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Towards sustainable water resources planning and pollution control: Inexact joint-probabilistic double-sided stochastic chance-constrained programming model

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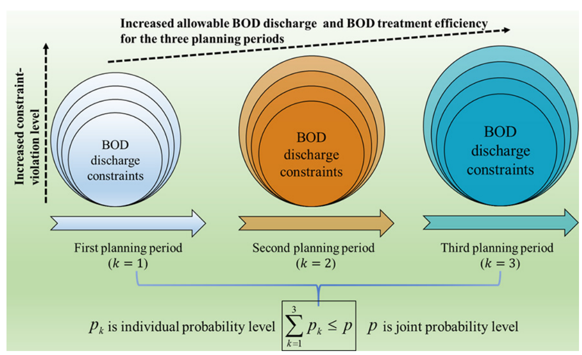
HIGHLIGHTS

- An inexact joint-probabilistic double-sided stochastic chance-constrained programming model is developed.
- A non-equivalent but sufficient linearization form of the developed model is presented.
- An increasing joint probability level leads to higher system benefits.
- A set of decreased individual probability levels results in the maximum system benefits at the same joint probability level.

GRAPHICAL ABSTRACT



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ABSTRACT

This study presents an inexact joint-probabilistic double-sided stochastic chance-constrained programming (IJDSCCP) model for sustainable water resources planning and pollution control in water quality management systems under uncertainty. Techniques of interval parameter programming (IPP), joint-probabilistic programming (JPP) and double-sided stochastic chance-constrained programming (DSCCP) are incorporated into a modeling framework. The IJDSCCP can not only address uncertainties presented as interval parameters and double-sided randomness (i.e. both left-hand and right-hand sides) that are characterized as normal distributions, but also examine the reliability level of satisfying the entire system constraints. It further improves upon conventional stochastic chance-constrained programming for handing random uncertainties in the left-hand and right-hand sides of constraints. Moreover, a non-equivalent but sufficient linearization form of the IJDSCCP is presented to solve such a problem. Then, the model is applied to a representative case for water resources planning and pollution control. The results including water resources planning solutions, pollution control plans and system benefits under the combinations of different joint and individual probability levels will be obtained. The solutions are expressed as combinations of deterministic, interval and distributional information, which can facilitate analysis of different forms of uncertainties. After investigating and comparing the variations of results, it is found that an increasing joint probability level can lead to higher system benefits, i.e., [13,841.68, 21,801.81] × 10^6 Yuan ($p = 0.01$, $p_1 = 0.0033$, $p_2 = 0.0033$ and $p_3 = 0.0033$), [14,150.26, 22,260.06] × 10^6 Yuan ($p = 0.05$, $p_1 = 0.0166$, $p_2 = 0.0166$ and $p_3 = 0.0166$) and [14,280.55, 22,415.52] × 10^6 Yuan ($p = 0.10$, $p_1 = 0.033$, $p_2 = 0.033$ and $p_3 = 0.033$). A set of decreased individual probability levels gives rise to the

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maximum system benefits at the same joint probability level. Furthermore, the results of the IJDSCCP are compared with a general interval-based optimization framework as well. Therefore, the results from the IJDSCCP are valuable for assisting managers in generating and identifying decision alternatives under different scenarios.

1. Introduction

Deterioration of water quality is currently one of the most pressing environmental issues in many parts of the world (Loecke et al., 2017). Globally, eutrophication (i.e., excess nutrients) has seriously affected water ecosystems in lakes, rivers, streams and estuaries, thereby threatening the safety of drinking water sources and leading to the formation of anoxic zones in lakes and coastal areas, such as the Black Sea in Eastern Europe, the Pearl River Delta in China, and the Gulf of Mexico and Chesapeake Bay in the United States (Diaz and Rosenberg, 2008; G. Liu et al., 2017; Schmidtko et al., 2017). Sources of pollution that cause water quality damage are generally divided into two categories: point sources and non-point sources. Point source refers to the discharge of sewage into water bodies through independent drainage systems (e.g. pipelines), such as municipal sewage treatment plants and industrial wastewater treatment plants. Conversely, pollution from non-point sources is often dispersed, such as agricultural runoff or urban runoff. From a global perspective, most nutrient pollutions come from non-point sources, mainly from agriculture (Ongley et al., 2010; Fanelli et al., 2018). Water pollution and ecological deterioration problems can undoubtedly impede socioeconomic growth. The system optimization model, as an effective means of water quality management, can comprehensively involve the management policies and interests of multiple participants to obtain optimal decision-making solutions for water quality management. Therefore, it is desired that effective strategies of water resources planning and pollution control be taken to facilitate sustainable development (Zhang et al., 2015).

Water quality management faces new challenges, including ever-increasing changes and the associated system uncertainties. For example, the uncertainties and complexities arising from the variations in hydrological processes and meteorological conditions, dynamics of pollution transport, interactions between pollutant loadings, and water bodies, the standard of environmental capacity should be taken into account (Babaeyan-Koopaei et al., 2003; Huang and Chang, 2003; Xu and Qin, 2010; Li et al., 2012; Y. Liu et al., 2017; McMillan et al., 2017). Therefore, many mathematical programming methods have been developed for addressing these problems (Wen and Lee, 1998; Qin et al., 2007; Kerachian and Karamouz, 2007; He et al., 2008; Qin and Huang, 2009; Pouget et al., 2012; Zhang et al., 2014; Liu et al., 2015; Zhang et al., 2015; Poff et al., 2016; Archibald and Marshall, 2018; Li et al., 2019). Generally, there are three main types of inexact mathematical methods, interval mathematical programming (IMP), stochastic mathematical programming (SMP) and fuzzy mathematical programming (FMP). These methods and their combinations have been extensively proposed to solve problems of environmental management and water quality management. As an effective SMP method to address random uncertainties, chance-constrained programming (CCP), initially introduced by Charnes and Cooper (1959), is suitable for occasions in the presence of an abundant source of information. Moreover, it is assumed that the constraints be satisfied at least probability $(1-\alpha)$ in a stochastic environment where α indicates the probability when system constraints are violated. This method does not require that all the system constraints should be totally satisfied. Theoretically, it has the following main advantages in: (1) allowing specific random constraints to be satisfied under given probability levels; (2) providing information on tradeoffs between the system benefits and system-failure risks and (3) enabling incorporation of other uncertain optimization methods into a general framework to enhance its applicability. Generally, the most-used CCP method is based on independent right-hand side randomness in system constraints (Zare and Daneshmand, 1995; Huang, 1998). For example, Zhu and Huang (2011) proposed a stochastic linear fractional programming approach for solid waste management and the right-hand side coefficients of the constraints are random. Zhou et al. (2017) developed a stochastic equilibrium chance-constrained programming model for supporting solid waste management in Dalian City, China. Wang et al. (2018) developed an inexact log-normal distribution-based stochastic chance-constrained model for agricultural water quality management. However, the CCP method still has some limitations that need further improvement.

Random uncertainties in fact may exist in both left-hand and right-hand sides of the constraints, thereby indicating that the above CCP cannot deal with such a difficulty due to the nonlinear forms of the deterministic equivalents. Therefore, double-sided stochastic chance-constrained programming (DSCCP) is considered as a promising method for better accounting for this difficulty (Liu, 2009; Zhang et al., 2018). For example, Zhang et al. (2018) developed a double-sided stochastic chance-constrained linear fractional programming model for irrigation water management under uncertainty. Moreover, the CCP also fails to examine and analyze the interrelationships among multiple constraints, which are required to be satisfied at a joint probability level (Li et al., 2009; Sun et al., 2013; Zhang et al., 2017). For example, when conducting long-term planning for specific problems, it is noted that the effects of changes of system components among different planning periods and their dynamic interrelationships need to be considered. Therefore, a joint-probabilistic programming (JPP) method is introduced to reflect the requirement of the satisfactory level. Besides, not all the collected data are good enough to be presented as precise random distributions, which affects the applicability of the CCP. Therefore, one potential direction to deal with this challenge is to introduce interval parameter programming (IPP) to roughly measure the uncertain information. In summary, in response to above concerns, techniques including DSCCP, JPP and IPP are incorporated into a general framework to formulate an inexact joint-probabilistic double-sided stochastic chance-constrained programming (IJDSCCP) model. Nevertheless, few applications of the IJDSCCP to water resources planning and pollution control have been reported.

Based on above analysis, the primary objective of this study is to develop a novel approach for water resources planning and pollution control problems in water quality management systems, namely, IJDSCCP. It is an improved method compared with general stochastic chance-constrained programming due to its handling of double-sided randomness. Moreover, a non-equivalent but sufficient linearization form of the IJDSCCP will be presented and proved. Therefore, it has the following advantages for effectively managing the water quality. (1) It can address uncertainties expressed as interval values and probability distributions; (2) it can examine the reliability level of satisfying the entire system constraints; (3) it can reduce computational burden for efficiently solving problems and (4) it can support in-depth analysis of the interrelationships among system benefits, violation probability level and constraints-violation risk level. A representative case will then be provided for demonstrating the applicability of the developed model. Therefore, the results (i.e. water resources planning solutions, pollution control plans and system benefits) under the combinations of different joint and individual probability levels will be obtained to facilitate further analysis and comparison.

2. Methodology

2.1. Double-sided stochastic chance-constrained programming

For a general linear programming problem, it is generally incapable of dealing with problems of both the left-hand and right-hand sides parameters within the constraints are present as mutually independent random parameters that follow normal distributions (μ is expected value and σ is standard variation), and the associated constraints are satisfied at a given probability $(1-p_i)$ (i.e. p_i is the probability that the constraints can be violated). Thus, a double-sided stochastic chance-constrained programming (DSCCP) model is proposed to address these concerns.

$$\max f = \sum_{j=1}^{n} c_j x_j \tag{1a}$$

subject to:

$$\Pr\left\{\sum_{j=1}^{n} a_{ij}(\omega) x_j \le b_i(\xi)\right\} \ge 1-p_i, i=1,2,\cdots,m$$

$$a_{ij}(\omega) \sim N\left(\mu_{a,ij}, \sigma_{a,ij}^2\right)$$

$$b_i(\xi) \sim N\left(\mu_{b,i}, \sigma_{b,i}^2\right)$$

$$x_j \ge 0, j=1,2,\cdots,m \tag{1e}$$

where f is the objective function; $a_{ij}(\omega)$ and $b_i(\xi)$ are the left-hand and right-hand sides parameters of the constraints, which all obey independent normal distributions; p_i ($p_i \in [0,1]$) is a predefined probability level for constraint i (i.e. violation probability), indicating the constraint is satisfied at the probability level of $(1-p_i)$ (Zhu and Huang, 2011); x_j is a decision variable; c_j is a parameter of the model; m is the number of constraints.

2.2. Inexact joint-probabilistic double-sided stochastic chance-constrained programming

When the probability distributions of parameters are unknown because of data quality, interval parameter programming can be used to address interval values when the upper and lower bounds are known. Moreover, it has difficulties in analyzing interactions among multiple constraints under joint probabilities. Since the requirement of satisfaction degree level is usually imposed on a set of the constraints as a whole, joint-probabilistic programming is incorporated into the optimization framework (Sun et al., 2013). Therefore, an inexact joint-probabilistic double-sided stochastic chance-constrained programming (IJDSCCP) model is developed to enhance its applicability. It is presented as follows:

$$\max f^{\pm} = \sum_{j=1}^{n} c_j^{\pm} x_j^{\pm} \tag{2a}$$

subject to:

$$\Pr\left\{\sum_{j=1}^{n} a_{ij}(\omega) x_j^{\pm} \le b_i(\xi)\right\} \ge 1-p_i, i=1,2,\cdots,m$$

$$\sum_{i=1}^{m} p_i \le p$$

$$a_{ij}(\omega) \sim N\left(\mu_{a,ij}, \sigma_{a,ij}^2\right)$$

$$b_i(\xi) \sim N\left(\mu_{b,i}, \sigma_{b,i}^2\right)$$

$$x_j^{\pm} \ge 0, j=1,2,\cdots,m \tag{2f}$$

where $f^{\pm}$ is the objective function; superscript '+' denotes the upper bound value; superscript '−' denotes the lower bound value; $a_{ij}(\omega)$ and $b_i(\xi)$ are the random parameters of left-hand and right-hand sides constraints; $x_j^{\pm}$ is an interval decision variable; $c_j^{\pm}$ is an interval parameter of the model; p is the joint probability level that is the sum of each significance level, indicating the overall violation probability of the system constraints.

2.3. Linearization form of IJDSCCP

According to Liu (2009), constraint Eq. (2b) can be converted into a nonlinear form and the set of feasible constraints is convex. Theoretically, the IJDSCCP model is still presented as a nonlinear form, which generally cannot be solved through traditional mathematical algorithms. In this study, a non-equivalent but sufficient approximated linearization is proposed to facilitate solving joint probabilistic double-sided chance constraints (Sun et al., 2013). The detailed derivation process is presented in Appendix I. Therefore, a linearization form of the IJDSCCP is formulated as follows:

$$\max f^{\pm} = \sum_{j=1}^{n} c_j^{\pm} x_j^{\pm} \tag{3a}$$

subject to:

$$\sum_{j=1}^{n} \mu_{a,ij} x_j^{\pm} + \Phi^{-1}(1-p_i)\left(\sum_{j=1}^{n} \sigma_{a,ij} x_j^{\pm} + \sigma_{b,i}\right) \le \mu_{b,i}, i=1,2,\cdots,m$$

$$\sum_{i=1}^{m} p_i \le p$$

$$a_{ij}(\omega) \sim N\left(\mu_{a,ij}, \sigma_{a,ij}^2\right)$$

$$b_i(\xi) \sim N\left(\mu_{b,i}, \sigma_{b,i}^2\right)$$

$$x_j^{\pm} \ge 0, j=1,2,\cdots,m \tag{3f}$$

Then, this linearization form of the IJDSCCP can be transformed into two deterministic submodels through the interactive algorithm method, including the upper and lower bounds of the objective function values. The solution process of the interactive algorithm method for solving the linearization form of IJDSCCP is shown in Appendix II. The framework of the IJDSCCP model is graphically illustrated in Fig. 1. Therefore, optimal solutions and desired objective function values under the combinations of different joint and individual probabilities (i.e. risk levels of constraint violating) can finally be obtained.

3. Application

3.1. Problem statement

Economic development is accompanied by increasingly serious water quality problems. Since zero-discharge of pollutants are

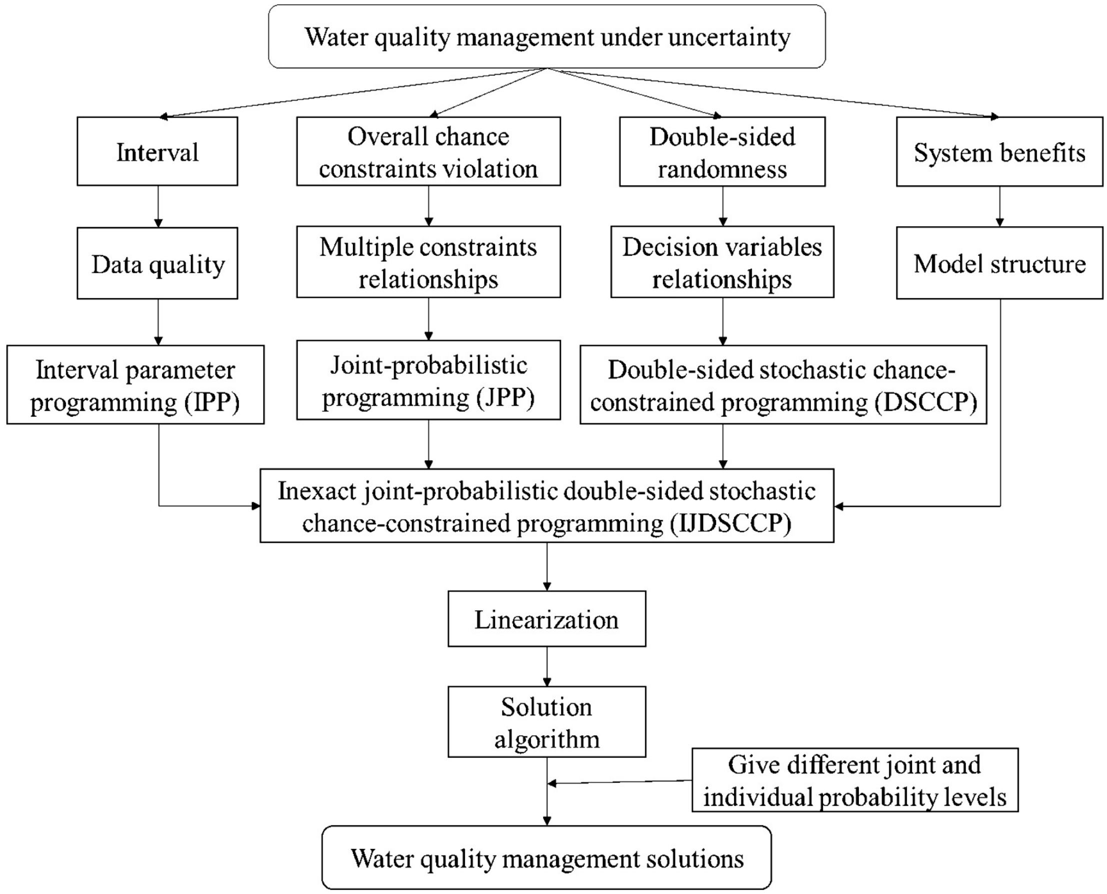

Fig. 1. The general framework of this study.

economically infeasible and technically unachievable under current technical conditions, managers should develop effective water resources planning solutions and pollutant control plans to concurrently balance environmental standards and economic benefits. In the process of decision-making on water quality management, managers need to consider various system factors, including temporal or spatial changes

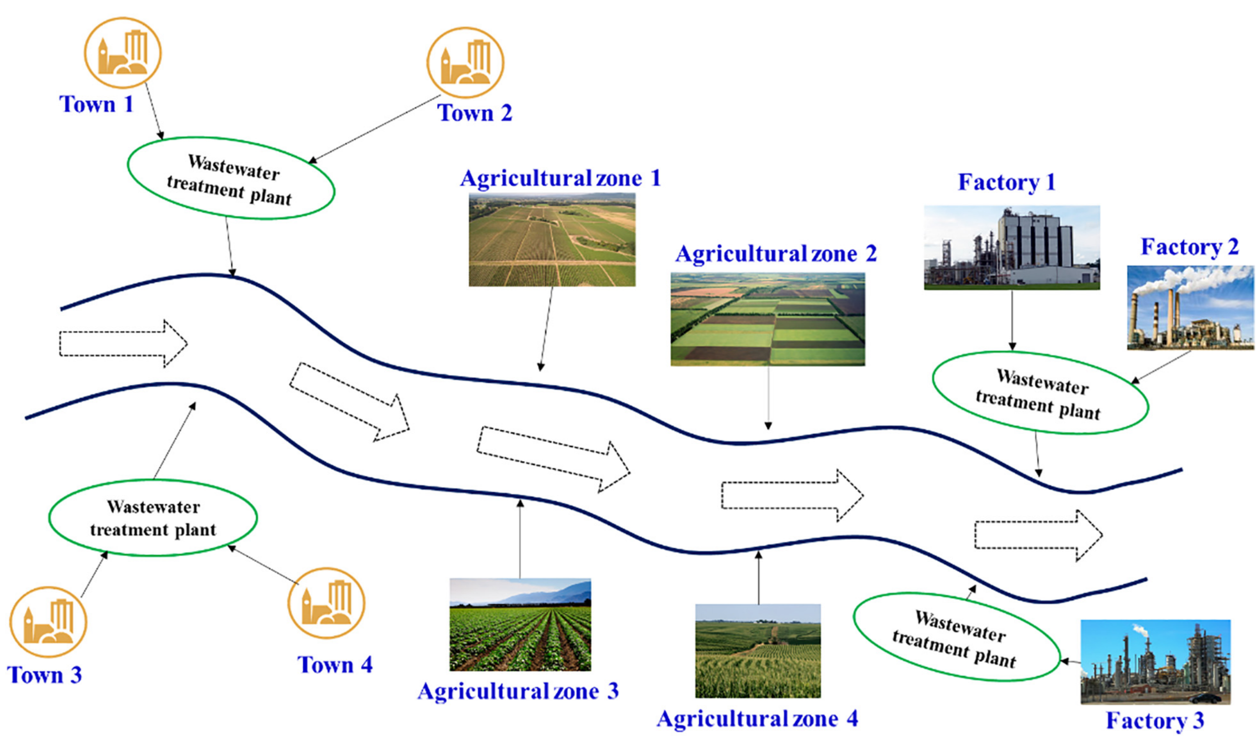

Fig. 2. Schematic diagram of the study system.

in input parameters, dynamic characteristics of system conditions, environmental factors, and policies and other uncertainties. Therefore, a representative case of water quality management is provided for demonstrating the applicability of the developed IJDSCCP model. Fig. 2 shows the schematic diagram of the study system. The main point sources typically include three industrial factories and four municipal towns, and non-point sources refer to the four agricultural zones due to applying fertilizers. These point sources and non-point sources are distributed on the same river section. In this study, the planning horizon is 15 years, including three 5 year periods (i.e., 1825 days). The objective is to maximize the economic benefits subject to the environmental constraints under uncertainty over the planning horizons.

Policies related to human activities (industrial, municipal, and agricultural activities) and pollutant discharge are critical to ensuring maximum system benefits and water quality. Biochemical oxygen demand (BOD) and total nitrogen (TN) are selected as water quality indicators and the sustainability can thus be achieved by setting allowable TN and BOD discharge (i.e. water environmental standards). In fact, the determination of the amount of allowable discharge is complicated, and it can be assumed that it is a random variable following a normal distribution (i.e. right-hand sides of constraints) (Li et al., 2012). Moreover, other uncertainties and complexities in the study system also include: (1) cost of wastewater treatment and fertilizer will fluctuate with market prices but within a certain range, which can be roughly expressed as interval values; (2) BOD treatment efficiency of the industrial factory, wastewater treatment plants are related to the operating conditions of the treatment equipment and they can be assumed to follow a normal distribution (i.e. left-hand sides of constraints); (3) the impact of the above-mentioned double-sided randomness chance constraints on the study system; (4) interrelationships among different joint and individual probabilities over multiple planning periods, and (5) input parameters or variables (e.g. unit benefits, water cost, irrigation quota, treatment cost, wastewater treatment capacity, chemical fertilizer applied) vary with different planning periods. Therefore, to address the above planning problem of water resources management and pollution control, the developed IJDSCCP model is applied.

3.2. Application of the IJDSCCP model

Accordingly, the proposed model can be formulated as follows:

3.2.1. Objective function

$$\max f^{\pm} = f_1^{\pm} - f_2^{\pm}$$

$$f_1^{\pm} = L_k \sum_{k=1}^{3} \left(\sum_{i=1}^{3} BI_{ik}^{\pm} PLI_{ik}^{\pm} + \sum_{m=1}^{4} BM_{mk}^{\pm} QM_{mk}^{\pm} \right) + 5 \sum_{k=1}^{3} \sum_{j=1}^{4} BA_{jk}^{\pm} PA_{jk}^{\pm}$$

$$f_2^{\pm} = L_k \sum_{k=1}^{3} \left(\sum_{i=1}^{3} PLI_{ik}^{\pm} \left(WI_{ik}^{\pm} CI_{ik}^{\pm} + FI_{ik}^{\pm} CW_k^{\pm} \right) + \sum_{m=1}^{4} QM_{mk}^{\pm} \left(WM_{mk}^{\pm} CM_{mk}^{\pm} + CW_k^{\pm} \right) \right) + 5 \sum_{k=1}^{3} \sum_{j=1}^{4} \left(CF_{jk}^{\pm} FA_{jk}^{\pm} + PA_{jk}^{\pm} IQ_{jk}^{\pm} CW_k^{\pm} \right) \quad (4c)$$

$f^{\pm}$ is the objective function representing net system benefits (10^6 Yuan); $f_1^{\pm}$ is the economic benefits resulting from different water users when water is delivered (10^6 Yuan); $f_2^{\pm}$ is costs of water use, wastewater treatment and agricultural production (10^6 Yuan); L_k is length of period k, $L_k = 365*5 = 1825$ day; k is planning period ($k = 1, 2, 3$); i is the number of industrial factories ($i = 1, 2, 3$); j is the number of agricultural zones ($j = 1, 2, 3, 4$); m is the number of municipal towns ($m = 1, 2, 3, 4$); $BI_{ik}^{\pm}$ is unit benefits of industrial production from factory i during period k (Yuan/ton); $BA_{jk}^{\pm}$ is unit benefits of agricultural production from zone j during period k (10^3 Yuan/ha); $BM_{mk}^{\pm}$ is unit benefits of municipal water supply from town m during period k (Yuan/m^3); $PLI_{ik}^{\pm}$, $PA_{jk}^{\pm}$ and $QM_{mk}^{\pm}$ are decision variables; $PLI_{ik}^{\pm}$ is the amount of production level of factory i during period k (ton/day); $PA_{jk}^{\pm}$ is planting area of zone j during period k (ha); $QM_{mk}^{\pm}$ is the amount of water supply to town m during period k (m^3/day);

$WI_{ik}^{\pm}$ is rate of wastewater generation of factory i during period k (m^3/ton); $CI_{ik}^{\pm}$ is wastewater treatment cost of factory i during period k (Yuan/m^3); $FI_{ik}^{\pm}$ is water consumption of per unit industrial production of factory i during period k (m^3/ton); $CW_k^{\pm}$ is water cost per unit of water supply during period k (Yuan/m^3); $FA_{jk}^{\pm}$ is the amount of fertilizer applied to agricultural zone j during period k (kg); $CF_{jk}^{\pm}$ is cost of purchasing fertilizer in agricultural zone j during period k (Yuan/ton); $IQ_{jk}^{\pm}$ is irrigation quota for agricultural zone j during period k (m^3/ha); $WM_{mk}^{\pm}$ is rate of wastewater discharge of municipal town m during period k (%); $CM_{mk}^{\pm}$ is cost of wastewater treatment of municipal town m during period k (Yuan/m^3).

3.2.2. Constraints

(1) Wastewater treatment capacity constraints:

$$PLI_{ik}^{\pm} WI_{ik}^{\pm} \le IPC_{ik}^{\pm}, \forall i, k$$

$$QM_{mk}^{\pm} WM_{mk}^{\pm} \le MPC_{mk}^{\pm}, \forall m, k \quad (4e)$$

In point sources, factories and municipal towns are the main sources of BOD discharge. Wastewater from each factory and town needs to be treated before being discharged into the river, and the final BOD discharge should be less than or equal to the discharge standards. The above constraints ensure that the wastewater treatment capacity of each point source is greater than the amount of wastewater generated by human daily life and industrial production. $IPC_{ik}^{\pm}$ is wastewater treatment capacity of factory i during period k (m^3/day); $MPC_{mk}^{\pm}$ is wastewater treatment capacity of municipal town m during period k (m^3/day).

Table 1
Input parameters of industrial factory for different planning periods.

Factory	$k = 1$	$k = 2$	$k = 3$	$k = 1$	$k = 2$	$k = 3$
	Unit benefits of industrial factory $BI_{ik}^{\pm}$ (Yuan/ton)			Water consumption of per unit industrial production $FI_{ik}^{\pm}$ (m^3/ton)		
$i = 1$	[1200, 1500]	[1250, 1600]	[1350, 1700]	[2.0, 3.0]	[3.0, 4.0]	[4.0, 5.0]
$i = 2$	[750, 900]	[800, 1000]	[850, 1100]	[2.5, 3.0]	[3.0, 4.0]	[3.5, 4.0]
$i = 3$	[700, 800]	[750, 850]	[800, 900]	[1.5, 2.5]	[2.0, 3.0]	[2.5, 3.5]
	Rate of wastewater generation $WI_{ik}^{\pm}$ (m^3/ton)			BOD concentration of discharged wastewater $IC_{ik}^{\pm}$ (kg/m^3)		
$i = 1$	[0.60, 0.70]	[0.55, 0.65]	[0.50, 0.60]	[1.25, 1.40]	[1.15, 1.30]	[1.05, 1.20]
$i = 2$	[0.55, 0.60]	[0.50, 0.55]	[0.45, 0.50]	[1.10, 1.20]	[1.00, 1.10]	[0.90, 1.00]
$i = 3$	[0.30, 0.40]	[0.25, 0.35]	[0.20, 0.30]	[0.95, 1.05]	[0.85, 0.95]	[0.75, 0.85]
	Wastewater treatment cost $CI_{ik}^{\pm}$ (Yuan/m^3)			BOD treatment efficiency $\lambda_{BOD,\, ik}$ (%)		
$i = 1$	[65, 75]	[55, 65]	[45, 55]	$(0.90, 0.01^2)$	$(0.93, 0.01^2)$	$(0.96, 0.01^2)$
$i = 2$	[55, 65]	[45, 55]	[35, 45]	$(0.90, 0.01^2)$	$(0.92, 0.01^2)$	$(0.95, 0.01^2)$
$i = 3$	[50, 60]	[40, 50]	[30, 40]	$(0.88, 0.01^2)$	$(0.92, 0.01^2)$	$(0.94, 0.01^2)$

Table 2
Input parameters of agricultural production for different planning periods.

Zone	k = 1	k = 2	k = 3	k = 1	k = 2	k = 3
	Unit benefits $BA_{jk}^{\pm}$ (10^3 Yuan/ha)			Nitrogen content of soil SN_j (%)		
j = 1	[20, 22]	[24, 28]	[28, 34]	[0.10, 0.12]		
j = 2	[15, 16]	[16, 18]	[17, 20]	[0.15, 0.18]		
j = 3	[17, 19]	[19, 23]	[20, 25]	[0.10, 0.13]		
j = 4	[16, 17]	[18, 19]	[19, 21]	[0.09, 0.12]		
	The amount of fertilizer applied $FA_{jk}^{\pm}$ (kg)			Average soil loss $SL_{jk}^{\pm}$ (ton/ha)		
j = 1	[270, 300]	[225, 240]	[150, 180]	[20, 22]	[16, 18]	[12, 14]
j = 2	[270, 300]	[225, 240]	[150, 180]	[16, 20]	[14, 16]	[12, 15]
j = 3	[270, 300]	[225, 240]	[150, 180]	[25, 30]	[20, 25]	[15, 20]
j = 4	[270, 300]	[225, 240]	[150, 180]	[18, 22]	[16, 18]	[12, 14]
	Cost of purchasing fertilizer $CF_{jk}^{\pm}$ (Yuan/ton)			Average runoff $RA_{jk}^{\pm}$ (mm)		
j = 1	[30, 35]	[25, 30]	[20, 25]	[800, 950]		
j = 2	[28, 33]	[24, 29]	[19, 24]	[800, 900]		
j = 3	[36, 40]	[32, 36]	[30, 34]	[750, 900]		
j = 4	[32, 35]	[29, 32]	[26, 29]	[850, 950]		
	Irrigation quota $IQ_{jk}^{\pm}$ (m^3/ha)			Dissolved nitrogen concentration in the runoff $DN_{jk}^{\pm}$ (mg/L)		
j = 1	[4000, 4500]	[3500, 4000]	[3000, 3500]	[2.5, 3.0]	[2.0, 2.5]	[1.5, 2.0]
j = 2	[3000, 3500]	[2800, 3300]	[2500, 3000]	[2.3, 2.5]	[2.0, 2.2]	[1.8, 2.0]
j = 3	[3300, 3500]	[3000, 3200]	[2800, 3000]	[2.6, 3.0]	[2.2, 2.8]	[2.0, 2.5]
j = 4	[3800, 4200]	[3600, 4000]	[3400, 3800]	[3.5, 3.8]	[2.6, 3.0]	[2.2, 2.8]
	Maximum allowable nitrogen loss $MNL_{jk}^{\pm}$ (ton/ha)			Tillable area $TA_{jk}^{\pm}$ (ha)		
j = 1	[0.12, 0.14]	[0.10, 0.12]	[0.08, 0.10]	[2800, 3000]		
j = 2	[0.15, 0.16]	[0.12, 0.14]	[0.10, 0.12]	[4000, 4500]		
j = 3	[0.11, 0.13]	[0.09, 0.11]	[0.06, 0.08]	[4500, 5000]		
j = 4	[0.10, 0.12]	[0.08, 0.10]	[0.06, 0.08]	[3800, 4000]		

(2) BOD discharge constraints:

$$\Pr\{PLI_{ik}^{\pm} WI_{ik}^{\pm} IC_{ik}^{\pm}(1-\lambda_{BOD,ik}) \le ABI_{ik}\} \ge 1-p_k, \forall i, k$$

$$\Pr\{QM_{mk}^{\pm} WM_{mk}^{\pm} MC_{mk}^{\pm}(1-\lambda_{BOD,mk}) \le ABM_{mk}\} \ge 1-p_k, \forall m, k \quad (4g)$$

The allowable discharge of pollutants is an essential factor in the control of water pollution, which can reflect the river's ability to receive water pollutants without destroying the basic functions of the water within a certain period of time. Due to the observation error of daily wastewater discharge, the allowable BOD discharge of industrial factories and municipal towns are allowed to be expressed as random numbers. Meanwhile, wastewater treatment efficiency is also assumed to be an independently random probability distribution.

$IC_{ik}^{\pm}$ and $MC_{mk}^{\pm}$ are BOD concentration of discharged wastewater that are generated from factory i and municipal town m during period k (kg/m^3); $\lambda_{BOD,\ ik}$ and $\lambda_{BOD,\ mk}$ are BOD treatment efficiency of factory i and municipal town m during period k (%); ABI_{ik} and ABM_{mk} are allowable BOD discharge for factory i and municipal town m during period k (kg/day).

(3) Joint-probabilistic constraints:

$$\sum_{k=1}^{3} p_k \le p \quad (4h)$$

p_k is an individual probability for each period k of constraints, and p is a joint probability that is the sum of all the planning periods.

(4) Total nitrogen (TN) discharge constraints:

$$\left(SN_j^{\pm} SL_{jk}^{\pm} + RA_{jk}^{\pm} DN_{jk}^{\pm}\right) PA_{jk}^{\pm} \le MNL_{jk}^{\pm} TA_{jk}^{\pm}, \forall j, k \quad (4i)$$

In practice, agricultural activities, as non-point sources, have more significant impact on river water pollution. These constraints consider TN losses from soil loss of farmland, including soil loss (loss of solid nitrogen) and surface runoff (loss of dissolved nitrogen). SN_j is nitrogen content of soil in the agricultural zone j (%); $SL_{jk}^{\pm}$ is average soil loss from agricultural zone j during period k (ton/ha); $RA_{jk}^{\pm}$ is average runoff from agricultural zone j during period k (mm); $DN_{jk}^{\pm}$ is dissolved nitrogen concentration in the runoff from agricultural zone j during period k (mg/L); $MNL_{jk}^{\pm}$ is the maximum allowable nitrogen loss in agricultural zone j during period k (ton/ha); $TA_{jk}^{\pm}$ is tillable area of agricultural zone j during period k (ha).

(5) Production level constraints:

$$PLI_{i,\ \min} \le PLI_{ik}^{\pm} \le PLI_{i,\ \max}, \forall i, k$$

$$PA_{j,\ \min} \le PA_{jk,\ \min}^{\pm} \le PA_{j,\ \max}, \forall j, k$$

$$QM_{m,\ \min} \le QM_{mk,\ \min}^{\pm} \le QM_{m,\ \max}, \forall m, k \quad (4m)$$

These constraints ensure that each decision variable is limited to a reasonable range. $PLI_{i,\ \min}$ and $PLI_{i,\ \max}$ are the minimum and maximum industrial production level of factory i (ton/day); $PA_{j,\ \min}$ and $PA_{j,\ \max}$ are the minimum and maximum planting area of agricultural zone j (ha); $QM_{m,\ \min}$ and $QM_{m,\ \max}$ are the minimum and maximum municipal water supply to town m (m^3/day).

Table 3
Input parameters of municipal water supply for different planning periods.

Town	k = 1	k = 2	k = 3	k = 1	k = 2	k = 3
	Unit benefits $BM_{mk}^{\pm}$ (Yuan/m^3)			Rate of wastewater discharge $WM_{mk}^{\pm}$ (%)		
m = 1	[40, 42]	[44, 48]	[48, 54]	[0.43, 0.48]	[0.40, 0.43]	[0.36, 0.40]
m = 2	[30, 32]	[32, 35]	[34, 38]	[0.68, 0.78]	[0.60, 0.65]	[0.52, 0.58]
m = 3	[35, 38]	[39, 43]	[40, 45]	[0.38, 0.42]	[0.35, 0.38]	[0.30, 0.35]
m = 4	[32, 34]	[35, 39]	[38, 42]	[0.72, 0.75]	[0.70, 0.73]	[0.68, 0.70]
	Wastewater treatment cost $CM_{mk}^{\pm}$ (Yuan/m^3)			BOD concentration of discharged wastewater $MC_{mk}^{\pm}$ (kg/m^3)		
m = 1	[25, 30]	[20, 25]	[15, 20]	[0.60, 0.65]	[0.55, 0.60]	[0.50, 0.55]
m = 2	[15, 20]	[10, 15]	[5, 10]	[0.70, 0.75]	[0.66, 0.72]	[0.60, 0.70]
m = 3	[22, 27]	[17, 22]	[12, 17]	[0.61, 0.64]	[0.59, 0.62]	[0.57, 0.60]
m = 4	[20, 25]	[15, 20]	[10, 15]	[0.95, 0.98]	[0.92, 0.95]	[0.90, 0.93]
	BOD treatment efficiency $\lambda_{BOD,\ mk}$ (%)					
m = 1	(0.90, 0.01^2)	(0.92, 0.01^2)	(0.95, 0.01^2)			
m = 2	(0.90, 0.01^2)	(0.92, 0.01^2)	(0.95, 0.01^2)			
m = 3	(0.90, 0.01^2)	(0.92, 0.01^2)	(0.95, 0.01^2)			
m = 4	(0.90, 0.01^2)	(0.92, 0.01^2)	(0.95, 0.01^2)			

(6) Planting area constraint:

$$PA^{\pm}_{jk} \le TA^{\pm}_{jk}, \forall j, k \quad (4n)$$

This constraint shows that the planting area is not greater than total tillable area (ha).

(7) Nonnegative constraints:

$$PLI^{\pm}_{ik}, FA^{\pm}_{jk}, PA^{\pm}_{jk}, QM^{\pm}_{mk} \ge 0, \forall i, j, m, k \quad (4o)$$

These constraints ensure that each production activity is positive to obtain feasible solutions.

Therefore, the model can be solved though the solution algorithm as described in the "Methodology" section.

3.2.3. Data preparation

Because this study is presented as a hypothetical water quality management system, the input data was primaryly determined by means of governmental reports and literature review, so as to progressively support water resources planning and pollution control for the purpose of sustainable development. Moreover, for data estimation, they are presented as intervals with upper and lower bounds, and normal distributions with mean values and standard deviations. Tables 1, 2 and 3 provide input parameters of industrial factory, agricultural production and municipal water supply at different planning periods. Table 4 presents wastewater treatment capacity and allowable BOD discharge for different planning periods. Table 5 lists the maximum and minimum production level of industrial factory, agricultural zone and municipal town. Note that all the input data are divided into three periods corresponding to three planning periods. Not all the data have the same changing trends due to the following reasons: (1) some parameters will become larger because of the economic development and technological progress while some parameters will become smaller with consideration of more stringent environmental standards and higher treatment efficiency; (2) some parameters have the same values, potentially indicating their lower uncertainty.

4. Results analysis

To examine and compare the risk of violating the constraints and obtain a range of decision solutions, nine scenarios with the combination of different joint and individual probability levels are provided. Table 6 shows the scenarios at representative joint and individual probability levels. In the developed IJDSCCP model, an increased joint probability level naturally represents a raised risk of violation and decreased strictness for satisfactory level of system constraints, and vice versa (Zeng et al., 2015). First, these scenarios are designed at the following three increased joint probability levels, that is: $p = 0.01$, 0.05 and 0.10. Second, for each joint probability level, three sets of individual probability levels are given: (1) the same levels are defined as scenarios A, i.e. 1A ($p = 0.01$, $p_1 = 0.0033$, $p_2 = 0.0033$ and $p_3 = 0.0033$), 2A and 3A, which basically suggests that violation levels for the chance constraints are the same during the three planning periods; (2) the increased levels are defined as scenarios B, i.e. 1B ($p = 0.01$, $p_1 = 0.002$, $p_2 = 0.003$ and $p_3 = 0.005$), 2B and 3B and (3) the decreased levels are similarly defined as scenarios C, i.e. 1C ($p = 0.01$, $p_1 = 0.005$, $p_2 = 0.003$ and $p_3 = 0.002$), 2C and 3C.

Table 4
Wastewater treatment capacity and allowable BOD discharge for different planning periods.

Factory	$k = 1$	$k = 2$	$k = 3$	$k = 1$	$k = 2$	$k = 3$
	Wastewater treatment capacity $IPC^{\pm}_{ik}$ (m^3/day)			Allowable BOD discharge ABI_{ik} (kg/day)		
$i = 1$	[450, 500]	[550, 600]	[650, 700]	(100, 5^2)	(110, 5^2)	(115, 5^2)
$i = 2$	[400, 450]	[450, 500]	[500, 550]	(55, 5^2)	(57, 5^2)	(62, 5^2)
$i = 3$	[270, 320]	[300, 350]	[330, 380]	(50, 5^2)	(55, 5^2)	(60, 5^2)
Town	$k = 1$	$k = 2$	$k = 3$	$k = 1$	$k = 2$	$k = 3$
	Wastewater treatment capacity $MPC^{\pm}_{mk}$ (m^3/day)			Allowable BOD discharge ABM_{mk} (kg/day)		
$m = 1$	[350, 400]	[400, 450]	[450, 500]	(26, 3^2)	(28, 3^2)	(30, 3^2)
$m = 2$	[450, 500]	[700, 750]	[850, 900]	(50, 3^2)	(52, 3^2)	(55, 3^2)
$m = 3$	[300, 350]	[350, 400]	[400, 450]	(35, 3^2)	(38, 3^2)	(40, 3^2)
$m = 4$	[550, 600]	[750, 800]	[950, 1000]	(60, 3^2)	(62, 3^2)	(65, 3^2)

Table 5
The maximum and minimum production level of industrial factory, agricultural zone and municipal town.

Location	Minimum	Maximum
Industrial factory (ton/day)		
$i = 1$	800	1200
$i = 2$	500	900
$i = 3$	800	1500
Agricultural zone (ha)		
$j = 1$	1000	1400
$j = 2$	1500	2000
$j = 3$	1850	2500
$j = 4$	1000	1800
Municipal town (m^3/day)		
$m = 1$	500	1200
$m = 2$	600	1800
$m = 3$	900	1500
$m = 4$	500	1600

4.1. Optimal solutions of industrial production, agricultural planting area and municipal water supply

The IJDSCCP method can effectively address both interval and probabilities uncertainties, and thus the solutions will include combinations of distributional information, interval and deterministic. The interval solutions can provide a range of decision space for managers, which will help them identify desired alternatives as well as facilitate further analyses of tradeoffs between system benefits and system-failure risks. The distributional information can demonstrate the variations of violation risk levels under different joint and individual probability levels. Fig. 3a–c shows the solutions of scenarios 1A, 1B and 1C. Among them, Fig. 3a presents the obtained results of industrial production levels in three scenarios, including interval and deterministic values. Overall for each scenario, the results would increase during three planning periods. Taking scenario 1A as an example, the amount of industrial production in factory 1 ($i = 1$) would be [642.9, 833.3] ton/day ($k = 1$), [846.2, 1090.9] ton/day ($k = 2$) and [1083.3, 1200.0] ton/day ($k = 3$) during the three planning periods. Similarly, those in factory 2 ($i = 2$) would be [452.0, 538.0] ton/day, [669.2. 809.7] ton/day and 900 ton/day in periods 1–3. Those in factory 3 ($i = 3$) would be [588.8, 867.7] ton/day, [857.1, 1400.0] ton/day and [1100.0, 1500.0] ton/day in periods 1–3, respectively. Additionally, it is noted that there is a deterministic value

Table 6
Scenarios at representative joint and individual probability levels.

Scenarios	p	p_1	p_2	p_3
1A	0.01	0.0033	0.0033	0.0033
1B	0.01	0.002	0.003	0.005
1C	0.01	0.005	0.003	0.002
2A	0.05	0.0166	0.0166	0.0166
2B	0.05	0.005	0.015	0.03
2C	0.05	0.03	0.015	0.005
3A	0.10	0.033	0.033	0.033
3B	0.10	0.02	0.03	0.05
3C	0.10	0.05	0.03	0.02

because both the lower and upper bounds approach the maximum industrial production level. Fig. 3c presents similar results for the municipal water supply plans in three scenarios. Generally, for the solutions of industrial production and municipal water supply, the overall increasing trends are mainly due to the assumed increase of waste treatment capacity and the amount of allowable BOD discharge (i.e. right-hand side constraints) but the decrease of wastewater generation rate and BOD concentration of discharged wastewater (i.e. left-hand side constraints). This has important practical implications for sustainable water resources management as these parameters are highly associated with continuous improvements of environmental standards and wastewater treatment technology over planning periods. However, the results of agricultural planting area show a different variation trend (see Fig. 3a). Overall for each scenario, the results, especially for the lower

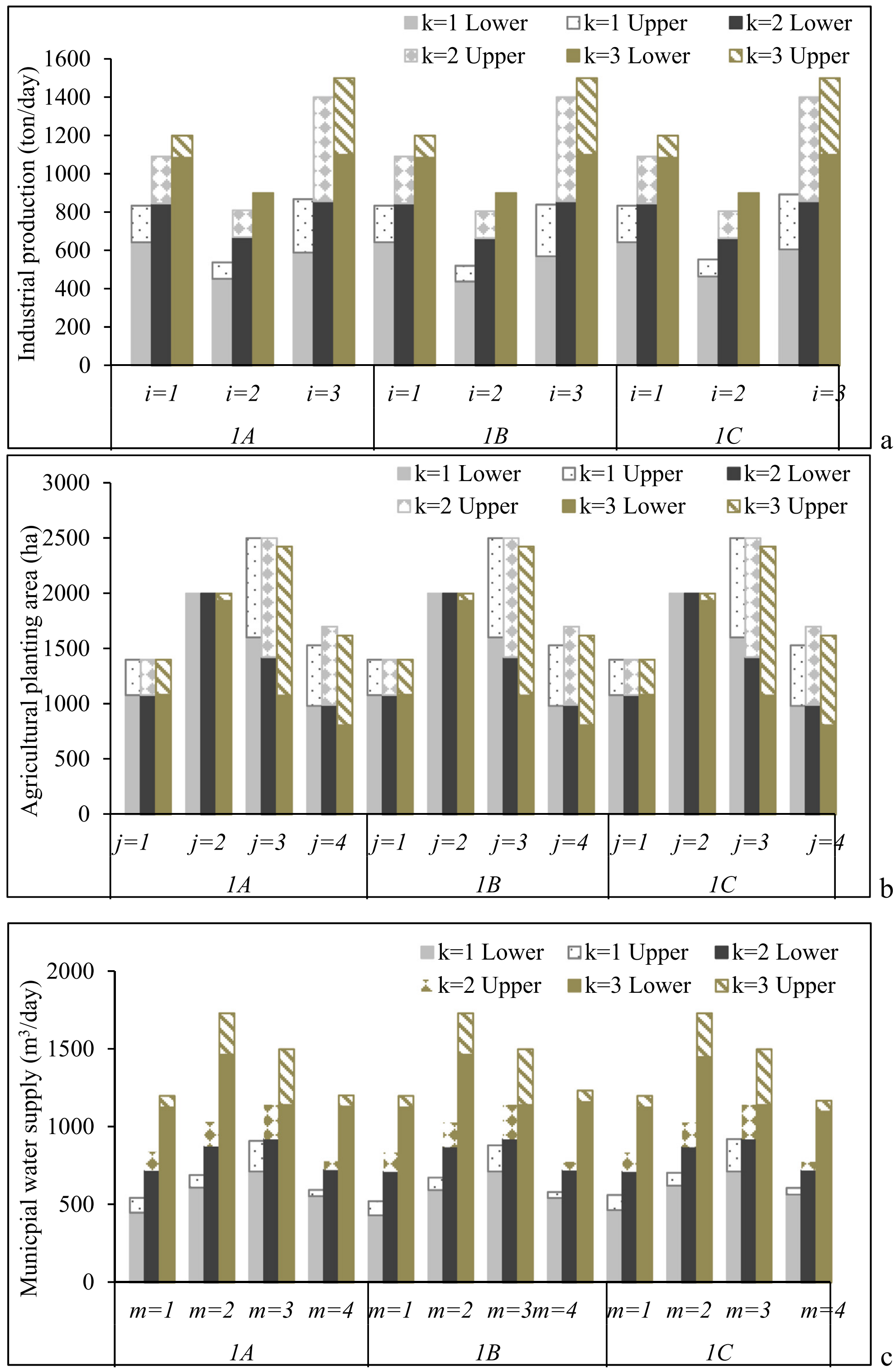

Fig. 3. Panels a, b, and c. Optimal solutions in the scenarios 1A, 1B and 1C during the three planning periods.

bounds, would decrease over the three planning periods. For example, in scenario 1A, agricultural planting area for zone 3 (j = 3) are [1601.9, 2500.0] ha, [1423.6, 2500.0] ha and [1075.7, 2424.2] ha in periods 1–3. This is mainly driven by the assumed decrease of the maximum allowable nitrogen loss in agricultural zone j during period k (i.e. $MNL_{jk}^{\pm}$) while the tillable area is kept the same during the three planning periods (i.e. $TA_{jk}^{\pm}$). These decreasing trends are likely to link to the national policy of returning farmland to forest.

By contrast, in scenarios 1A, 1B and 1C at the same joint probability, almost all the results during three planning periods are the same, especially in the third periods because of system constraints, that is, the maximum production level and municipal water supply amount are

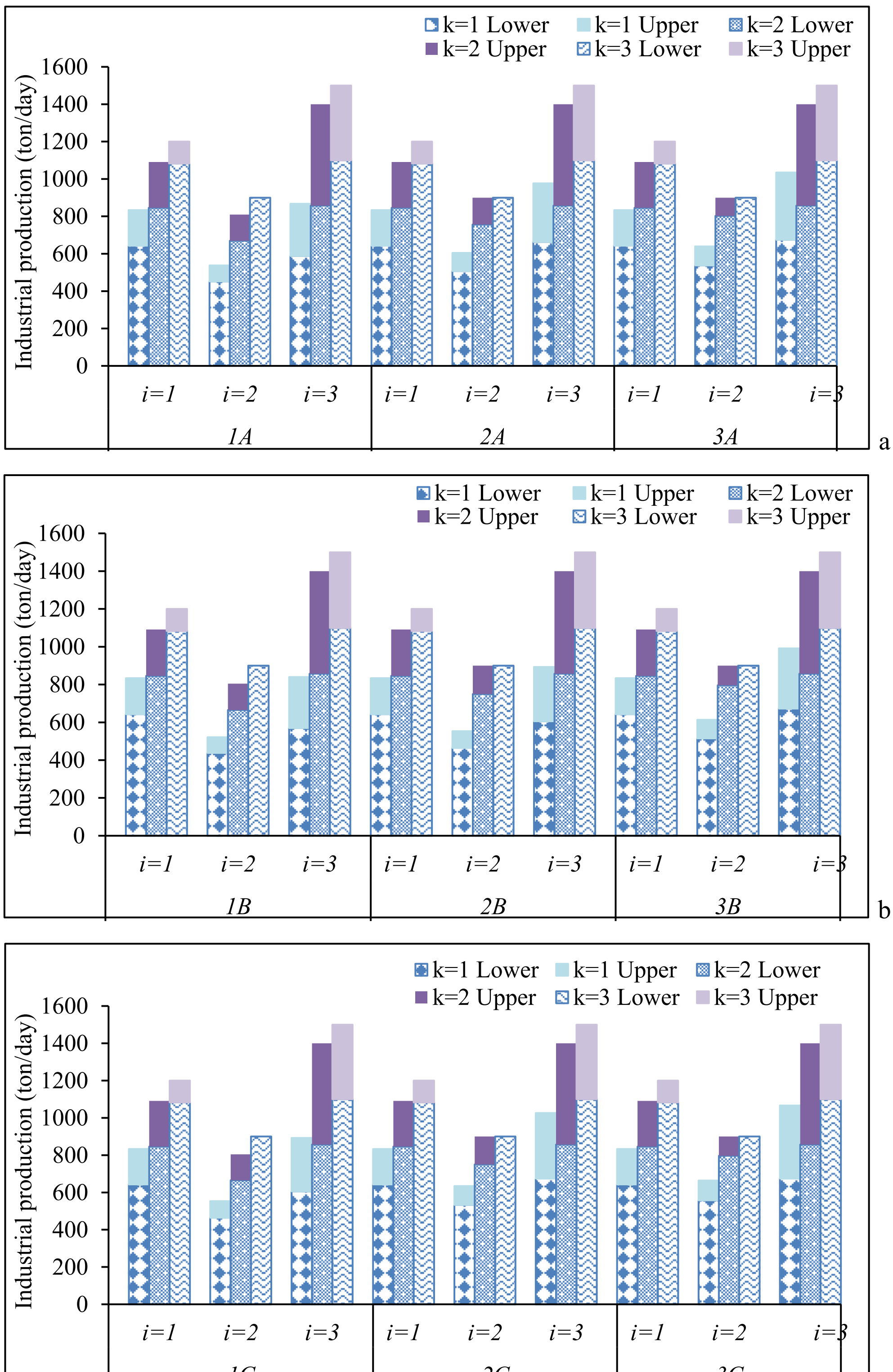

Fig. 4. Panels a, b, and c. Solutions of industrial production in three periods at different joint probability levels.

fixed. For example, in scenario 1A for factory 2, the solutions are [452.0, 538.0] ton/day, [669.2. 809.7] ton/day and 900 don/day in periods 1–3. The results in scenario 1B are [437.8, 521.0] ton/day, [665.0, 804.7] ton/day and 900.0 ton/day and in scenario 1C are [464.8, 553.2] ton/day, [665.0, 804.7] ton/day and 900 ton/day in three planning periods. Meanwhile, the amount of municipal water supply for town 2 are [609.1, 691.0] m^3/day, [873.8, 1032.7] m^3/day and [1465.5, 1730.8] m^3/day in scenario 1A, [594.7, 674.6] m^3/day, [869.6, 1027.7] m^3/day and [1465.5, 1730.8] m^3/day in scenario 1B as well as [622.1, 705.7] m^3/day, [869.6, 1027.7] m^3/day and [1449.1, 1730.8] m^3/day in scenario 1C, respectively. The results in scenario 1C are not less than that of scenario 1B, which indicates that the different sets of individual probability have different effects on solutions. Moreover, for other cases, the results may present uncertain trends in three scenarios. For instance, the solutions of factory 1 among three scenarios and planning periods are kept the same, that is, [642.9, 833.3] ton/day, [846.2, 1090.9] ton/day and [1083.3, 1200.0] ton/day. This indicates that the effects of individual probability levels on the results may be uncertain even at the same joint probability level. Additionally, the solutions of agricultural planting area in scenarios 1A, 1B and 1C are kept the same since there is no consideration of the random uncertainty and chance constraints in the agricultural water quality management system.

Fig. 4a–c presents the solutions of industrial production in three periods at different joint probability levels (e.g. scenarios 1A, 2A, and 3A) resulting from the IJDSCCP model. As a whole, optimal solutions at each scenario (i.e. from 1A to 3C) for each factory would increase during periods 1–3. The reason for this has already been mentioned above. In comparison, based on the solutions of scenarios 1A–3A, 1B–3B and 1C–3C at the same individual probability settings but increased joint probability levels, it is clear that both the lower and upper bounds would be higher at a higher joint probability level. Taking factory 2 as an example, the solutions in scenario 1B are [437.8, 521.0] ton/day, [665.0, 804.7] ton/day and 900.0 ton/day in periods 1–3. While those results in scenarios 2B and 3B are [464.8, 553.2] ton/day, [750.1, 900.0] ton/day and 900 ton/day and [515.3, 613.2] ton/day, [796.3, 900.0] ton/day and 900 ton/day, respectively. Obviously, a higher joint probability level corresponds to these increasing trends of industrial productions. Similarly, optimal solutions of municipal water supply among four towns show the same trends. In other words, the above results indicate that joint probability level would have more impacts on the solutions than individual probability levels. In general, an increased joint probability level means an increased admissible risk of violating the system constraints and a decreased satisfaction degree level of the constraints, which will naturally result in a decreased strictness for the constraints and a more relaxed decision domain (i.e. smaller left-hand side constraints while bigger right-hand side constraints) and a lower system reliability level (Zhu and Huang, 2011; Sun et al., 2013).

Table 7
Seven scenarios at the same joint probability level $p = 0.01$.

Scenarios	p	p_1	p_2	p_3
1A	0.01	0.0033	0.0033	0.0033
1B	0.01	0.002	0.003	0.005
1C	0.01	0.005	0.003	0.002
1D	0.01	0.003	0.005	0.002
1E	0.01	0.003	0.002	0.005
1F	0.01	0.002	0.005	0.003
1G	0.01	0.005	0.002	0.003

4.2. System benefits

Optimal solutions for different representative scenarios corresponding to the combinations of joint probability and individual probability levels are different, thereby leading to different system benefits. The objective function is to maximize the system benefits over the planning periods. Fig. 5 presents system benefits at different probability levels from scenarios 1A to 3C. As optimal solutions have obviously increasing trends along with increasing joint probability levels, both the upper and lower bounds of system benefits would accordingly increase as well. For example, system benefits resulting from the IJDSCCP model would be [13,841.68, 21,801.81] × 10^6 Yuan in scenario 1A, while those in scenarios 2A and 3A would be [14,150.26, 22,260.06] × 10^6 Yuan and [14,280.55, 22,415.52] × 10^6 Yuan, respectively. Generally, higher joint probability level lead to higher system benefits, indicating joint probability level has a significant effect on the system benefits. When the joint probability level is raised, it is expected that the decision domain is expanded to easily achieve more system benefits but an increased constraint-violation risk should be noted. Therefore, the results can support in-depth analysis of the interrelationships among system benefits, violation probability level and constraints-violation risk level.

There is an important distinction among these three scenarios, that is, the same, increased and decreased individual probability levels. This means that violation levels for the chance constraints are the same (scenario 2A), increasing (scenario 2B) and decreasing (scenario 2C) over the first, second and third planning periods accordingly. For the objective functions at the same joint probability level with three sets of individual probability levels (e.g. scenarios 2A, 2B and 2C), it can be clearly seen that the biggest system benefits are achieved in scenario 2C (i.e. [14,238.82, 22,373.01] × 10^6 Yuan), followed by 2A (i.e. [14,150.26, 22,260.06] × 10^6 Yuan) and 2B (i.e. [14,014.28, 22,055.04] × 10^6 Yuan). The results indicate that violation level of the first period plays a major role in overall system benefits because the individual probability level of scenario 2C is $p_1 = 0.03$, $p_2 = 0.015$ and $p_3 = 0.005$ ($p = 0.05$). Moreover, the difference values among these scenarios are [88.56, 112.95] × 10^6 Yuan between scenarios 2C

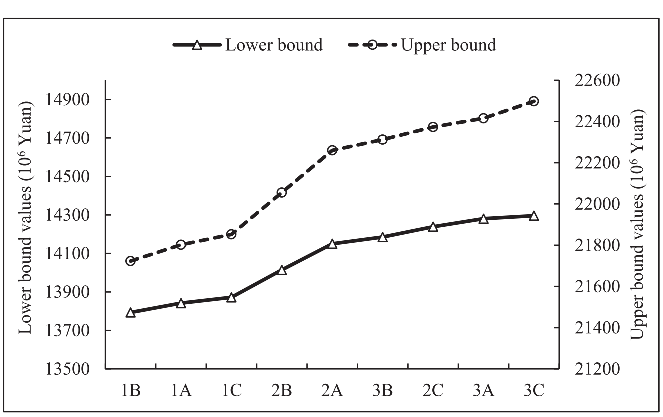

Fig. 5. System benefits at different probability levels from scenarios 1A to 3C.

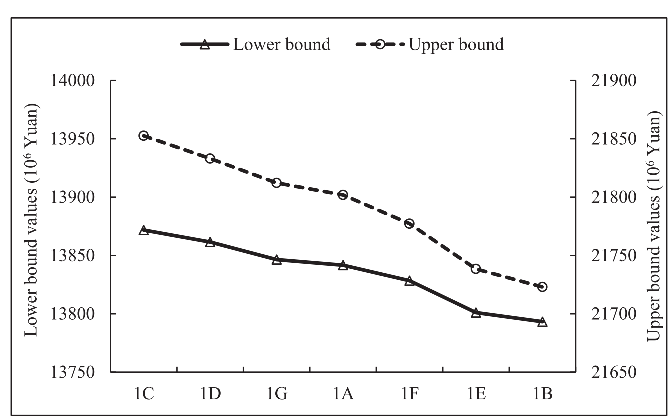

Fig. 6. System benefits resulting from scenarios 1A to 1G.

Table 8
Solutions of the ILP.

	k = 1	k = 2	k = 3
Factory	Industrial production levels (ton/day)		
$i = 1$	[637.8, 833.3]	[846.2, 1090.9]	[1083.3, 1200.0]
$i = 2$	[399.3, 475.2]	[588.3, 711.8]	900.0
$i = 3$	[517.6, 762.8]	[857.1, 1400.0]	[1100.0, 1500.0]
Zone	Agricultural planting area (ha)		
$j = 1$	[1079.0, 1400.0]	[1080.7, 1400.0]	[1083.2, 1400.0]
$j = 2$	2000.0	2000.0	[1932.4, 2000.0]
$j = 3$	[1601.9, 2500.0]	[1423.6, 2500.0]	[1075.7, 2424.2]
$j = 4$	[980.9, 1530.1]	[991.5, 1699.2]	[806.2, 1617.8]
Town	Municipal water supply (m^3/day)		
$m = 1$	[384.6, 465.2]	[615.6, 722.0]	[1091.0, 1200.0]
$m = 2$	[555.6, 630.3]	[791.9, 935.9]	[1330.1, 1730.8]
$m = 3$	[697.6, 808.9]	[921.1, 1142.9]	[1142.9, 1500.0]
$m = 4$	[510.2, 548.3]	[661.6, 712.5]	[1013.9, 1078.5]

and 2A, and [135.98, 205.02] × 10^6 Yuan between scenarios 2A and 2B. By contrast, these difference values are [130.29, 155.46] × 10^6 Yuan between scenarios 3A and 2A, and [308.58, 458.25] × 10^6 Yuan between scenarios 2A and 1A. Such a difference shows that joint probability levels have more impacts on the system benefits than individual probability levels.

There is an interesting phenomenon that system benefit in scenario 2C ([14,238.82, 22,373.01] × 10^6 Yuan) is slightly greater than that in scenario 3B ([14,185.13, 22,312.22] × 10^6 Yuan). Strictly speaking, a higher joint probability level would naturally lead to higher system benefits. Closer examination of the scenario setting reveals that the first individual probability level of scenario 2C ($p_1 = 0.03$) is greater than 3B ($p_1 = 0.02$). This can be further explained by the system constraints of the first planning period are so stringent that even minor changes in the violation probability will result in the disturbance of the results. However for the third period, its system constraints may have less impacts on the final solutions and system benefits due to more relaxed decision domain.

In summary, the above analysis shows that the IJDSCCP model can effectively reflect the uncertainty in water resources planning and pollution control of the water environment system, and thus obtain a series of optimal solutions containing potential options for industrial production, agricultural planting and municipal water supply. From the methodological perspective, the model can readily address random and interval uncertainties, which means that comparisons can be made under different violation-constraint risk levels when uncertain parameters can be described as double-sided randomness and discrete intervals. In addition, it can effectively reflect the interrelationships among multiple constraints during the planning periods. More importantly, from the practical perspective, managers can determine the final solutions in the form of the interval according to the actual situations. An acceptable violation-constraints risk level should be discussed and decided by stakeholders based on the specific system conditions (e.g. system benefits, system failure risks and watershed pollution status). Therefore, the obtained solutions can better assist in managing the water quality problems and further support sustainable development of water resources.

5. Discussion

5.1. Scenario analysis of the same joint probability level

To investigate the effect of the different individual probability levels at the same joint probability level on the constraints and solutions, Table 7 lists the seven scenarios at the same joint probability level $p = 0.01$. Based on the results of permutations and combinations, there are six sets of individual probability levels at the same joint probability level due to three planning periods. Moreover, there is one more scenario where individual probability levels are equally kept all the same as one third of the joint probability level. Therefore, Fig. 6 shows the system benefits resulting from scenarios 1A to 1G. As shown in the figure, the variations of the individual probability level may lead to different results, indicating the sensitivity of uncertain information. The results show that scenario 1C with a decreased individual probability level would lead to the maximum system benefits (i.e. [13,871.76, 21,852.52] × 10^6 Yuan) while an increased one results in the minimum system benefits (i.e. [13,793.21, 21,722.95] × 10^6 Yuan). Additionally, system benefits in scenario 1A are in the middle of the seven scenarios. Therefore, three sets of individual probability level (e.g. A, B and C) are verified as the representative scenarios. The above results can provide a decision domain of the system benefits for managers and then assist them in identifying the solutions based on their risk preferences and judgments.

5.2. Effectiveness and reliability of the model

In contrast, for the violation probability $p_k = 0$ without relaxation of the constraints, the IJDSCCP model can be transformed into a general interval linear programming (ILP) model. Table 8 shows the solutions of the ILP. Obviously, it can only provide a single solution under an extreme scenario of system conditions. Moreover, the system benefits of the ILP are [13,530.00, 21,339.64] × 10^6 Yuan, which is slightly lower than that of the IJDSCCP model. This is mainly because the conservative attitudes towards the system conditions without constraints-violation risks would narrow down decision space (e.g. decreased treatment capacity and/or increased waste generation rates). Therefore, there is an advantage that the IJDSCCP model can generate more flexible solutions corresponding to different predefined scenarios. Compared with the ILP model, the distinction between them is that the IJDSCCP model can reflect more information about the tradeoffs among system benefits, violation-constraints risk levels and system reliability levels. The above analysis demonstrates that the IJDSCCP model can provide more reliable solutions under different violation probability levels, corresponding to their specific risk levels.

The model can also be solved through a deterministic model by letting random uncertain parameters be equal to their mean values without standard deviations and averaging the lower and upper bounds of interval values. That means that all the input parameters associated with the objective function and the constraints are replaced by deterministic values in the model, which can be readily solved by conventional linear programming methods (Zhang et al., 2009). Therefore, Table 9 shows the solutions through solving the deterministic model. Accordingly, the results are fixed values rather than interval values, which can be regarded as a special case in the solutions of the IJDSCCP

Table 9
Solutions of the deterministic model.

	k = 1	k = 2	k = 3
Factory		Industrial production levels (ton/day)	
$i = 1$	800.0	958.3	1200.0
$i = 2$	739.1	900.0	900.0
$i = 3$	842.9	1083.3	1420.0
Zone		Agricultural planting area (ha)	
$j = 1$	1400.0	1400.0	1400.0
$j = 2$	2000.0	2000.0	2000.0
$j = 3$	2159.1	2036.4	1850.0
$j = 4$	1227.5	1300.7	1143.9
Town		Municipal water supply (m^3/day)	
$m = 1$	824.2	1024.1	1200.0
$m = 2$	678.6	1160.0	1590.9
$m = 3$	900.0	1027.4	1307.7
$m = 4$	782.3	1083.9	1413.0

model. Moreover, the objective function value is $18{,}744 \times 10^6$ Yuan and thus the decision alternative will be restricted to a single solution. Obviously, replacing interval and random uncertainties with their corresponding deterministic values will undoubtedly lose some uncertain but valuable information, thereby reducing the effectiveness and flexibility of the obtained solutions (Dai et al., 2014). Thus, compared with the deterministic model, the effectiveness of the IJDSCCP model is expected to be increased.

5.3. Controllability and efficiency of the model

In this study, the basic principle to address the double-sided randomness of the constraints is the approximation of nonlinear expressions (Sun et al., 2013). Namely, the IJDSCCP model is solved through substituting the nonlinear form of the constraints with the corresponding non-equivalent but liner ones. The solution process is also mathematically proved based on the Theorems 1 and 2. Therefore, the controllability of such a general interval-based nonlinear programming model can be highly strengthened due to its approximated linearization form. It can be readily solved by conventional interval mathematical programming with two deterministic submodels. Moreover, the main purpose of approximation is to simplify computational process and improve computational efficiency and the obtained results are almost same (Zhang et al., 2018). However, it should be noted that the original decision space may be narrowed down because this is a sufficient but not necessary linearization form of the IJDSCCP model, resulting in systematic errors. In spite of that, it is quite an innovative and effective method to solve the IJDSCCP model.

6. Conclusions

In this study, an inexact joint-probabilistic double-sided stochastic chance-constrained programming (IJDSCCP) model is developed for sustainable water resources planning and pollution control in water quality management problems. This approach is a hybrid of interval parameter programming, joint-probabilistic programming and double-sided stochastic chance-constrained programming. To reduce computational burden, a non-equivalent but sufficient linearization form of the IJDSCCP is provided and proved in a straightforward manner. It can deal with uncertainties including interval and double-sided randomness. Moreover, it is capable of reflecting chance-constraints with double-sided randomness and interrelationships with dynamic feature among multiple constraints during three planning periods.

The developed model is then tested by a representative case for water quality management. Based on the results, the dynamic interactions among pollutant loading and water environments are reflected during three planning periods. The results including water resources planning solutions, pollution control plans and system benefits under the combinations of different joint and individual probability levels can readily facilitate further analysis and comparisons. It is noted that the assumed amount of wastewater discharge is decreased and wastewater treatment efficiency and capacity are increased for the purpose of sustainable development. Although this study is presented for water quality and water environmental system management, the novel model can also be extended into many other environmental problems under uncertainties. However, the applicability of model should be enhanced by integrating fuzzy set theory where fuzzy information exists. Because water quality management usually involves various physical processes, additional parameter and optimization of the water quality simulation model should be incorporated into the system optimization model to further demonstrate its utility.

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Appendix I. The derivation process of linearization form of IJDSCCP

Theorem 1. *Eq. (2b) is equivalent to Eq. (5a). That is, assume that the stochastic vectors a_{ij} and b_i pertain to be independently normally distributed random variables, then Eq. (2b) can be hold if and only if*

$$\sum_{j=1}^{n} \mu_{a,ij} x_j + \Phi^{-1}(1-p_i)\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2} \le \mu_{b,i}, i = 1, 2, \cdots, m \tag{5a}$$

where Φ is the standardized normal distribution function.

Proof. Since a_{ij} and b_i are independently random variables following normal distributions, the variable: $y_i = \sum_{j=1}^{n} a_{ij} x_j - b_i, i = 1, 2, \cdots, m$ is also a normally distributed random variable with the following expected value and variance:

$$\mu_{y,i} = \sum_{j=1}^{n} \mu_{a,ij} x_j - \mu_{b,i}$$

$$\sigma_{y,i}^2 = \sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2 \tag{5c}$$

Then, the quantity: $\frac{\sum_{j=1}^{n} a_{ij} x_j - b_i - \left(\sum_{j=1}^{n} \mu_{a,ij} x_j - \mu_{b,i}\right)}{\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2}}$ must follow a standardized normal distribution.

Then, the inequality $\sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1, 2, \cdots, m$ is equivalent to:

$$\frac{\sum_{j=1}^{n} a_{ij} x_j - b_i - \left(\sum_{j=1}^{n} \mu_{a,ij} x_j - \mu_{b,i}\right)}{\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2}} \le -\frac{\left(\sum_{j=1}^{n} \mu_{a,ij} x_j - \mu_{b,i}\right)}{\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2}} \tag{5d}$$

Thus, the chance constraint Eq. (2b) can be converted into:

$$\Pr\left\{ z \le -\frac{\left(\sum_{j=1}^{n} \mu_{a,ij} x_j - \mu_{b,i}\right)}{\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2}} \right\} \ge 1 - p_i, i = 1, 2, \cdots, m \tag{5e}$$

where z is defined as the random variable that follows a standardized normal distribution. Therefore, the chance constraint Eq. (5e) can be hold if and only if

$$\Phi^{-1}(1-p_i) \le -\frac{\left(\sum_{j=1}^{n} \mu_{a,ij} x_j - \mu_{b,i}\right)}{\sqrt{\sum_{j=1}^{n} \sigma_{a,ij}^2 x_j^2 + \sigma_{b,i}^2}}, i = 1, 2, \cdots, m \tag{5f}$$

That is, the deterministic equivalent of chance constraint is Eq. (5a). Theorem 1 is thus proved.

Since Eq. (5a) is nonlinear, it will inevitably increase computational burden when solving problems. As an alternative solution method, an approximated linearization form of Eq. (5a) is proposed (see Theorem 2).

Theorem 2. *Eq. (6a) is a sufficient condition for Eq. (6a) when $p_i \le 0.5$ and $x_j \ge 0$. That is,*

$$\sum_{j=1}^{n}\mu_{a,ij}x_j+\Phi^{-1}(1-p_i)\left(\sum_{j=1}^{n}\sigma_{a,ij}x_j+\sigma_{b,i}\right)\le\mu_{b,i}, i=1,2,\cdots,m \tag{6a}$$

Proof. Based on the following inequality,

$$\sqrt{\sum_{j=1}^{n}(t)^2}\le\sum_{j=1}^{n}(t), t\in R, t\ge 0 \tag{6b}$$

when $\left(\sum_{j=1}^{n}\sigma_{a,ij}x_j+\sigma_{b,i}\right)\in R$ and $\left(\sum_{j=1}^{n}\sigma_{a,ij}x_j+\sigma_{b,i}\right)\ge 0$, we have

$$\sqrt{\sum_{j=1}^{n}\sigma_{a,ij}^2x_j^2+\sigma_{b,i}^2}\le\left(\sum_{j=1}^{n}\sigma_{a,ij}x_j+\sigma_{b,i}\right) \tag{6c}$$

Moreover, when $(1-p_i)\ge 0.5$, we have

$$\Phi^{-1}(1-p_i)\ge 0 \tag{6d}$$

Thus, from inequalities Eqs. (6c) and (6d), we have

$$\sum_{j=1}^{n}\mu_{a,ij}x_j+\Phi^{-1}(1-p_i)\sqrt{\sum_{j=1}^{n}\sigma_{a,ij}^2x_j^2+\sigma_{b,i}^2}\le \sum_{j=1}^{n}\mu_{a,ij}x_j+\Phi^{-1}(1-p_i)\left(\sum_{j=1}^{n}\sigma_{a,ij}x_j+\sigma_{b,i}\right), i=1,2,\cdots,m \tag{6e}$$

Based on the above analysis, if inequality Eq. (6a) holds (sufficient condition), then from inequality Eq. (6e), we have Eq. (5a).

Therefore, Eq. (2b) can be transformed into a non-equivalent but sufficient linearization form in Eq. (7a) according to Theorems 1 and 2.

$$\sum_{j=1}^{n}\mu_{a,ij}x_j+\Phi^{-1}(1-p_i)\left(\sum_{j=1}^{n}\sigma_{a,ij}x_j+\sigma_{b,i}\right)\le\mu_{b,i}, i=1,2,\cdots,m \tag{7a}$$

Appendix II. Solutions for solving the linearization form of IJDSCCP

An interactive algorithm method is used to obtain the upper and lower bounds submodels of the linearization form of IJDSCCP model. Because the objective f^+ corresponds to the upper bound objective function value, thus the following submodel can be reformulated:

$$\max f^+=\sum_{j=1}^{j_1}c_j^+x_j^++\sum_{j=j_1+1}^{n}c_j^+x_j^- \tag{8a}$$

subject to:

$$\left(\sum_{j=1}^{j_1}\mu_{a,ij}x_j^++\sum_{j=j_1+1}^{n}\mu_{a,ij}x_j^-\right)+\Phi^{-1}(1-p_i)\left(\left(\sum_{j=1}^{j_1}\sigma_{a,ij}x_j^++\sum_{j=j_1+1}^{n}\sigma_{a,ij}x_j^-\right)+\sigma_{b,i}\right)\le\mu_{b,i}, i=1,2,\cdots,m$$

$$\sum_{i=1}^{m}p_i\le p$$

$$a_{ij}(\omega)\sim N\left(\mu_{a,ij},\sigma_{a,ij}^2\right)$$

$$b_i(\xi)\sim N\left(\mu_{b,i},\sigma_{b,i}^2\right)$$

$$x_j^+\ge 0, j=1,2,\cdots,j_1$$

$$x_j^-\ge 0, j=j_1+1,j_1+2,\cdots,n \tag{8g}$$

Therefore, optimal solutions of the upper bound submodel can be obtained, including $x_{j,\,opt}^+$ for $j=1$ to j_1, $x_{j,\,opt}^-$ for $j=j_1+1$ to n and f_{opt}^+. Accordingly, the following lower bound submodel corresponds to f^- can be reformulated:

$$\max f^-=\sum_{j=1}^{j_1}c_j^-x_j^-+\sum_{j=j_1+1}^{n}c_j^-x_j^+ \tag{9a}$$

subject to:

$$\left(\sum_{j=1}^{j_1}\mu_{a,ij}x_j^-+\sum_{j=j_1+1}^{n}\mu_{a,ij}x_j^+\right)+\Phi^{-1}(1-p_i)\left(\left(\sum_{j=1}^{j_1}\sigma_{a,ij}x_j^-+\sum_{j=j_1+1}^{n}\sigma_{a,ij}x_j^+\right)+\sigma_{b,i}\right)\le\mu_{b,i}, i=1,2,\cdots,m$$

$$\sum_{i=1}^{m}p_i\le p$$

$$a_{ij}(\omega)\sim N\left(\mu_{a,ij},\sigma_{a,ij}^2\right)$$

$$b_i(\xi)\sim N\left(\mu_{b,i},\sigma_{b,i}^2\right)$$

$$x_j^-\le x_{j,opt}^+, j=1,2,\cdots,j_1$$

$$x_j^+\ge x_{j,opt}^-, j=j_1+1,j_1+2,\cdots,n \tag{9g}$$

Optimal solutions of $x_{j,\,opt}^-$ for $j=1$ to j_1, $x_{j,\,opt}^+$ for $j=j_1+1$ to n and f_{opt}^- can be obtained. By combining the solutions of the upper and lower bounds of submodels, the final solutions of the IJDSCCP model are presented as a set of interval values: $f_{opt}^\pm=[f_{opt}^-, f_{opt}^+]$ and $x_{j,\,opt}^\pm=[x_{j,\,opt}^-, x_{j,\,opt}^+]$.

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