITSP. Five  $\alpha$  values were defined for  $\alpha$ -cut method, i.e.  $\alpha = \{1, 0.75, 0.5, 0.25, 0\}$  to divide IFS into dual intervals.

### 4.1. Production frontier estimation and shadow price of irrigation water

The technical inefficiency  $\mu$  is not the real inefficiency as we adopted multiplication model as production function. The Farrell inefficiency and efficiency were defined as true output inefficiency and efficiency measure by  $\exp(\mu_i)$  and  $\exp(-\mu_i)$  (Kuosmanen and Kortelainen, 2010). Table 5 describes the expected inefficiency, the associated Farrell output efficiency and inefficiency, as well as shadow price of irrigation water. Fig. 5 shows the trend of the Farrell efficiency from three subareas during 2001–2015. Overall, the efficiencies of three subareas have been growing from 2001 to 2015, among which GT has achieved 1 since 2012. Among three subareas, GT possessed highest mean technical efficiency while the mean efficiency of GZ was the lowest. The Farrell efficiency of GZ in 2015 was 0.66, which implies that if the inputs were used efficiently, the inputs could be scaled down by 34% while maintaining the same output level. Therefore, local managers should pay more attention to the production process in GZ and take action to improve its production efficiency. Aimed at the optimization model, the inefficiencies  $\mu$  were expressed as interval numbers considering the possible fluctuation, namely [0.26, 0.49], [0.14, 0.37] and [0, 0.03] for GZ, LZ, and GT, respectively.

Regarding shadow price of water, GZ had the higher mean price than other two subareas and the price in 2015 reached up to 12.94 Yuan/m<sup>3</sup>, inferring that irrigation water resources severely restrained local agricultural production. Although the shadow prices of irrigation water in LZ and GT are relatively low, the mean water prices are higher than the market price in Zhangye, where the agricultural volumetric water price was 0.092–0.15 Yuan/m<sup>3</sup> until 2015 according to the government document. Besides, there exists some zero values in earlier years, especially in LZ and GT. One possible reason may be that they had relatively less water stress thus the marginal benefits of water was low. Another most-likely reason may lie in the non-volumetric pricing methods of irrigation water in the early period adopted in China. The irrigation water expense was calculated by per unit irrigated area before and

**Table 2**  
The descriptive statistics of inputs and output in production frontier estimation.

		Unit	Mean	Min	Max	Standard deviation (S.D.)
GZ	Gross agricultural value	10 <sup>6</sup> Yuan	1973.15	592.57	3621.05	1000.72
	Irrigation area	10 <sup>3</sup> ha	50.65	45.60	62.54	6.14
	Agricultural electricity	10 <sup>6</sup> KWh	88.71	49.99	149.09	26.37
	Dosage of agricultural fertilizer	10 <sup>3</sup> t	36.35	32.37	43.07	3.59
	Agricultural water	10 <sup>6</sup> m <sup>3</sup>	480.16	422.00	565.40	49.30
LZ	Gross agricultural value	10 <sup>6</sup> Yuan	799.00	245.68	1421.11	392.89
	Irrigation area	10 <sup>3</sup> ha	20.77	15.77	27.93	4.35
	Agricultural electricity	10 <sup>6</sup> KWh	21.06	8.46	34.05	7.61
	Dosage of agricultural fertilizer	10 <sup>3</sup> t	13.43	7.99	17.78	2.84
	Agricultural water	10 <sup>6</sup> m <sup>3</sup>	305.41	208.73	361.86	41.96
GT	Gross agricultural value	10 <sup>6</sup> Yuan	1206.38	421.97	2185.09	558.29
	Irrigation area	10 <sup>3</sup> ha	21.91	19.96	40.38	5.16
	Agricultural electricity	10 <sup>6</sup> KWh	27.11	17.54	38.91	6.03
	Dosage of agricultural fertilizer	10 <sup>3</sup> t	9.29	6.04	12.38	2.31
	Agricultural water	10 <sup>6</sup> m <sup>3</sup>	261.46	220.97	309.68	30.77



**Table 3**  
The available irrigation water resources under different hydrological years and corresponding probabilities ( $10^6 \text{ m}^3$ ).

	Probability	GZ	LZ	GT
Low	0.37	(430.3, 440.0, 448.0; 408.6, 440.0, 489.8)	(202.0, 206.5, 210.3; 194.8, 206.5, 229.9)	(245.9, 251.4, 256.0; 236.5, 251.4, 279.9)
Medium	0.3	(476.9, 502.3, 528.0; 460.1, 502.3, 537.6)	(223.9, 235.8, 247.8; 215.9, 235.8, 252.3)	(272.5, 287.0, 301.7; 262.9, 287.0, 307.2)
High	0.33	(544.8, 557.7, 571.0; 526.4, 557.7, 609.9)	(255.7, 261.8, 268.0; 247.1, 261.8, 286.3)	(311.3, 318.7, 326.3; 300.8, 318.7, 348.5)

**Table 4**  
The benefits of crop production, predetermined irrigation targets, irrigation quota and the yield per hectare of different crops in three subareas.

	Spring wheat	Maize	Economic crops
The benefits of crops ( $10^3$ Yuan/ha)			
GZ	(18.2, 18.8, 19.3; 15.8, 18.8, 21.2)	(17.8, 19.0, 20.2; 17.6, 19.0, 21.6)	(159.3, 173.0, 186.6; 148.2, 173.0, 196.9)
LZ	(17.8, 18.0, 18.4; 15.5, 18.0, 21.2)	(17.0, 18.4, 19.3; 16.8, 18.4, 20.1)	(150.1, 173.7, 185.6; 147.7, 173.7, 217.3)
GT	(16.1, 17.7, 19.2; 14.1, 17.7, 22.7)	(16.9, 18.5, 19.4; 15.7, 18.5, 21.8)	(158.0, 168.6, 173.6; 127.8, 168.6, 215)
The predetermined irrigation targets ( $10^3$ ha)			
GZ	[3.60, 5.70]	[43.45, 51.84]	[14.29, 17.78]
LZ	[0.65, 1.31]	[19.43, 22.29]	[6.79, 9.38]
GT	[5.88, 7.76]	[16.89, 19.14]	[13.66, 16.65]
The irrigation quota ( $10^3 \text{ m}^3/\text{ha}$ )			
GZ	[5.5, 6.0]	[6.9, 8.1]	[8.5, 9.0]
LZ	[6.3, 6.9]	[7.0, 8.4]	[8.4, 9.1]
GT	[5.4, 5.7]	[7.8, 8.6]	[7.6, 8.9]
The yield per hectare (kg/ha)			
GZ	[8033.8, 8858.9]	[8081.1, 8700.7]	[41123.3, 50256.6]
LZ	[7267.8, 8841.1]	[6950.8, 8220.0]	[37088.6, 100384.9]
GT	[7716.8, 9447.6]	[7284.3, 8422.6]	[28239.8, 41882.8]

**Table 5**  
The descriptive statistics of expected inefficiency, Farrell efficiency and inefficiency, and shadow price.

		Expected inefficiency $\mu_i$	Farrell efficiency $\exp(-\mu_i)$	Farrell inefficiency $\exp(\mu_i)$	Water shadow price (Yuan/ $\text{m}^3$ )
GZ	Mean	0.79	0.49	2.96	2.96
	S.D.	0.42	0.17	2.91	2.91
	Min	0.26	0.18	0.00	0.00
	Max	1.73	0.77	12.94	12.94
LZ	Mean	0.44	0.65	1.58	0.24
	S.D.	0.19	0.13	0.30	0.71
	Min	0.14	0.47	1.15	0.00
	Max	0.76	0.87	2.13	2.61
GT	Mean	0.31	0.76	1.43	0.53
	S.D.	0.29	0.20	0.44	1.09
	Min	0.00	0.43	1.00	0.00
	Max	0.85	1.00	2.34	2.67

farmers would not stop irrigating until the marginal product of irrigation equals zero (Shen and Lin, 2017). Considering the precious value of limited water resources in the middle researches of HRB, water price reform should be deepened to achieve high-efficient water use. However, if the second reason dominated the shadow price results, the shadow price in earlier years may be imperfect to evaluate the true value of water in this particular case study. Even so, the shadow price was still better than the real water price. Since the situation is rare that the valuable resources were wasted for a long term, the method can make bigger contribution in other cases especially when the market controls price. Taking all these into account, the shadow prices in 2015 were adopted in the optimization model, i.e. 12.94, 2.61, 2.67 Yuan/ $\text{m}^3$  for GZ, LZ, and GT.

4.2. Crop area planning strategies

Interval decision can be obtained from IFITSP for each particular

$\alpha$ , containing all the elements whose membership is equal or bigger than  $\alpha$ . Then a curve was fitted based on the results from all  $\alpha$ -cut values. Fig. 6 describes the modified irrigation targets (the modified first-stage decisions) based on the predetermined irrigation targets from local managers. The optimal irrigation targets for crops in three subareas were calculated through the equation:  $[AT_{ij}^{\alpha\pm}]^{\pm} = AT_{ij}^{\alpha} + [t_{ij}^{\alpha\pm}]^{\pm} \cdot \Delta AT_{ij}^{\alpha}$ . The results are presented as IFS in Fig. 6 and 1-nonmembership function was chosen to clearly demonstrate the relationship between membership and non-membership. Particularly, when the membership coincides with 1- nonmembership, the IFS will degrade into traditional FS. Similarly, if the triangle membership becomes two vertical lines, the FS will degrade into a conventional simple interval. Finally, if the membership curve is a vertical line, a certain number is generated without any uncertainty.

The ordinate in Fig. 6 indicates the predetermined irrigation targets. Compared with the interval of predetermined irrigation targets, the optimal irrigation targets were more compact after

considering the influence from random events. Instead of a certain number, the restrained intervals were capable to offer decision makers more flexible references compared with traditional ITSP solving method (Li and Guo, 2015; Zhang et al., 2017). Even though, certain modified targets were still made for spring wheat and economic crops in GZ and GT, which means the uncertainty in all parameters of the IFITSP has no impact on the irrigation targets of these crops in GZ and GT. Besides, the irrigation targets of spring wheat and economic crops in GZ and GT were set to the biggest predetermined values, implying that they made relatively high profits when consuming same amount of water thus the model gave them priority in water supply. The results of spring wheat in LZ can be treated as traditional simple interval  $[1.17, 1.31] \times 10^3$  ha, implying that the fuzzy feature of benefits and water availability could make no difference during the whole decision process and the irrigation targets stood close to the biggest predetermined targets  $1.31 \times 10^3$  ha. The space between the membership and 1-

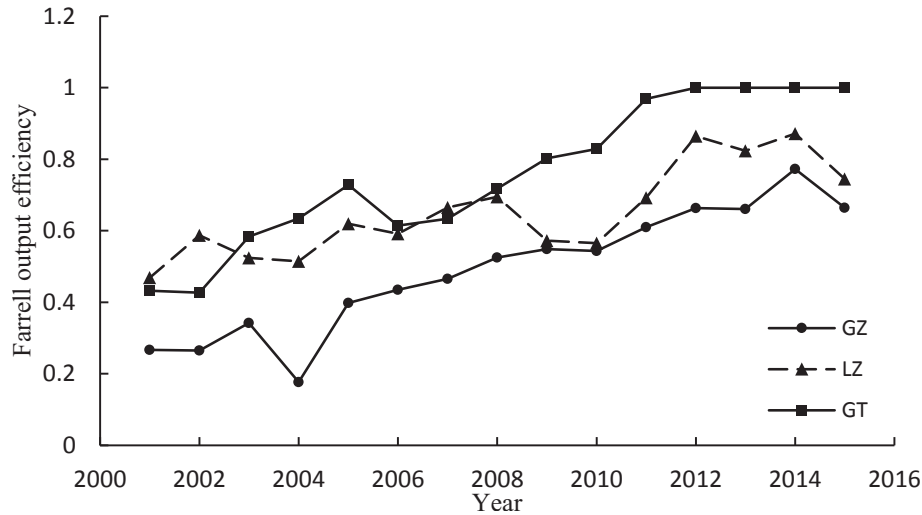


Fig. 5. The trend of Farrell output efficiency during 2001–2015.

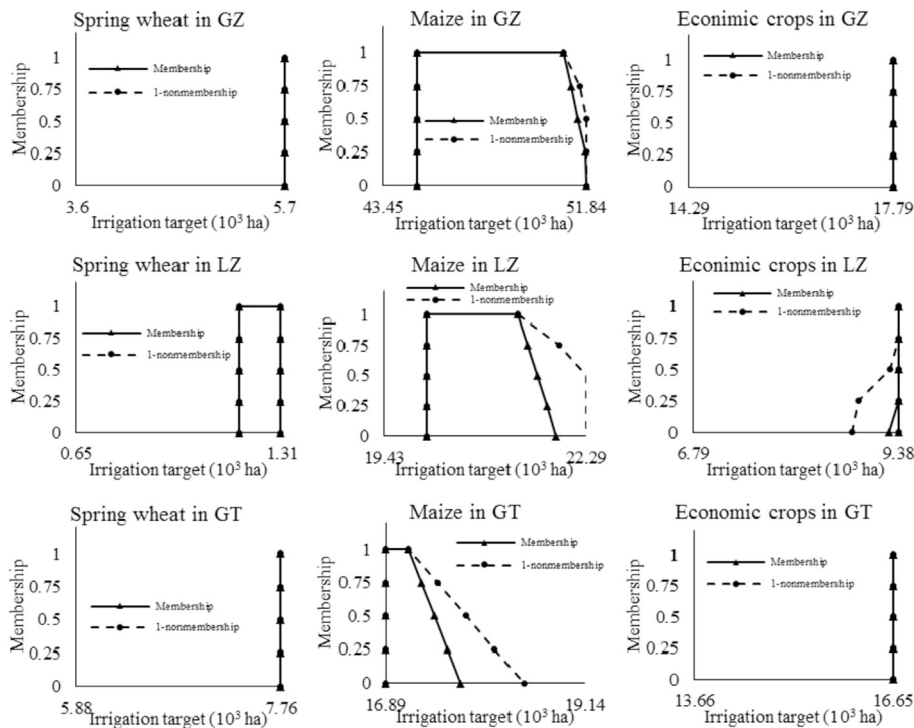


Fig. 6. The modified irrigation targets (first-stage decision) of three crops in subareas.

nonmembership in irrigation targets of maize in three subareas and economic crops in LZ represents the hesitancy of managers. The bigger the space is, the more uncertainties the decisions contain, which means more risk. Therefore, the decision makers should pay more attention towards the crops with high hesitancy.

Figs. 7–9 show the optimal irrigation area of spring wheat, maize and economic crops in three subareas under different hydrological years, respectively (the second-stage decisions). With the increase of available water resources from low to high level year, the irrigation area grows gradually until the maximum modified irrigation targets are achieved. Meanwhile, the interval range and hesitancy space in spring wheat and economic crops in three subareas also decrease with the growth of available water

the membership distribution gradually approaches to the modified targets (the first-stage decisions), which means the uncertainty of the optimal decision drops with the water supply increase. The low level year had biggest uncertainty in spring wheat and economic crops in all subareas among three hydrological years. This circumstance may result from the high targets optimized by the IFITSP and when the water supply is insufficient, the system would suffer from big water scarcity penalty. Particularly in low hydrological year, spring wheat and maize in three areas had a more uncertain upper bound and a stable lower bound, which means the system preferred to guarantee the least irrigation area of spring wheat and maize in three regions. On the contrary, the economic crops were guaranteed a more exact upper bound, indicating the

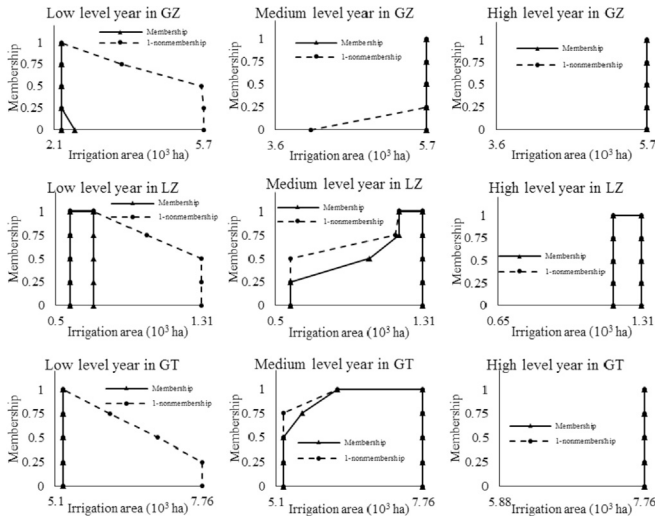


Fig. 7. The optimal irrigation area of spring wheat in three subareas under different hydrological years.

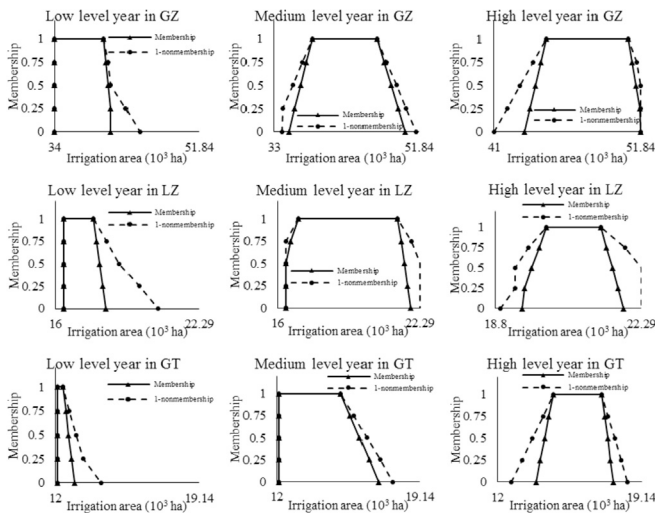


Fig. 8. The optimal irrigation area of maize in three subareas under different hydrological years.

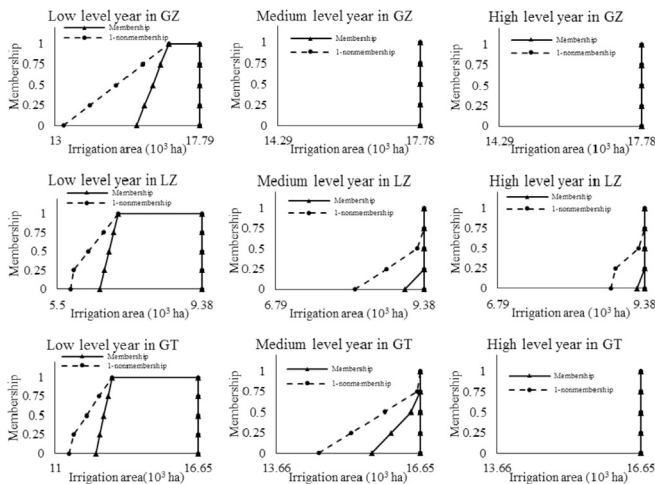


Fig. 9. The optimal irrigation area of economic crops in three subareas under different hydrological years.

economic crops possessed higher profits from unit water consumption. The tradeoff between basic food production and high profits of the economic crops could be inferred from this result feature. The cultivation of crops is significantly motivated by profits on the premise of basic need. When the water resources is limited, the farmers and managers tend to guarantee food demand in advance while when water resources is sufficient, the high profits are pursued. Involving the tradeoff of crop production, the developed model proved to be suitable to the practice.

Therefore, according to the uncertainty (including the interval range and the hesitancy space) of first and second-stage results, the sensitivity of crops planting area could be analyzed. The sensitivity of modified irrigation targets of maize in all three subareas is relatively high among three types of crops, indicating they are likely to be impacted by the uncertain input parameter. Besides, the low level year possessed huge sensitivity among three kinds of hydrological years. Therefore, more attention should be paid to the sensitive crops especially when the water availability is low and the situation is pessimistic.

4.3. Optimal objectives

Fig. 10 reflects the total obtainable benefits considering all the probability of random events. The optimal benefits can be expressed as an IFS. The space between membership and 1-nonmembership lines denotes the hesitancy of the IFS. With the increase of membership, the hesitancy of objective decreases. When membership equals 0, the objective value is  $[[6200, 7416], [9627, 10906]]$  million Yuan with the biggest hesitancy. This means after taking the negative factor into account with biggest fuzzy uncertainty, the system can obtain at least 6200 million Yuan benefits, which can be regarded as the conservative decision from pessimistic decision makers. By contrast, the objective of 7416 million Yuan is the conservative decision from optimistic decision makers. The upper bound is similar. The IFS objective results can provide a more professional reference involving the influence of different attitudes from various decision makers. This reference can provoke thinking of managers with subjective preference and contribute to rational decision.

Fig. 11 demonstrates the relationship between water deficiency penalty and water residual penalty. Note that the lower and upper bounds do not mean the real bounds of water deficiency and residual penalty values but the lower-bound and upper-bound submodels when solving IFITSP. The lower-bound submodel

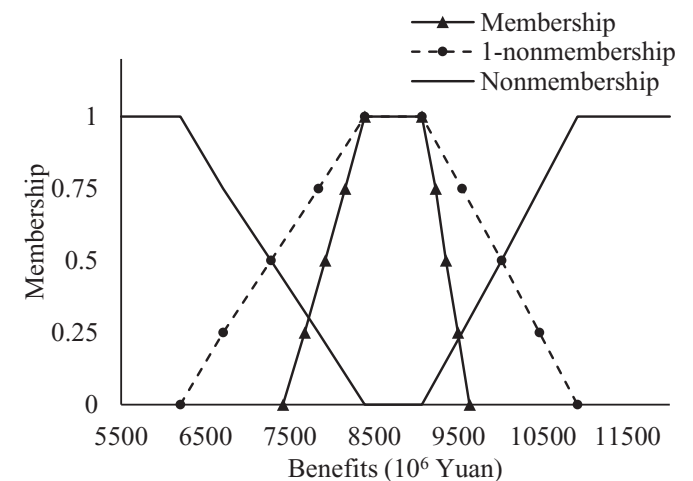


Fig. 10. The optimal obtainable benefits from IFITSP.

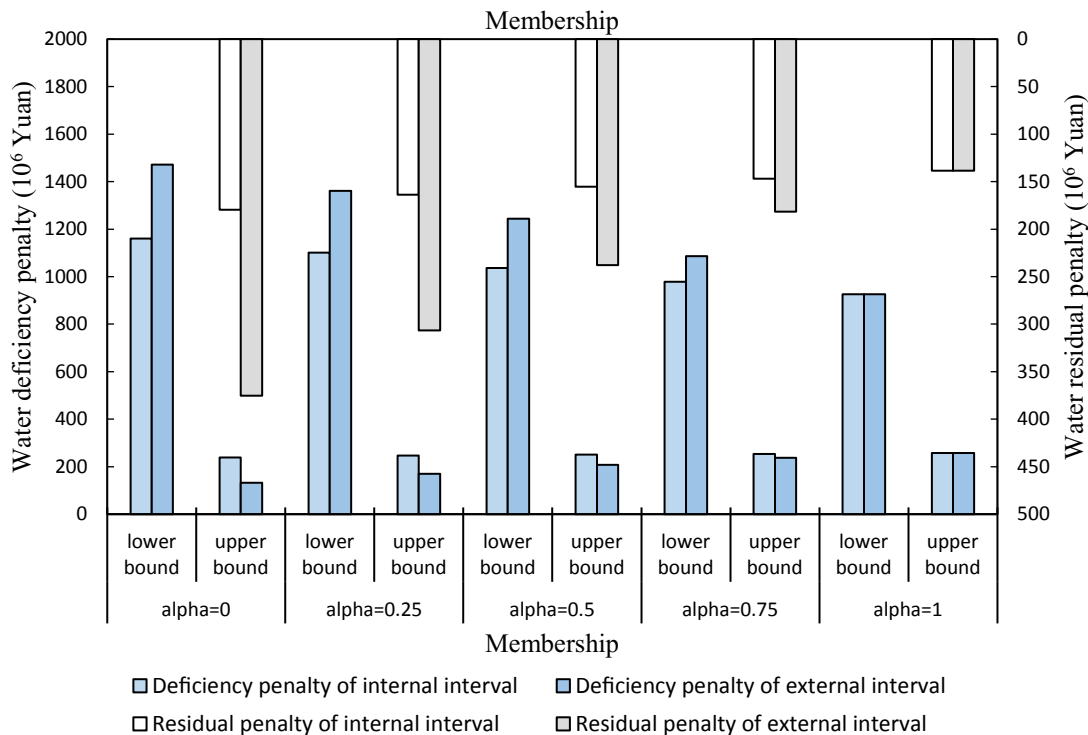


Fig. 11. The water deficiency penalty and its corresponding water residual penalty.

corresponds to the lower-bound of benefits objective and available water resources, thus creating relatively high water deficiency and none water residual. The upper-bound submodel is associated with the upper-bound water availability and produces the upper-bound benefits. With  $\alpha$  increasing from 0 to 1, the disposable water resources of lower-bound model grows and the water scarcity decreases with the corresponding water surplus penalty being zero. With  $\alpha$  increasing from 0 to 1, the hesitancy of IFS decreases thus the results from internal and external submodels approach to be the same. Regarding upper-bound submodel, there are more available water and water residual dominates the penalty. Note that the water deficiency and residual penalty coexist in the upper-bound submodel solutions. The reason may lie in the influence of random events. As the random events are unknown when the irrigation targets are made, the tradeoff between low and high hydrological levels must be considered. In order to guarantee the overall interests, there are water deficiency in low level year and water surplus in high level year. The tradeoff does not exist in lower-bound model due to the serious water deficiency even in high level years.

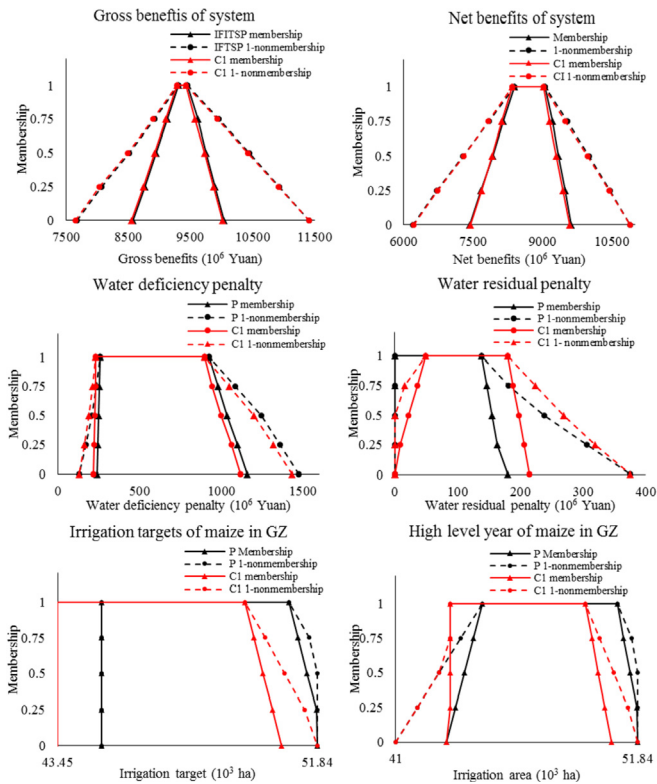
4.4. Model comparisons

As we have mentioned before, the previous studies on TSP of resources planning have some common shortages including ignoring resources surplus, rarely combined with IFS, and lack of robust solving approach. Therefore, model physical meaning and mathematical programming method including uncertainty representation and solving method were separately considered when comparing the IFITSP with other previous approaches.

As mentioned in Section 1, the recourse penalty caused by resources scarcity has been popular in previous research while the wasting of precious resources has not drawn enough attention. Therefore, as a comparison of model physical meaning, an IFITSP focusing on penalty of water deficiency was adopted without the

model component of water waste penalty in this study. The same parameters and solving method were applied to the IFITSP focusing on water deficiency. For convenience, the IFITSP considering the water wasting used in this paper is called P and the IFITSP without water waste for comparison is called C1. The gross benefits denoted the total income from crop production by irrigation without any penalty while the net benefits reflecting the total income minus the water deficiency and surplus penalty.

From Fig. 12, the gross benefits and net benefits had little difference between P and C1. However, the penalty from water deficiency and residual was different, which means P and C1 gave different irrigation targets and irrigation area under different hydrological years. According to Fig. 12, the water deficiency penalty of P is larger than that from C1 while the membership distribution shared similar shape, which denoting the uncertainty of model inputs had similar influence on model P and C1. Then we can infer that the uncertain degree of water deficiency was independent with whether the water surplus penalty was considered or not. However, it would influence the uncertainty of water residual penalty. From Fig. 12, the water waste caused by C1 was bigger than that by P except for the external interval when  $\alpha = 0$ . It can be easily understood that when the water availability is poor, the results of irrigation area from P and C1 will be the same as the water residual of C1 equals zero as well as P shown in Fig. 11. However, when the water supply is adequate, the water waste penalty will have a great influence on model performance. The modified irrigation targets of maize in GZ and irrigation area in high level year of maize in GZ are demonstrated in Fig. 12 as examples for detailed decision strategy. The modified irrigation targets of maize in GT tend to be smaller for C1 as model C1 focused on the water scarcity penalty and in order to reduce the water deficiency penalty in low hydrological year, small irrigation targets were suggested. However, over-concern of water shortage in low level year would aggravate water wasting in high level year, which is detrimental to conservation-oriented agricultural development.

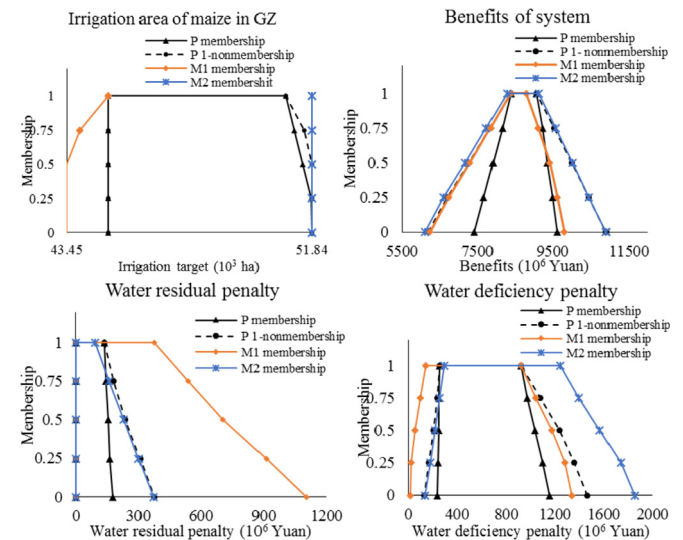


**Fig. 12.** The results of water deficiency penalty, water waste penalty, the modified irrigation targets of maize in GZ, and irrigation area in high level year of maize in GZ.

The other comparison was conducted aimed at uncertain programming approach. The IFITSP proposed in this paper was compared with conventional ITSP formulated as model (4) which was solved by two different methods. One solving method (M1) was similar as the developed method in this study, namely that the worst-case submodel (lower-bound submodel particularly for the case study) was solved first followed by the best-case submodel, but a certain first-stage decision variable was generated only from the worst-case submodel, which is similar with the robust stepwise interactive algorithm (RSIA) (Fan et al., 2015; Fan and Huang, 2012). The other one (M2) was the opposite, i.e. the best-case submodel (upper-bound submodel) was solved first and the first-stage decision was passed to the worst-case submodel, which is consistent with the widely used method (Huang and Loucks, 2000; Li et al., 2010). Note that the first-stage decision is a certain value from model (4) by above two solving methods. The external interval of IFS used in model (7) was applied into ITSP. The results comparison of IFITSP and ITSP by two solving approaches can be exhibited in Fig. 13.

The irrigation targets of maize in GT were taken as examples for comparison. For the worst-case submodel was solved first, the irrigation targets derived from M1 was relatively conservative while these from M2 was optimistic, which can be seen in Fig. 13. The results of P were between the targets from M1 and M2, covering the possible strategies from pessimistic to optimistic decision makers. The benefits of system, water deficiency penalty and residual penalty demonstrate the similar trend. Besides, considering the hesitancy of membership from traditional FS, the results of IFITSP are more abundant and suitable for multilateral decision.

Therefore, following advantages can be drawn up according to the comparison with previous studies. 1) Although the gross and net benefits had few differences whether the surplus penalty of



**Fig. 13.** The compared results of irrigation targets of maize in GZ, benefits, water deficiency penalty and water waste penalty from IFITSP and ITSP by two solving methods.

water resources was involved or not, the water surplus penalty would be remarkable decreased when the surplus penalty was considered by adjusting the crop planting structure. Thus the developed planning thinking could help to reducing the resources surplus risk. 2) The proposed IFITSP combined IFS and traditional ITSP, considering the ambiguity of membership in FS. The obtained results can reflect the hesitancy of decision makers towards the membership of input IFSs. 3) The established solving method can not only solving the IFITSP, but improve the traditional solving approach of ITSP. The improved solving approach of ITSP can mitigate the subjective preferences between optimism and pessimism for the first-stage decision.

### 5. Conclusions

Aimed at popular TSP for resources planning, this study was conducted to improve model performance from two aspects: model components and programming approach. In order to define a clear recourse penalty, the technical inefficiency from production frontier estimation was introduced. Meanwhile, instead of focusing on the penalty caused by resources scarcity, the penalty of resources surplus was emphasized alongside with the scarcity and shadow price of resources was evaluated to represent the resources residual value. Then IFS theory was applied to characterize the uncertainty of fuzzy membership and an IFITSP was established for resources planning. In order to solve the complex uncertain programming, an integrated solving approach was proposed based on the concept of IFS and HFS, the  $\alpha$ -cut method of FS, robust stepwise interactive algorithm, dual-interval programming and robust two-step method. Besides, an improved solving approach was generated regarding the traditional ITSP to generate a better first-stage decision modification to offer decision makers a more flexible choice.

The IFITSP was applied in a real case study in northwest China to schedule agricultural cultivation scale based on limited water resources. The results show that: 1) the modified irrigation targets of maize in three subareas possessed highest uncertainty among three kinds of crops, 2) the irrigation area of low hydrological year was most uncertain especially for spring wheat and economic crops among three level years, which should draw managers' attention to the likely risk associated with subjective preference, 3) the lower

bound of spring wheat and maize was guaranteed due to their function of basic survival while the upper bound of economic crops was preferred due to the profit pursue of decision makers. The results are capable of offering diverse decision references to decision makers with different subjective preference and dealing with the tradeoff between basic food production and economic benefits as well as among different hydrological years. The generated model framework in this paper is particularly suitable for decision maker group with great diversity.

Aimed at resources planning problem based on TSP, the developed IFITSP in this study has three advantages. 1) Based on production frontier and shadow price, the recourse penalty caused by resources deficiency and surplus was quantified scientifically, clarifying the physical meaning of recourse and reducing the influence of subjective judgment. 2) From the perspective of subjective influence during decision making process, the IFS was introduced to express the uncertainty in fuzzy membership and an IFITSP was exhibited to improve resources planning integrating the IFS theory and the traditional ITSP. 3) An integrated solving approach was proposed coupling the previous uncertain programming methods and a modified robust ITSP solving method. The decision strategies possess high diversity and robustness, providing a more flexible and reliable decision references for managers. However, there exist some limits about the proposed method. The production frontier estimation requires a relatively big data basis. It does not apply to the area where the statistical data are scarce. Moreover, the discreteness of random sets may lead to the loss of distributional information while it is the most convenient way. The combination of more advanced TSP method and the developed IFITSP structure is desired for further exploration.

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