Journal of Hydrology 564 (2018) 467–475

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Research papers

Double-sided stochastic chance-constrained linear fractional programming model for managing irrigation water under uncertainty

Chenglong Zhang[a], Bernard A. Engel[b], Ping Guo[a,*], Xiao Liu[a], Shanshan Guo[a], Fan Zhang[a], Youzhi Wang[a]

[a] Center for Agricultural Water Research in China, College of Water Resources and Civil Engineering, China Agricultural University, Beijing 100083, China
[b] Department of Agricultural and Biological Engineering, Purdue University, West Lafayette, IN 47907, USA

ARTICLE INFO

This manuscript was handled by Marco Borga, Editor-in-Chief, with the assistance of Baptiste François, Associate Editor

Keywords:
Irrigation management
Linear fractional programming
Chance-constrained programming
Double-sided randomness
Water productivity

ABSTRACT

A double-sided stochastic chance-constrained linear fractional programming (DSCLFP) model is developed for managing irrigation water under uncertainty. The model is developed by incorporating double-sided stochastic chance-constrained programming (DSCCP) into a linear fractional programming (LFP) optimization framework. It can address ratio optimization problems with double-sided randomness (i.e. both left-hand and right-hand sides). More importantly, it also improves upon the existing stochastic chance-constrained programming for handing random uncertainties in the left-hand and right-hand sides of constraints simultaneously. A non-equivalent but sufficient linearization form of the DSCLFP is provided and proved, which will greatly reduce the computational burden. Then, the model is applied to a case study in Yingke Irrigation District (YID) in the middle reaches of the Heihe River Basin, northwest China. Four confidence levels (e.g. α_i = 0.85, 0.90, 0.95 and 0.99) are provided to examine and compare the results. The objective function values are slightly decreased from 5.284 Yuan/m^3 to 5.276 Yuan/m^3 when α_i level is raised from 0.85 to 0.99. The results from the DSCLFP can identify desired irrigation water allocation plans under the objective function of maximizing water productivity under different confidence levels. Therefore, the results can provide tradeoffs among water productivity, confidence level and constraint-violation risk level. Moreover, comparisons with double-sided stochastic chance-constrained linear programming (DSCLP) model and deterministic model are introduced to highlight advantages and feasibility of the developed model. Therefore, these results can provide decision-support for managers in arid areas.

1. Introduction

Under the pressures of the increasing water demands and the shortages of water supply, sustainable water management is becoming an issue of great significance for water managers throughout the world, especially in arid areas dominated by irrigated agriculture (Elliott et al., 2014; Kang et al., 2017). There is a concern that the water productivity need to be addressed in irrigation water management problems due to water scarcity. In other words, this is a conflicting objective to maximize the system benefits with minimum irrigation water use. Generally, water productivity is defined as a ratio representing the unit of outputs (e.g. crop yield, economic benefits) per unit of irrigation water (Barker et al., 2003). Such a ratio optimization problem can be quantitatively solved by linear fractional programming (LFP) method, which is effectively used to account for conflicting objectives and reflect system efficiency (Lara and Stancu-Minasian, 1999; Gómez et al., 2006; Zhu and Huang, 2011; Stancu-Minasian, 2012; Zhang and Guo, 2018). Compared with traditional methods, the LFP is superior to them to compare multiple objective directly through the original magnitudes (Guo et al., 2014). Particularly, it's adopted in the management problems that need to compare two magnitudes (e.g. output/input).

Moreover, another concern is the inherent uncertainty in practical applications. For example, spatial and temporal changes in surface runoff and groundwater, fluctuations of market prices effected by various stochastic factors (Li et al., 2010), which are closely related to input parameters and hardly quantified accurately. Additionally, the interrelationships among these uncertain factors and economic implications may cause challenges in planning irrigation water management due to system complexities (Tan et al., 2011; Li et al., 2012). Therefore, it is imperative to develop novel method to deal with these

* Corresponding author at: Center for Agricultural Water Research in China, College of Water Resources and Civil Engineering, China Agricultural University, Tsinghuadong Street No. 17, Beijing 100083, China.
E-mail addresses: engelb@purdue.edu (B.A. Engel), guop@cau.edu.cn (P. Guo).

https://doi.org/10.1016/j.jhydrol.2018.07.024
Received 1 May 2018; Received in revised form 31 May 2018; Accepted 11 July 2018
Available online 11 July 2018

concerns and generate irrigation water allocation plans for supporting sustainable irrigation water management.

Previously, three main types of inexact mathematical programming methods were proposed including interval-parameter linear programming (ILP), fuzzy mathematical programming (FMP) and stochastic mathematical programming (SMP) to address uncertainties. Among them, SMP method is exclusively for handling random variables expressed as known probability density functions (PDFs) in the model's input information (Gu et al., 2013; Guo et al., 2014; Zhang et al., 2017). This means that when uncertain parameters can be described, results can be presented as comparisons under different reliability levels. As a major type of SMP, chance-constrained programming (CCP) initially developed by Charnes and Cooper (1959) allows violation of system constraints, indicating that not all of the constraints must be rigorously satisfied. Thus, the random constraints can be hold at least a certain probability α, where $\alpha \in [0, 1]$ is defined as confidence level given by the decision-makers. This will normally increase the system benefits to a certain extent at a compromise of environmental capacity in irrigation water management problems. Generally, due to the nonlinear forms in solving CCP problems, it's commonly employed in the case of the independent right-hand side randomness in constraints (Huang, 1998; Zhu and Huang, 2011; Nemirovski, 2012). For example, Huang (1998) proposed an inexact CCP method for water quality management where stochastic uncertainties exist in the right-hand side of constraints. Zhu and Huang (2011) developed a stochastic linear fractional programming for solid waste management where the right-hand side coefficients are random for all α values. Moreover, the CCP methods incorporated with the randomness in the left-hand side of the constraints have also been developed. For example, Cao et al (2011) developed an interval left-hand-side CCP method for regional air quality management. Sun et al (2013) developed an inexact joint-probabilistic left-hand-side CCP model for solid waste management. Zhang et al (2017) proposed an interval multistage joint-probabilistic CCP model with left-hand-side randomness for crop area planning. However, in practice, random uncertainties may exist in both left-hand side and right-hand side of the constraints, which will thereby lead to the above-mentioned CCP methods not account for such a difficulty. Therefore, based on the assumption of both left-hand and right-hand sides of randomness have Gaussian distributions and $\alpha \geq 0.5$ (α is the satisfaction degree of constraints) (Roubens and Teghem, 1991; Huang, 1998), a double-sided stochastic chance-constrained programming (DSCCP) method is useful for solving above difficulties (Liu, 2009). Although the algorithm of the DSCCP can be directly proved by mathematical proof, its nonlinear forms will inevitably intensify challenges in practical applicability. Furthermore, the DSCCP method can hardly account for ratio optimization problems and few scholars handled double-sided random/stochastic issues with LFP models. Nevertheless, few applications of the DSCCP method to irrigation water management are conducted.

Therefore, the objective of this study is to develop a novel method to address ratio optimization problems in the case of double-sided random/stochastic uncertainties in the constraints. It is an attempt to develop such a double-sided stochastic chance-constrained linear fractional programming (DSCLFP) model by incorporating the DSCCP into a LFP framework for supporting irrigation water management under uncertainty. It can improve upon the existing stochastic chance-constrained programing by dealing with double-sided randomness. Moreover, to reduce computational burden and improve computational efficiency, a non-equivalent but sufficient linearization form of DSCLFP model will be provided and proved for comparisons. To demonstrate its applicability, the developed model will be applied to a case study to manage irrigation water to different crops in the Yingke Irrigation District (YID) in the middle reaches of the Heihe River Basin, northwest China. Several scenarios associated with confidence levels will be analyzed for examining and comparing the variations of results.

2. Methodology

2.1. Linear fractional programming (LFP)

In a linear fractional programming (LFP) problem, it is generally considered as a functional relationship expressed as a ratio between two variables in the numerator and denominator (Charnes and Cooper, 1962). The numerator represents the change in total cost and other variables while the denominator represents the volume changes that may lead to the change of the related cost coefficients. Therefore, LFP can deal with bi-objectives problems and reflect system efficiency. It can be formulated as follows:

$$\max f = \frac{CX + \alpha}{DX + \beta} \tag{1a}$$

subject to:

$$AX \leqslant B \tag{1b}$$

$$X \geqslant 0 \tag{1c}$$

where A is a real $m \times n$ matrix; X and B are column vectors with n and m components respectively; C and D are row vectors with n components; α and β are constant terms. The LFP model can address deterministic ratio-optimization problems, but it is not able to deal with complexities when uncertainties exist in the constraints of optimization model (Zhu and Huang, 2011).

2.2. Double-sided stochastic chance-constrained programming (DSCCP)

In a linear programming model, when both left-hand (a_{ij}) and right-hand side (b_j) parameters in the constraints are to be independently normally distributed random variables (μ is expected value and σ is standard variation), and the constraints are hold at a certain level of probability α. Then, the double-sided stochastic chance-constrained programming (DSCCP) method can be adopted as follows:

$$\max f = \sum_{j=1}^{n} c_j x_j \tag{2a}$$

subject to:

$$\Pr\left\{\sum_{j=1}^{n} a_{ij} x_j \leqslant b_i\right\} \geqslant \alpha_i,\ i = 1, 2, \cdots, m \tag{2b}$$

$$a_{ij}(\omega) \sim N(\mu_{a,ij}, \sigma_{a,ij}^2) \tag{2c}$$

$$b_i(\xi) \sim N(\mu_{b,i}, \sigma_{b,i}^2) \tag{2d}$$

$$x_j \geqslant 0, j = 1, 2, \cdots, n \tag{2e}$$

where f is the objective function; x_j is decision variable; b_i and c_j are input parameters; α_i ($\alpha_i \in [0, 1]$) refers to a given level of probability for constraint i, representing the satisfaction degree level of constraints; m is the number of constraints. According to Liu (2009), constraint (2b) can be transformed into a nonlinear form and the set of feasible constraints is convex (see Theorem 1):

Theorem 1. Eq. (2b) is equivalent to Eq. (3a). That is, if a_{ij} and b_i are assumed to be independently normally distributed random variables, then Eq. (2b) can be hold if and only if

$$\sum_{j=1}^{n} \mu_{a,ij} x_j + \Phi^{-1}(\alpha_i)\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2} \leqslant \mu_{b,i},\ i = 1, 2, \cdots, m \tag{3a}$$

where Φ is the standardized normal distribution function.

Proof. Since a_{ij} and b_j are independently random variables following normal distributions, let's introduce the variable: $y_i = \sum_{j=1}^{n} a_{ij} x_j - b_i$, $i = 1, 2, \cdots, m$, it also follows normal distribution. Its

expected value and variance are presented as follows:

$$\mu_{y,i} = \sum_{j=1}^{n} \mu_{a,ij}x_j - \mu_{b,i} \tag{3b}$$

$$\sigma_{y,i}^2 = \sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2 \tag{3c}$$

Then, the expression: $\frac{\sum_{j=1}^{n} a_{ij}x_j - b_i - \left(\sum_{j=1}^{n} \mu_{a,ij}x_j - \mu_{b,i}\right)}{\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2}}$ must be standardized normally distributed.

Accordingly, the associated inequality $\sum_{j=1}^{n} a_{ij}x_j \leqslant b_i$, $i = 1, 2, \cdots, m$ is equivalent to:

$$\frac{\sum_{j=1}^{n} a_{ij}x_j - b_i - \left(\sum_{j=1}^{n} \mu_{a,ij}x_j - \mu_{b,i}\right)}{\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2}} \leqslant -\frac{\left(\sum_{j=1}^{n} \mu_{a,ij}x_j - \mu_{b,i}\right)}{\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2}} \tag{3d}$$

Thus, the chance constraint (2b) can be converted into:

$$\Pr\left\{ z \leqslant -\frac{\left(\sum_{j=1}^{n} \mu_{a,ij}x_j - \mu_{b,i}\right)}{\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2}} \right\} \geqslant \alpha_i,\ i = 1, 2, \cdots, m \tag{3e}$$

where z is defined as the standardized normally distributed random variable. Therefore, the chance constraint (3e) can be satisfied if and only if

$$\Phi^{-1}(\alpha_i) \leqslant -\frac{\left(\sum_{j=1}^{n} \mu_{a,ij}x_j - \mu_{b,i}\right)}{\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2}},\ i = 1, 2, \cdots, m \tag{3f}$$

Finally, we have the deterministic equivalent of chance constraint (3a). The Theorem 1 is proved.

Because Eq. (3a) is presented as nonlinear form, it will intensify computational burden when solving problems. As an alternative solution method, an approximated linearization form of Eq. (3a) is proposed (see Theorem 2).

Theorem 2. Eq. (4a) is a sufficient condition for Eq. (3a) where $\alpha_i \geqslant 0.5$ and $x_j \geqslant 0$.

$$\sum_{j=1}^{n} \mu_{a,ij}x_j + \Phi^{-1}(\alpha_i)\left(\sum_{j=1}^{n} \sigma_{a,ij}x_j + \sigma_{b,i}\right) \leqslant \mu_{b,i},\ i = 1, 2, \cdots, m \tag{4a}$$

Proof. Based on the following inequality,

$$\sqrt{\sum_{j=1}^{n} (t)^2} \leqslant \sum_{j=1}^{n} (t),\ t \in R,\ t \geqslant 0 \tag{4b}$$

when $\left(\sum_{j=1}^{n} \sigma_{a,ij}x_j + \sigma_{b,i}\right) \in R$ and $\left(\sum_{j=1}^{n} \sigma_{a,ij}x_j + \sigma_{b,i}\right) \geqslant 0$, we have

$$\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2} \leqslant \left(\sum_{j=1}^{n} \sigma_{a,ij}x_j + \sigma_{b,i}\right) \tag{4c}$$

Moreover, when $\alpha_i \geqslant 0.5$, we have

$$\Phi^{-1}(\alpha_i) \geqslant 0 \tag{4d}$$

Thus, from inequalities (4c) and (4d), we have

$$\sum_{j=1}^{n} \mu_{a,ij}x_j + \Phi^{-1}(\alpha_i)\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2} \leqslant \sum_{j=1}^{n} \mu_{a,ij}x_j + \Phi^{-1}(\alpha_i)\left(\sum_{j=1}^{n} \sigma_{a,ij}x_j + \sigma_{b,i}\right),\ i = 1, 2, \cdots, m \tag{4e}$$

Understandably, if inequality (4a) holds (sufficient condition), then from inequality (4e), we have (3a).

Therefore, according to Theorems 1 and 2, when $\alpha_i \geqslant 0.5$ and $x_j \geqslant 0$, Eq. (2b) can be transformed into a non-equivalent but sufficient linearization form in (5a).

$$\sum_{j=1}^{n} \mu_{a,ij}x_j + \Phi^{-1}(\alpha_i)\left(\sum_{j=1}^{n} \sigma_{a,ij}x_j + \sigma_{b,i}\right) \leqslant \mu_{b,i},\ i = 1, 2, \cdots, m \tag{5a}$$

2.3. Double-sided stochastic chance-constrained linear fractional programming (DSCLFP)

In this study, a DSCLFP model is developed in response to ratio optimization problems with double-sided randomness in the constraints. It incorporates DSCCP model into LFP optimization framework. The developed DSCLFP model is written as follows:

$$\max\ f = \frac{CX + \alpha}{DX + \beta} \tag{6a}$$

subject to:

$$\sum_{j=1}^{n} \mu_{a,ij}x_j + \Phi^{-1}(\alpha_i)\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2} \leqslant \mu_{b,i},\ i = 1, 2, \cdots, m \tag{6b}$$

$$a_{ij}(\omega) \sim N(\mu_{a,ij}, \sigma_{a,ij}^2) \tag{6c}$$

$$b_i(\xi) \sim N(\mu_{b,i}, \sigma_{b,i}^2) \tag{6d}$$

$$x_j \geqslant 0,\ j = 1, 2, \cdots, n \tag{6e}$$

Alternatively, by using inequality (5a) substituting constraint (6b), we have a linearization form of DSCLFP model.

$$\max f = \frac{CX + \alpha}{DX + \beta} \tag{7a}$$

subject to:

$$\sum_{j=1}^{n} \mu_{a,ij}x_j + \Phi^{-1}(\alpha_i)\left(\sum_{j=1}^{n} \sigma_{a,ij}x_j + \sigma_{b,i}\right) \leqslant \mu_{b,i},\ i = 1, 2, \cdots, m \tag{7b}$$

$$a_{ij}(\omega) \sim N(\mu_{a,ij}, \sigma_{a,ij}^2) \tag{7c}$$

$$b_i(\xi) \sim N(\mu_{b,i}, \sigma_{b,i}^2) \tag{7d}$$

$$x_j \geqslant 0,\ j = 1, 2, \cdots, n \tag{7e}$$

2.4. Solution method

The framework of the developed DSCLFP model is graphically presented in Fig. 1. The detailed solution method can be further summarized as follows:

Step 1: Formulate the DSCLFP model.

Step 2: Acquire the model input parameters including deterministic values and independently normally distributed random variables (probabilistic distributions).

Step 3: Convert stochastic chance constraints into deterministic ones through the DSCCP method by giving a certain confidence level (α_i) for each constraint i.

Step 4: Reformulate the deterministic DSCLFP model.

Step 5: Solve the deterministic model and obtain solutions.

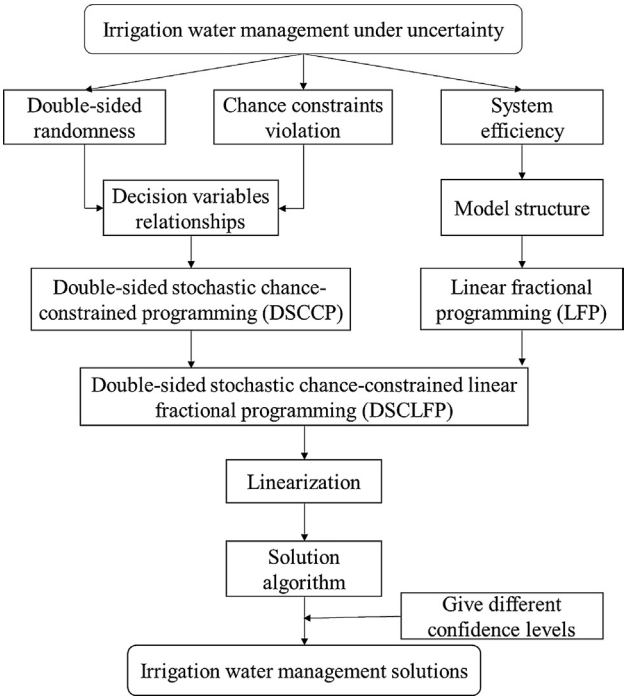

Fig. 1. The framework of study.

Step 6: Repeat steps 3–5 under different confidence levels and obtain final decision solutions.

3. Case study

3.1. Study area

The Yingke Irrigation District (YID) is one of the three major irrigation areas in the middle reaches of the Heihe River Basin. It's located in Zhangye City, Gansu Province, northwest China (100°17′–100°34′ E, 38°50′–38°58′ N) (Fig. 2). It's a highly developed agricultural zone with a typical inland arid climate where 68% of the overall area (13147 ha) are arable lands that particularly need to be irrigated. The mean annual temperature is 7.0 °C. The annual sunshine hours are over 3000 h and the frost-free period is around 140 days. The mean annual precipitation is merely 125 mm and annual ET is about 1200 mm. However, over 80% of the precipitation is concentrated between July and September. In the YID, the main types of crops are maize (e.g. field maize and seed maize), spring wheat and economic crops because of their overwhelmingly larger proportion of the total planting area (Jiang et al., 2015). Meanwhile, economic crops mainly refers to vegetables in the study area. The growth period of spring wheat is from April to July, and the growth period of maize is from April to September. Soil texture is dominated by sandy loam and loam. Agricultural water consumptions including surface water and groundwater, and surface water is mostly used for agricultural irrigation (i.e. accounting for more than 90%). Moreover, traditional irrigation patterns such as flood and furrow methods are commonly adopted, which is a cost-effective way but with a lower irrigation water efficiency. Groundwater pumping can compensate insufficient part of surface water due to seasonal variations and untimely events. Agricultural irrigation water is physically transported to field crops through a multi-layered canal system from water sources, including one main canal, and eleven sub-canals. Among them, all the main canal, nearly 97% of secondary canals and 60% of tertiary canals have been lined (Jiang et al., 2016). Therefore, in the YID, the irrigation water use coefficients of surface water and groundwater are 0.52, 0.60, respectively.

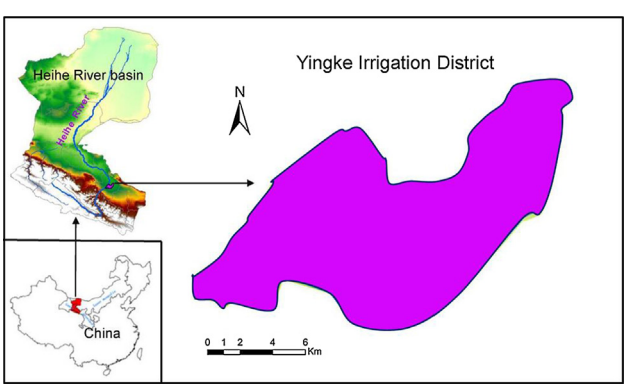

Fig. 2. The study area.

3.2. Problem statement

A manager is responsible for allocating limited irrigation water resources to four types of crops. Due to irrigation water shortages and little precipitation in arid areas, there is a growing competition among different water users (crops). Moreover, crop water production function (CWPF) is selected as the basis of irrigation planning because it can describe mathematical relationships between crop production (i.e. crop yield or dry biomass) and water use (i.e. evapotranspiration). CWPFs for different crops are generally expressed as polynomial function, and linear CWPFs are selected because their simple form can facilitate the further promotion of the study model. Table 1 presents the linear CWPFs for the study crops. These CWPFs are obtained by fitting the experimental data of crop yields and actual evapotranspiration. Table 2 presents the basic input data of the study area and crops, including crop planting area, crop price, and cost of per unit of irrigation surface water and groundwater, effective precipitation and maximum evapotranspiration. These related parameters are acquired from government reports and statistical data.

In fact, there are some uncertain factors in agricultural systems. For example, surface water and groundwater availabilities, and the rates of surface water and groundwater loss during conveyance usually show randomness. In response to these existing problems, rational assumptions and simplifications of input parameters are needed to analyze the management system and tackle uncertainties. Thus, it is assumed that there are sufficient historical records for determining the random distributions, namely, normal distributions (see Figs. 3 and 4). Specifically, the surface water and groundwater availabilities are $N(10195, 1200^2) \times 10^4\,m^3$ and $N(4300, 1000^2) \times 10^4\,m^3$. The rates of surface water and groundwater loss during conveyance are $N(0.25, 0.0167^2)$ and $N(0.15, 0.0167^2)$. Therefore, an optimization model integrating linear CWPFs is developed for effective managing irrigation water to allocate them to different crops under uncertainty.

3.3. Application of the DSCLFP model

In this study, the difficulties of optimizing irrigation water allocation include: (1) how to deal with double-sided random uncertainties in the system; (2) how to address system efficiency (i.e. water

Table 1
Linear crop water production functions.

Crop	Linear CWPFs
Field maize	$Y = 0.7243ET + 6746.6$
Seed maize	$Y = 0.5216ET + 6658.5$
Spring wheat	$Y = 0.6061ET + 3688.7$
Economic crops	$Y = 0.4875ET + 1308.5$

Note: Y is crop yield (kg/ha); ET is actual evapotranspiration (m^3/ha).

Table 2
Basic related input parameters.

Crop	A_j(ha)	NB_j(Yuan/kg)	P_e(m^3/ha)	$ET_{max,j}$(m^3/ha)	CS_j(Yuan/m^3)	CG_j(Yuan/m^3)	CP_j(Yuan/m^3)	SC_j(Yuan/ha)
Field maize	2230	2.42	903.50	7818	0.16	0.40	0.45	300
Seed maize	4554	5.02		7856	0.16	0.40	0.60	300
Spring wheat	2886	2.04		5842	0.16	0.40	0.45	300
Economic crops	2876	3.58		5030	0.16	0.40	0.60	0

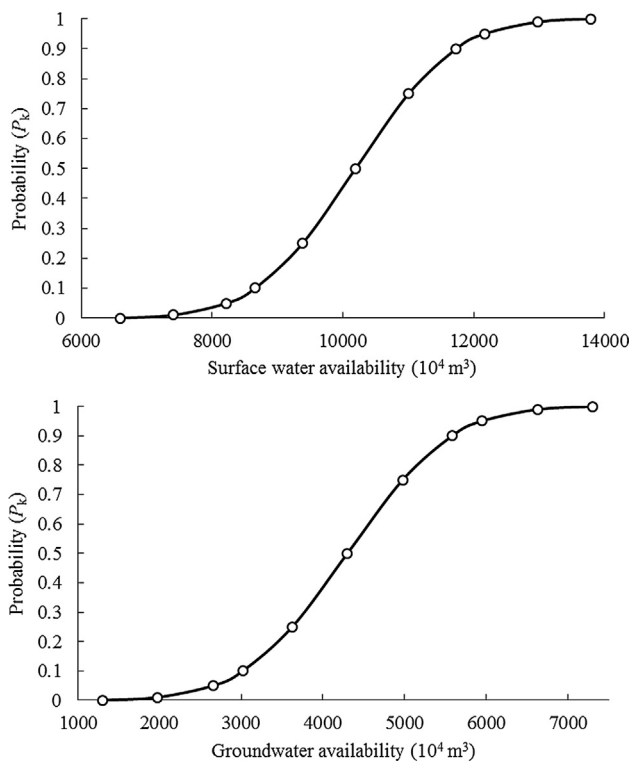

Fig. 3. Cumulative distribution of the surface water and groundwater availabilities (10^4 m^3).

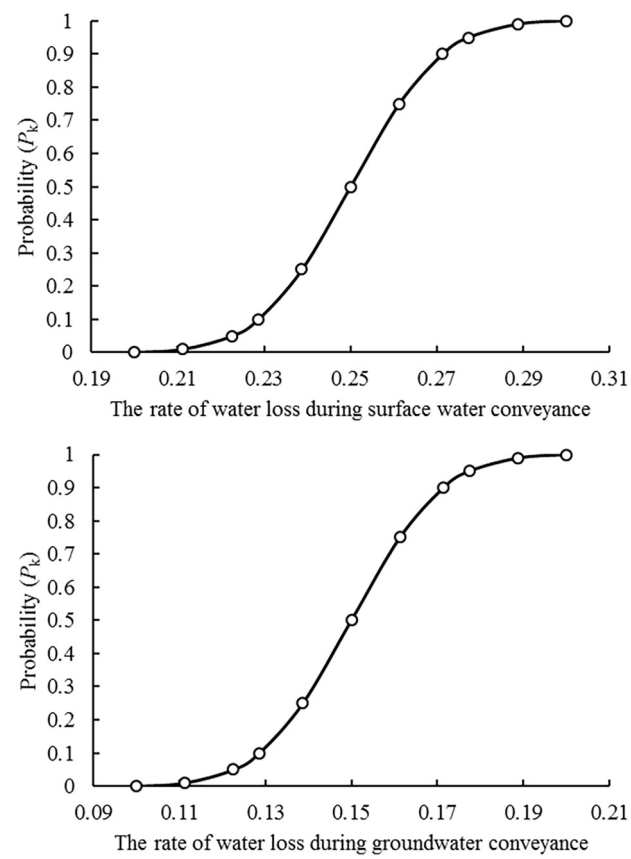

Fig. 4. Cumulative distribution of the rates of water loss during surface water and groundwater conveyance.

productivity); (3) how to allocate irrigation water to different crops to achieve maximum system efficiency and (4) how to identify optimal irrigation water allocation solutions under given confidence levels. Therefore, a double-sided stochastic chance-constrained linear fractional programming (DSCLFP) model is developed under uncertainty. The objective is to obtain maximum system benefits per unit of the allocated irrigation water. Meanwhile, a series of constraints should be provided by involving all the relationships between the decision variables and system conditions. The DSCLFP model is formulated as follows:

Objective function:

$$\max f = \frac{\text{Net system benefits}}{\text{Total irrigation water amount}} = \frac{\sum_{j=1}^{4} A_j((NB_j - CP_j)[a_j(SW_j + GW_j + P_e) + b_j] + SC_j) - \sum_{j=1}^{4} A_j(CS_j SW_j/\eta_s + CG_j GW_j/\eta_g)}{\sum_{j=1}^{4} A_j(SW_j/\eta_s + GW_j/\eta_g)} \tag{8a}$$

Net system benefits represent the total system benefits minus costs of crop productions, irrigated surface water and groundwater use. Total irrigation water amount represents the irrigated surface water and groundwater use through irrigation water allocation solutions. There is an assumption that the amount of irrigation water demands during the whole growth period is the sum of irrigation water and effective precipitation due to higher management level (Tong and Guo, 2013).

where f is objective function (Yuan/m^3), its physical meaning is to maximize economic water productivity in irrigation water management problems; j denotes the crop types ($j = 1$ for field maize, $j = 2$ for seed maize, $j = 3$ for spring wheat, $j = 4$ for economic crops); A_j is planting area of crop j (ha); NB_j is crop price of crop j (Yuan/kg); CP_j is the cost of crop production for crop j, including all the costs such as seed, fertilizer, pesticides, machinery, harvesting and other costs (Yuan/kg); a_j and b_j are the empirical coefficients of the linear CWPFs for crop j; SW_j and GW_j are the decision variables denoting the amount of irrigated surface water and groundwater for crop j (m^3/ha); P_e is the effective precipitation of the study area (m^3/ha); SC_j is subsidies for food crops per unit of area (Yuan/ha); CS_j and CG_j are the cost of surface water and groundwater use per unit of irrigation water (Yuan/m^3); η_s and η_g are the comprehensive irrigation water use coefficients of surface water and groundwater.

Constraints:

(1) Surface water availability constraints

$$\Pr\left\{\sum_{j=1}^{4}(1+\lambda_s)A_j SW_j \leqslant Q_s \eta_s \beta_s\right\} \geqslant \alpha_i, \forall\, i \tag{8b}$$

(2) Groundwater availability constraints

$$\Pr\left\{\sum_{j=1}^{4}(1+\lambda_g)A_j GW_j \leqslant Q_g\eta_g\beta_g\right\} \geqslant \alpha_i,\ \forall\ i \tag{8c}$$

Constraints (8b and 8c) are stochastic chance constraints, indicating that such constraints can be satisfied at a certain confidence level α_i and the admissible risk of constraint violating $(1-\alpha_i)$.

λ_s and λ_g are the rates of surface water and groundwater loss during water conveyance that are presented as random variables following normal distributions, respectively. Q_s and Q_g are the surface water and groundwater availabilities, which are also expressed as random variables following normal distributions ($10^4\,m^3$). β_s and β_g are the proportion of surface water and groundwater used for irrigation. In this study, $\beta_s = 0.9$ and $\beta_g = 0.9$ in the above constraints.

(3) Irrigation water demand constraints

$$ET_{\min,j} \leqslant SW_j + GW_j + P_e \leqslant ET_{\max,j},\ \forall\ j \tag{8d}$$

where $ET_{\min,j}$ and $ET_{\max,j}$ denote the minimum and maximum evapotranspiration for each crop j, indicating the fluctuation of actual irrigation water requirements for crops. Thus, constraint (8d) means that each crop should be irrigated within a certain range between maximum and minimum evapotranspiration.

(4) Non-negative constraints

$$SW_j \geqslant 0,\ GW_j \geqslant 0,\ \forall\ j \tag{8e}$$

Therefore, by solving the model based on the solution algorithms (see Section 2.4), we have solutions under different confidence levels: f_{opt}, $SW_{j,opt}$ and $GW_{j,opt}$.

4. Results and discussion

4.1. Results analysis

Four confidence levels ($\alpha_i = 0.85$, 0.90, 0.95 and 0.99) are introduced to investigate the satisfaction degree level of constraints and compare the results of the DSCLFP model. Accordingly, the risk levels of violating the constraints are presented as $(1-\alpha_i)$, which can be interpreted as significance level or probability of constraints violation. Table 3 presents optimal solutions resulting from the DSCLFP model under four confidence levels. The results show that the available groundwater is firstly utilized for irrigation because the allocated groundwater is greater than that of the allocated surface water. For example, when $\alpha_i = 0.99$, optimal irrigation water allocation for four crops are: 1063.58 m^3/ha (surface water) and 1551.02 m^3/ha (groundwater) for field maize, 828.75 m^3/ha and 1802.95 m^3/ha for seed maize, 0 and 1725.40 m^3/ha for spring wheat, and 357.78 m^3/ha and 1002.22 m^3/ha for economic crops. This can be explained by groundwater has a higher irrigation water use coefficient and a lower rate of water loss during water conveyance in contrast with surface water (i.e. $\eta_g > \eta_s$ and $\lambda_g < \lambda_s$). In terms of different confidence levels, when α_i is increased from 0.85 to 0.99, the overall trend of allocated surface water is raised but allocated groundwater is decreased for field maize, seed maize and economic crops. Spring wheat is an exception due to unchanged results. Meanwhile, the sum of allocated surface water and groundwater is exactly equal to the difference between their minimum crop water requirements and effective precipitation. Obviously, it is evident that those crops will be water-deficient, which can be attributed to the efficiency-oriented objective function for achieving the maximum system benefits with the minimum irrigation amount.

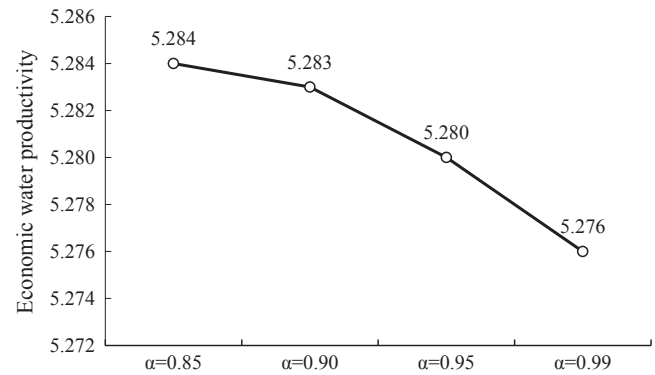

Fig. 5. The objection function values of the DSCLFP model under different confidence levels (Yuan/m^3).

Fig. 5 presents the objection function values of the DSCLFP model under different confidence levels. The results show that a high α_i level bring a lower ratio objective. For example, the objectives are decreased from 5.284 Yuan/m^3 to 5.276 Yuan/m^3 when α_i level is raised from 0.85 to 0.99. The variation trend of this ratio objective is in fact consistent with the findings of previous similar research of a stochastic linear fractional programming approach (Zhu and Huang, 2011). In their paper, when p_i is increased, then the ratio objective would be increased accordingly. This is because the p_i level means the probability that the chance constraints can be violated, which is contrary to the confidence level mentioned in this paper. The α_i level means the satisfaction degree level of the constraints (i.e. $\alpha_i = 1-p_i$), and the ratio objective represents the water productivity. Moreover, a higher α_i level corresponds to a lower constraint-violation risk level. Therefore, the results can support in-depth of the interrelationship among the objective value, α_i level and constraint-violation risk level. In fact, an increased α_i level can lead to an increased strictness for the system conditions and then a narrower decision space. However, it is notable that an increased α_i level makes the system more reliable. Thus, an acceptable and suitable level should be decided based on attitudes of managers and stakeholders associated with conservative or positive choices.

The above analysis indicates that the DSCLFP model can effectively address random information in its framework. It can also generate a range of optimal solutions under different confidence levels. However, it is worth mentioning that the DSCLFP model has computational burden due to nonlinear forms of constraints, which will limit the scope of application of this method. It is thus desirable to develop a computationally tractable solution algorithm to improve computational efficiency and promote practical applications.

Table 3
Optimal solutions resulting from the DSCLFP model under four confidence levels (m^3/ha).

Crop	α = 0.85		α = 0.90		α = 0.95		α = 0.99	
	SW	GW	SW	GW	SW	GW	SW	GW
Field maize	1039.93	1574.67	1044.49	1570.11	1051.19	1563.41	1063.58	1551.02
Seed maize	780.45	1851.25	789.76	1841.94	803.43	1828.27	828.75	1802.95
Spring wheat	0	1725.40	0	1725.40	0	1725.40	0	1725.40
Economic crops	327.27	1032.73	333.15	1026.85	341.79	1018.21	357.78	1002.22

Note: SW and GW denote the allocated surface water and groundwater.

Table 4
Optimal solutions obtained from the linearization form of the DSCLFP model under different confidence levels (m^3/ha).

Crop	α = 0.85		α = 0.90		α = 0.95		α = 0.99	
	SW	GW	SW	GW	SW	GW	SW	GW
Field maize	1039.96	1574.63	1044.53	1570.07	1051.23	1563.37	1063.65	1550.95
Seed maize	780.51	1851.19	789.834	1841.87	803.53	1828.17	828.89	1802.81
Spring wheat	0	1725.40	0	1725.40	0	1725.40	0	1725.40
Economic crops	327.32	1032.68	333.20	1026.8	341.86	1018.14	357.87	1002.13

Note: SW and GW denote the allocated surface water and groundwater.

4.2. Comparison with linearization form of the DSCLFP

In response to this concern, a non-equivalent but sufficient linearization form of the DSCLFP is proposed and proved through Theorems 1 and 2. Table 4 presents the results obtained from the linearization form of the DSCLFP model under different confidence levels. Compared with Table 3, the results of the two models are almost the same. This also verifies the feasibility of the linearization form of the DSCLFP model. Therefore, the linearization form of the DSCLFP model can be regarded as a reasonable one and can enhance its practical implementations for future promotion for address random uncertainties.

In terms of errors occurred by the linearization of the DSCLFP model, which theoretically occurred in the process of the linearization from nonlinear forms into non-equivalent linear one (Eq. (4b). When the nonlinear left-hand side random coefficients are replaced by linear forms, the left-hand side coefficients in the constraints become greater, which may lead to an increased strictness for the constraints and thus narrow down the feasible decision space to some extent. However, it has an advantage of a highly efficient computational process to greatly reduce computational time (Sun et al., 2013; Zhang et al., 2017). Therefore, it should be an innovative and effective method to address the DSCLFP model.

4.3. Comparison with double-sided stochastic chance-constrained linear programming (DSCLP)

When the managers put more emphases on economic returns under the objective function of maximizing system benefits, the DSCLFP model can be transformed into a linear one, i.e. double-sided stochastic chance-constrained linear programming (DSCLP). With the same input parameters of deterministic and random information, results of the DSCLP model can be obtained under given confidence levels. Table 5 shows optimal solutions of the DSCLP model under different α_i levels. Obviously, more surface water is utilized for irrigation in this case because its lower cost of irrigation water use in contrast with groundwater. This results can be explained by the objective function of the benefit-oriented DSCLP model. With α_i level increases, the variation trends of allocated surface water is decreased for field maize and seed maize and stays unchanged for spring wheat and economic crops. This is contrary to the trend of efficiency-oriented DSCLFP model. Especially, when α_i level is increased from 0.85 to 0.99, water productivity resulting from the DSCLP model will be increased from 2.808 Yuan/m^3 to 2.848 Yuan/m^3, which is considerably lower than that from the DSCLFP model. Such a significant difference between the DSCLFP and DSCLP can be further demonstrated in the following analysis of the system benefits and total irrigation water use.

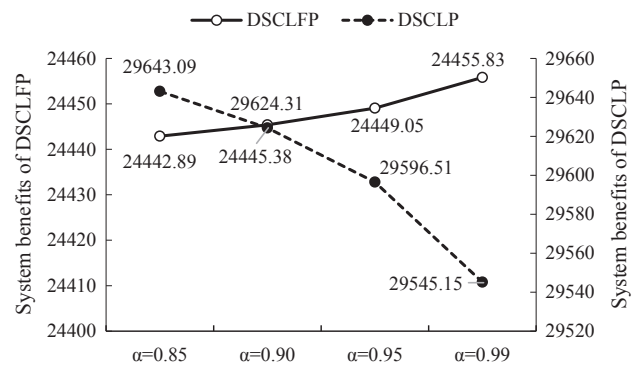

Fig. 6. System benefits resulting from the DSCLFP and DSCLP models under different confidence levels (10^4 Yuan).

Fig. 6 presents the system benefits resulting from two models under a range of α_i levels. Apparently, the DSCLP model brings approximately 20% higher system benefits than the DSCLFP model. For example, the system benefits from the DSCLP model are 29643.09 × 10^4 Yuan when α_i = 0.85, 29624.31 × 10^4 Yuan when α_i = 0.90, 29596.51 × 10^4 Yuan when α_i = 0.95 and 29545.15 × 10^4 Yuan when α_i = 0.99. A higher confidence level leads to a lower system benefits due to an increasing strictness of constraints, which will lower the system-failure risk and increase reliability level. Accordingly, achieving these system benefits is inevitably at the expense of large amounts of irrigation water resources. Fig. 7 presents the total irrigation water use obtained from two models under different confidence levels. The total irrigation water use of the DSCLP model is much greater than that of the optimal-ratio model. Comparison between the two models shows that the DSCLP model makes full use of irrigation water resources but the DSCLFP model illustrates the water-saving potential of the YID. Therefore, the DSCLFP model is clearly exemplified by the current results to utilize irrigation water in an efficient manner.

4.4. Comparison with deterministic model

Without consideration of random information and uncertainties in

Table 5
Optimal solutions of the DSCLP model under different α_i levels (m^3/ha).

Crop	α = 0.85		α = 0.90		α = 0.95		α = 0.99	
	SW	GW	SW	GW	SW	GW	SW	GW
Field maize	4049.46	0	3964.22	0	3838.11	0	3605.12	0
Seed maize	2584.63	4367.87	2599.85	4352.65	2622.36	4330.14	2663.91	4288.59
Spring wheat	1725.40	0	1725.40	0	1725.40	0	1725.40	0
Economic crops	4126.50	0	4126.50	0	4126.50	0	4126.50	0

Note: SW and GW denote the allocated surface water and groundwater.

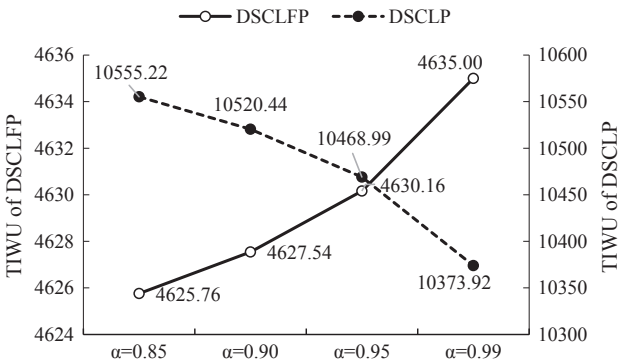

Fig. 7. Total irrigation water use from the DSCLFP and DSCLP models under different confidence levels (10^4 m^3).

Table 6
Results of the LFP model.

Crop	SW	GW
Field maize	1420.89	1193.71
Seed maize	2631.7	0
Spring wheat	1148.78	576.62
Economic crops	645.21	714.79
System benefits (10^4 Yuan)	24927.86	
Total irrigation water use (10^4 m^3)	4972.168	
Water productivity (Yuan/m^3)	5.013	

Note: SW and GW denote the allocated surface water and groundwater (m^3/ha).

the model, the DSCLFP model can be directly simplified into a deterministic linear fractional programming (LFP) model. Thus, only singe solution can be generated by solving this model (see Table 6). Moreover, this solution can be regarded as a special case in the solutions resulting from the DSCLFP model. In such a case, the flexibility of the solution is reduced. On the one hand, there are many uncertain factors in the irrigation water resources planning problems, the traditional LFP model has some limitations in solving practical problems. The obtained results through the newly developed model can provide managers with more reliable and reasonable decision-making recommendations. In addition, the LFP model can only provide a set of results, while the DSCLFP model can generate more results based on the system conditions and managers' attitudes. This will undoubtedly provide managers with more reference information when making decision plans. For example, under a lower confidence level, the system can achieve greater water productivity but at the same time it must withstand higher constraint-violation risks. On the contrary, a lower water productivity will be achieved with a reduced risk level. In summary, the DSCLFP model not only has an enhanced applicability than the LFP model, but it can also be used to better handle trade-offs between economics, environment, and system reliability, and to provides more effective choices for managers.

5. Conclusions

A double-sided stochastic chance-constrained linear fractional programming (DSCLFP) model has been developed for supporting irrigation water management problems under uncertainty. It can address ratio optimization problems (i.e. water productivity) associated with double-sided stochastic constraint-violation, where double-sided stochastic chance-constrained programming (DSCCP) is incorporated into a linear fractional programming (LFP) framework. It thus improves upon the existing stochastic chance-constrained programming by addressing double-sided randomness simultaneously. Meanwhile, a non-equivalent but sufficient linearization form of the DSCLFP model is presented to reduce computational burden. Therefore, the DSCLFP model has the following advantages in: (1) reflecting the ratio objective of water productivity, (2) providing solutions under different confidence levels and (3) supporting in-depth analysis of interrelationships among water productivity, confidence level and constraint-violation risk as well as reliability level.

A case study is provided for demonstrating the applicability of the developed DSCLFP model, which is used to allocate limited irrigation water resources to different crops in YID, northwest China. Optimal solutions are useful for supporting managers to decide desirable choices under different system conditions. Moreover, comparison with linearization form of the DSCLFP model shows the effectiveness of the innovative method. Comparisons with double-sided stochastic chance-constrained linear programming (DSCLP) model and deterministic model can clearly highlight and reflect advantages and feasibility of the developed model. Therefore, some meaningful findings are summarized into following three aspects to prove worthiness of proposed study. First, the results indicate that a high α_i level bring a lower water productivity, which will correspond to managers' preferences regarding the tradeoff between the water productivity and system constraint-violation risk. Accordingly, the results under different confidence levels refer to different irrigation water management solutions and risk-violation levels. Based on these results, managers can adjust their policies in response to uncertain information. Second, to pursue a higher water productivity will certainly improve irrigation water use efficiency but water deficits undoubtedly occur in crop growth due to less allocated irrigation water as well. From a positive perspective, the water-saving potential of the YID is enormous, the saved water can be transferred to other purposes for achieving higher ecological values and economic benefits. Third, the linearization process of model can greatly enhance practical applicability for address random uncertainties.

Although this study is the attempt for supporting irrigation water management problems, the algorithms and results suggest that it can be extended to other resources and environmental management problems. Moreover, this study is based on the assumption of independently normally random variables, which may have limitations when encountering more unidentified probability distributions. Thus, the DSCLFP model still has space for future improvements. It can be potentially enhanced by incorporating techniques of Monte Carlo stochastic simulation, uncertainty analysis, fuzzy theory and dynamic programming into its framework for tackling more complex applications.

Acknowledgements

This research was supported by National Natural Science Foundation of China (91425302) and National Key Research and Development Programs of Thirteen Five Plan (2016YFC0400207). We gratefully acknowledge financial funding from the China Scholarship Council, which supported the first author as a visiting Ph.D. student at Purdue University in the USA. The authors are grateful to the editors and the anonymous reviewers for their insightful and constructive comments and suggestions for improving the paper.

References

Barker, Randolph, Dawe, David, Inocencio, Arlene, 2003. Economics of water productivity in managing water for agriculture. Limits Opportunities Improv. Water Prod. Agric. 19–36.
Cao, M.F., Huang, G.H., He, L., 2011. An approach to interval programming problems with left-hand-side stochastic coefficients: an application to environmental decisions analysis. Expert Syst. Appl. 38 (9), 11538–11546.
Management Science, Chance-constrained programming, pp. 73–79.
Charnes, A., Cooper, W.W., 1962. Programming with linear fractional functionals. Naval Res. Logistics (NRL) 9 (3–4), 181–186.
Elliott, J., Deryng, D., Müller, C., et al., 2014. Constraints and potentials of future irrigation water availability on agricultural production under climate change. Proc. Natl. Acad. Sci. 111 (9), 3239–3244.
Gómez, T., Hernández, M., León, M.A., Caballero, R., 2006. A forest planning problem solved via a linear fractional goal programming model. Forest Ecol. Manage. 227

(1–2), 79–88.
Gu, J., Huang, G.H., Guo, P., et al., 2013. Interval multistage joint-probabilistic integer programming approach for water resources allocation and management. J. Environ. Manage. 128 (20), 615–624.
Guo, P., Chen, X., Li, M., et al., 2014. Fuzzy chance-constrained linear fractional programming approach for optimal water allocation. Stoch. Env. Res. Risk Assess. 28 (6), 1601–1612.
Huang, G.H., 1998. A hybrid inexact-stochastic water management model. Eur. J. Oper. Res. 107 (1), 137–158.
Jiang, Y., Xu, X., Huang, Q., Huo, Z., Huang, G., 2015. Assessment of irrigation performance and water productivity in irrigated areas of the middle Heihe River basin using a distributed agro-hydrological model. Agric. Water Manage. 147, 67–81.
Jiang, Y., Xu, X., Huang, Q., Huo, Z., Huang, G., 2016. Optimizing regional irrigation water use by integrating a two-level optimization model and an agro-hydrological model. Agric. Water Manage. 178, 76–88.
Kang, S., Hao, X., Du, T., et al., 2017. Improving agricultural water productivity to ensure food security in China under changing environment: from research to practice. Agric. Water Manag. 179, 5–17.
Lara, P., Stancu-Minasian, I., 1999. Fractional programming: a tool for the assessment of sustainability. Agric. Syst. 62 (2), 131–141.
Li, W., Li, Y.P., Li, C.H., Huang, G.H., 2010. An inexact two-stage water management model for planning agricultural irrigation under uncertainty. Agric. Water Manag. 97 (11), 1905–1914.
Li, Y.P., Huang, G.H., Nie, S.L., 2012. A mathematical model for identifying an optimal waste management policy under uncertainty. Appl. Math. Model. 36 (6), 2658–2673.
Liu, B., 2009. Theory and practice of uncertain programming. Springer, Berlin.
Nemirovski, A., 2012. On safe tractable approximations of chance constraints. Eur. J. Oper. Res. 219 (3), 707–718.
Roubens, M., Teghem Jr, J., 1991. Comparison of methodologies for fuzzy and stochastic multi-objective programming. Fuzzy Sets Syst. 42 (1), 119–132.
Stancu-Minasian, I.M., 2012. Fractional programming: theory, methods and applications. Springer Science & Business Media.
Sun, W., Huang, G.H., Lv, Y., Li, G., 2013. Inexact joint-probabilistic chance-constrained programming with left-hand-side randomness: an application to solid waste management. Eur. J. Oper. Res. 228 (1), 217–225.
Tong, F., Guo, P., 2013. Simulation and optimization for crop water allocation based on crop water production functions and climate factor under uncertainty. Appl. Math. Model. 37 (14–15), 7708–7716.
Tan, Q., Huang, G., Cai, Y., 2011. Robust planning of environmental management systems with adjustable conservativeness under compound uncertainty. J. Environ. Eng. 138 (2), 208–222.
Zhang, C., Guo, P., 2018. FLFP: a fuzzy linear fractional programming approach with double-sided fuzziness for optimal irrigation water allocation. Agric. Water Manag. 199, 105–119.
Zhang, C., Li, M., Guo, P., 2017. An interval multistage joint-probabilistic chance-constrained programming model with left-hand-side randomness for crop area planning under uncertainty. J. Cleaner Prod. 167, 1276–1289.
Zhu, H., Huang, G.H., 2011. SLFP: A stochastic linear fractional programming approach for sustainable waste management. Waste Manage. (Oxford) 31 (12), 2612–2619.