An interval multi-objective fuzzy-interval credibility-constrained nonlinear programming model for balancing agricultural and ecological water management

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ABSTRACT

This study presents an interval multi-objective fuzzy-interval credibility-constrained nonlinear programming (IMFICNP) model combined with spatial water requirement of ecological vegetation (SEWR) estimation for solving the problem of allocation of agricultural and ecological water in irrigation districts under uncertainties. Through techniques of remote sensing (RS) and geographic information system (GIS), the ecological vegetation is subdivided into three types including forest land, grassland and shrubland and the water requirement of ecological vegetation is extended from site-specific sample to spatial decision-making unit (DMU), which provides a set of spatial data for input parameters of constraints. The IMFICNP model can be formulated through combination of interval parameter programming, multi-objective programming and fuzzy-interval credibility-constrained programming, which can handle the conflicts of multiple objectives under uncertainties such as single uncertainty (interval and fuzzy parameters) and dual-uncertainties (fuzzy-interval sets), and finally generate optimal water allocation schemes for crop and ecological vegetation under different credibility levels. The interval quadratic crop water production functions (IQCWPFPs) are introduced to express the nonlinear relationships between crop yield and irrigation amount. Then, this model is applied to a case study of Huangyang Irrigation District (HID) in Shiyang River Basin to demonstrate its applicability. The results indicate that a higher credibility level is accompanied by less amount of water allocation and lower system benefit. The amount of water allocation at the DMU is dominated by planting area of crops and ecological vegetation, but there are few exceptions that optimal solutions are determined by the economic value. In addition, SEWR enables to reflect spatial heterogeneity of the DMU at a larger scale. IMFICNP model can coordinate conflicts among multiple objectives and it can tackle the violation of system constraints with fuzzy-interval sets. Therefore, these results can effectively balance the agricultural and ecological water management in irrigation districts, and provide valuable basis for the sustainable development of arid and semi-arid areas.

1. Introduction

Global water resources are under the crisis of water shortages due to negative effects of global warming, rapid population growth, serious ecological pollution and industrial market expansions (Cheng et al., 2014; He and Winde, 2018). For example, in arid and semi-arid areas of Northwest China, the problems of water shortages and low water use efficiency exacerbate the contradictions among agricultural irrigation, industrial production, municipal use and ecological water requirement, which limits the development of regional agriculture and industries (Han and Zhang, 2013; Ren et al., 2016; Wang et al., 2016; Zhang et al., 2018a). Obviously, the fragile and unbalanced ecological environment may cause severe consequences such as land desertification, desert locust infestation and even huge sandstorm (D’Odorico et al., 2013; Showler et al., 2021; Wang et al., 2021a, 2021b). Therefore, how to coordinate different competing water users and allocate the limited water to agricultural irrigation and ecological vegetation rationally to ensure the sustainable development of agricultural farmlands and...
ecological environments, have become a major challenge for decision makers and an urgent aim of China’s 14th Five-Year Plan (Cai and Rosegrant, 2004; Wang et al., 2021a, 2021b).

Optimization water allocation model is an effective tool to provide reasonable water allocation scheme. In fact, water allocation requires to consider different perspectives and positions of decision makers, which are often in conflicts and hard to coordinate. Therefore, multi-objective optimization model has been proven to coordinate various factors and providing optimal solutions in water resources management. For example, Zhang et al. (2019) established a model of irrigation water management considering the conditions of canal system, and optimized three objectives of economic benefits, canal water seepage and water allocation in multiple irrigation districts. Li et al. (2019) studied optimal allocation of water, energy and land resources based on six objectives: economy, environment, energy, food safety, resource allocation and land policy. In addition, uncertain information often appears in the process of water resources management such as water supply, water requirement of crops and vegetation, crop prices and production costs (Marques et al., 2005; Zhang and Guo, 2017; Guo et al., 2019). Thus, uncertain methods including interval parameter programming, fuzzy mathematical programming and stochastic mathematical programming are used to describe uncertainty and solve them (Maqsood et al., 2005; Nikoo et al., 2012; Kaviani et al., 2015; Xu et al., 2019; Dadmand et al., 2020; Lalehzari and Kerachian, 2021). For instance, Zhang et al. (2015) proposed water management and pollution control schemes based on fuzzy credibility-constrained programming to support the allocation of surface water and groundwater. Lu et al. (2016) formulated a credibility-constrained optimization model to measure a fuzzy event, and the model was applied to agricultural irrigation management in south central China. Xu et al. (2020) established a stochastic programming model with water scarcity risk, upstream flood risk and downstream flood risk as objectives to obtain robust risk-averse plans for multisreservoir system under multiple risks. Among above methods, fuzzy credibility-constrained programming can tackle violated constraints that they are satisfied at a predefined credibility level in a fuzzy environment. However, in real-world problems, single uncertainty (e.g. interval and fuzzy values) is difficult to cope with complex parameters. For example, a triangular fuzzy number can be presented as $b = (b_1, b_2, b_3)$, where $b_1$ is the least possible value, $b_2$ is the main value and $b_3$ is the highest possible value. In some cases, when the boundary of fuzzy sets is unclear and $b_1$, $b_2$ and $b_3$ cannot be specified as deterministic value but with a range between the upper and lower bound, thus fuzzy-interval sets, as a type of dual-uncertainties, need to be used instead of fuzzy sets to solve such a complex uncertainty in the model. Analogous to traditional fuzzy credibility-constrained programming, fuzzy-interval credibility-constrained programming was developed to tackle the constraint that existing violation probability under dual-uncertain conditions based on fuzzy-interval sets. For example, Zhang et al. (2018a, 2018b) used fuzzy-interval sets to represent the uncertainty of water supply and precipitation, and employed fuzzy-interval credibility-constrained programming method to optimize the monthly crop water allocation. Yue et al. (2020) optimized the water allocation scheme and system benefit, and fuzzy-interval sets were introduced to reflect water supply and crop water demand. However, the majority of above methods were used to optimize agricultural water allocation, few studies on ecological water allocation considering ecological water requirement have been undertaken.

Ecological water requirement is the minimum water consumption under the condition of maintaining sustainability and stability of the regional ecosystem, restoring fragile ecosystem and providing the maximum ecological services (Su and Kang, 2003). Previous studies of ecological water requirement were divided into in-canal and out-canal ones (Sajedipour et al., 2017), and the out-canal ecological water requirement generally refers to the ecological water requirement of natural vegetation (Wang and Cheng, 2002; Wang et al., 2005).

Traditional methods are commonly used to estimate the water requirement of ecological vegetation, such as area quota method, water balance method and evapotranspiration method (Su and Kang, 2003; Wang et al., 2005; Yang et al., 2012; Yang et al., 2020). These methods only obtain results based on the site-specific sample points, and these results cannot represent spatial characteristics of the whole study area accurately. Therefore, techniques of remote sensing (RS) and geographic information system (GIS) are employed to estimate the water requirement of ecological vegetation. It has ability in improving data accuracy, identifying the crops or vegetation information more accurately, and reflecting the spatial heterogeneity. However, most of studies still cannot fully reflect spatiotemporal variability and can only use spatial information on a specific element, such as identification of vegetation types or vegetation coefficients (Yuan et al., 2016; Zolfagharnajd et al., 2017; Chi et al., 2018). To overcome the limitations of previous studies, a novel concept that spatializes all elements of estimating water requirement of ecological vegetation based on Normalized Difference Vegetation Index (NDVI) and Potential Evapotranspiration (PET) is introduced to estimate spatial water requirement of ecological vegetation (SEWR). This method provides a good basis for the estimation of SEWR and ecological water allocation.

Therefore, in response to above concerns, this study aims to develop an interval multi-objective fuzzy-interval credibility-constrained nonlinear programming (IMFICNP) model combined with SEWR estimation for solving the problem of allocation of agricultural and ecological water in irrigation districts under uncertain conditions. The main content of this study includes the following sections: (1) establishment of IMFICNP model through integration of the methods of interval parameter programming, multi-objective programming and fuzzy-interval credibility-constrained programming into an optimization model; (2) formulation of the concept of SEWR that spatializes all elements of estimating water requirement of ecological vegetation based on NDVI and PET. The SEWR of three types of ecological vegetation (forest land, grassland and shrubland) was obtained based on decision-making unit (DMU); (3) introduction of the IQCWPFs to express the nonlinear relationships between crop yield and irrigation amount; (4) combination of the IMFICNP model with the SEWR estimation and applied it to Huangyang Irrigation District (HID) in the Shiyang River Basin for managing agricultural and ecological water allocation. The proposed model can tackle the conflicts of multiple objectives under single and dual uncertainties including interval, fuzzy and fuzzy-interval values. It can also handle violated constraints that they are hold at a predefined credibility level. Moreover, this study comes up with a new method for SEWR estimation, and provides effective and reliable information for ecological water monitoring in irrigation districts. Therefore, optimal schemes can help alleviate water conflicts between agricultural and ecological sectors in irrigation districts and these findings are helpful for decision-makers to manage limited water resources in arid and semi-arid areas. The framework of study system is shown in Fig. 1.

The paper is organized as follows. Section 2 describes methodology of estimation of the SEWR, calculation of eco-economic water productivity, the formulation of IMFICNP model and its solution process. Section 3 provides problem statement, modeling framework and data collection of study area. The results analysis and discussions are presented in Section 4 and we conclude the paper in Section 5.
will be. In consequence, RS data with high resolution as much as possible should be chosen.

When soil evaporation is not considered and only vegetation transpiration is considered, the vegetation transpiration coefficient could be approximated as the following equation (Choudhury et al. (1994)):

$$T_c = \frac{V_{I_{\text{mean}}} - V_{I_{\text{min}}}}{V_{I_{\text{max}}} - V_{I_{\text{min}}}}$$  \hspace{1cm} (1)

where $T_c$ is the transpiration coefficient of vegetation; $V_I$ is the vegetation index; $V_{I_{\text{max}}}, V_{I_{\text{min}}, V_{I_{\text{mean}}}}$ are the maximum, minimum and average values of the vegetation index respectively.

$NDVI$ has the most sensitive and relevant characteristics when differentiating and identifying vegetation types (Zhou, 2014), so this study selects $NDVI$ as the approximate calculation variable of $T_c$.

2.1.2. Spatialized reference crop evapotranspiration ($ET_0^{'}$)

Generally, $PET$ data by RS has errors in values, and the actual crop evapotranspiration is often overestimated. Therefore, it is very important to carry out fitting and correction for $PET$ data. There is an assumption that the reference crop evapotranspiration ($ET_0$) calculated by Penman-Monteith Eq. (P-M) is highly accurate (Tang et al., 2019). Based on this assumption, it is widely used in the calculation of $ET_0$. However, due to the limitation of meteorological stations for data collection, $ET_0$ obtained by traditional methods cannot represent the difference of crop evapotranspiration in complex surface land conditions. In other words, traditional methods cannot reflect the temporal and spatial variability of crop evapotranspiration. Therefore, this study will linearly fit and correct the $ET_0$ data obtained from several meteorological stations with the $PET$ data by RS, which can not only extend the traditional $ET_0$ to the spatial scale, but also ensure the accuracy of the $PET$ data. The form of the fitting equation is as follows:

$$ET_0^{'} = a \cdot PET + \theta$$  \hspace{1cm} (2)

where $ET_0^{'}$ is the spatialized reference crop evapotranspiration (mm); $a$ is the coefficient of first order; $\theta$ is the constant.

2.1.3. Estimation of SEWR

According to the above steps, it is possible to obtain key parameters for estimation of SEWR, namely, the grid-scale $T_c$ and $ET_0^{'}$. Similar to the area quota method, the estimated results of SEWR at each grid can be expressed as:

$$W_h = 0.1 \cdot ET_0^{'} \cdot T_c \cdot A_h$$  \hspace{1cm} (3)

where $W_h$ is the water requirement of ecological vegetation of $h$th grid ($10^4$ m$^3$); $ET_0^{'}$ is the spatialized reference crop evapotranspiration of $h$th grid (mm); $T_c$ is the transpiration coefficient of vegetation of $h$th grid; $A_h$ is the area of $h$th grid (km$^2$).

When obtaining the water requirement of ecological vegetation of the entire district through the grid-scale in a small-scale area, the regional water requirement of ecological vegetation is often underestimated due to insufficient accuracy or lack of measurement of RS data. Therefore, if the grid water requirement of ecological vegetation is
added directly, the result will be smaller than the actual water requirement of ecological vegetation. In order to estimate the water requirement of ecological vegetation correctly and reflect the spatial variability, the study area is divided into multiple DMU according to the actual situation, such as geographical condition, vegetation distribution and other characteristics. Each DMU contains some grids, by extracting and averaging the value of water requirement of ecological vegetation of each grid, then the average water requirement of ecological vegetation of each DMU is obtained. By multiplying the average water requirement of ecological vegetation and area of each DMU, the water requirement of ecological vegetation of each DMU is obtained, thereby obtaining the SEWR of study area.

2.2. Calculation of eco-economic water productivity (EEWP)

The eco-economic water productivity (EEWP) refers to the ratio of the output value to the water consumption of ecological vegetation. It is proposed to describe the relationship between the amount of irrigation water and the benefits of ecological vegetation, and its expression is:

\[
EV_k = \frac{PV_k}{EM_k A_k}
\]

where \(EV_k\) is the EEWP (CNY/m²); \(PV_k\) is the output value (CNY); \(EM_k\) is the irrigation quota of vegetation (m³/hm²); \(A_k\) is the area of vegetation (hm²).

Ecological service value is generally used to describe the direct or indirect benefits of an area’s ecological environment. However, to some extent, only an empirical relationship between the equivalent factors of ecological service value and land area is established, and the selection of equivalent factor is subjective. EEWP can bridge the relationship between ecological benefits and water input objectively, which makes the equivalent factor is subjective. EEWP can bridge the relationship between ecological benefits and water input objectively, which makes the equivalent factor is subjective.

2.3. Interval multi-objective fuzzy-interval credibility-constrained nonlinear programming (IMFeCNP)

2.3.1. Interval multi-objective programming (IMP)

Interval parameter programming (IPP) is an effective method to solve interval parameters or variables in objective functions and constraints in optimization models, and can give a relatively stable solution. The typical IPP model is:

\[
\begin{align*}
\max f^i &= c^i X^i \\
\min f^i &= c^i X^i \\
X^i &= [a^i, b^i] \\
of &\geq 0
\end{align*}
\]

where \(f^i\) is the objective function; \(a^i, b^i\) are the interval coefficients; \(X^i\) is the interval decision variable. “±” is the upper bound and “−” is the lower bound.

Due to the different positions of different decision makers, there will be several trade-offs and conflicting objectives, thus multi-objective programming (MOP) is emerged. Aiming at the problem of multiple objectives under uncertainty, an interval multi-objective programming (IMOP) method is proposed. It is written as follows:

\[
\begin{align*}
F^i(X^i) &= \left[\max f^i_1(X^i_1), \max f^i_2(X^i_2), \cdots, \max f^i_n(X^i_n)\right] \\
\sum a^i_n X^i_n &= b^i \\
of &\geq 0
\end{align*}
\]

where \(f^i_n(X^i_n)\) is the objective function; \(n\) is the number of objective functions; \(u\) is the number of decision variables; \(a^i_n\) is the interval coefficient; \(X^i_n\) is the interval decision variables.

2.3.2. Fuzzy-interval credibility-constrained programming (FICP)

When the uncertain problem needs to describe the risk and probability of violation, and there are fuzzy parameters in the constraints, thus fuzzy credibility-constrained programming (FCCP) method is proposed. This method can provide decision makers with more feasible results under different credibility levels based on credibility measure. The FCCP model is expressed as follows:

\[
\begin{align*}
\max f &= c X \\
Cr(aX \leq \hat{b}) &\geq \lambda \\
of &\geq 0
\end{align*}
\]

where \(f\) is the objective function; \(c\) is the coefficient; \(\hat{b}\) is the fuzzy coefficient; \(X\) is the decision variable; \(Cr\) is the credibility measure; \(\lambda\) is the credibility level.

FCCP can deal with the above problems effectively, but cannot deal with problems under more complex uncertainty. Especially when the boundary of fuzzy parameters is unclear, fuzzy-interval sets need to be used instead of fuzzy sets to solve more complex uncertainty features in the model. The formation process of fuzzy-interval sets is shown in Fig. 2. Therefore, the FICP model is expressed as follows:

\[
\begin{align*}
\max f &= c X \\
Cr(aX \leq \hat{b}) &\geq \lambda \\
of &\geq 0
\end{align*}
\]

where \(\hat{b}\) is the fuzzy-interval sets.

2.3.3. IMFeCNP

Once the uncertain model contains nonlinear multiple optimization objectives, interval parameters, fuzzy parameters and fuzzy-interval sets in constraints, and has to provide guidance for decision makers according to the credibility level, to combine the above programming methods to formulate an interval multi-objective fuzzy-interval credibility-constrained nonlinear programming (IMFeCNP) model is needed. Its representation is presented as follows:

\[
\begin{align*}
F^i(X^i) &= \left[\max f^i_1(X^i_1), \max f^i_2(X^i_2), \cdots, \max f^i_n(X^i_n)\right] \\
\sum a^i_n X^i_n &= b^i \\
\sum c^i_n X^i_n &= d^i \\
of &\geq 0
\end{align*}
\]

where \(c^i_n\), \(d^i_n\) is the interval coefficient.

It is assumed that the fuzzy-interval sets in the above model are triangular fuzzy numbers:

\[
\hat{b} = (b^i_1, b^i_2, b^i_3) = \{[b^i_1, b^i_2], [b^i_2, b^i_3], [b^i_3, b^i_1]\}
\]

The credibility-constrained programming can be transformed into:

\[
\begin{align*}
\sum a^i_n x^i_n &= b^i \\
\sum a^i_n x^i_n &= b^i \\
\sum a^i_n x^i_n &= b^i \\
\sum a^i_n x^i_n &= b^i
\end{align*}
\]
Generally, the credibility-constrained programming model should avoid the medium and high risks in real situation, so the credibility confidence level should be 0.5–1.0. According to the above equations, we have:

\[
Cr \left\{ \sum_{i=1}^{v} a_i^1 x_i^1 \leq b^1 \right\} = \frac{2^{b^1 - a^1} \sum_{i=1}^{v} a_i^1 c_i^1}{2(b^1 - a^1)} \geq \lambda
\]

\[
\rightarrow \sum_{i=1}^{v} a_i^1 x_i^1 \leq b^1 + (1 - 2\lambda)(b^1 - b^1_+)
\]

Thus, the IMFICNP model is transformed into IMP model, which can be solved by common model solutions.

2.4. Model solution process

The IMFICNP model can tackle the uncertainties that may exist in the objective functions and constraints in multi-objective problems. At the same time, by designing different credibility levels, IMFICNP model can provide decision-makers with decision schemes containing different violation risks. It is also convenient for flexible transformation and concrete analysis under different situations.

The solution processes of IMFICNP model are summarized as follows:

Step 1: Formulate the IMFICNP model, and transform the model into the upper and lower bound sub-models according to the characteristics of interval parameters. According to Eq. 5, let \( c_u \) be the \( u \)th member of vector \( c^- \), \( x_u \) be the \( u \)th member of matrix \( x^- \), \( x_u \) be the \( u \)th member of vector \( X^- \). Assuming that the quantity of positive numbers are \( s_1 \), the quantity of negative numbers are \( s_2 \), and \( s_1 + s_2 = t \) in uncertain coefficients \( c_u \). Let the first \( s_1 \) coefficients be the positive numbers, thus we have \( c_u, x_u > 0 \) for \( u = 1, 2, \ldots, t \).

Assuming that \( c_u \) has the same sign in sub-models. The lower bound sub-model is expressed as:

\[
\begin{aligned}
&\max f^- = \sum_{a=1}^{n} c_1^- x_1^- + \sum_{a=n+1}^{N} c_1^- x_1^- \\
&\sum_{a=1}^{n} \left| a_{u1} \right| \text{sign}(a_{u1}) x_1^- + \sum_{a=n+1}^{N} \left| a_{u1} \right| \text{sign}(a_{u1}) x_1^- \leq b_1^- \quad \forall v \\
&x_1^u \geq 0, \ u = 1, 2, \ldots, t
\end{aligned}
\]

Solving the above model, we can obtain \( f_{opt^ {-}} \), \( x_{opt^-} \) for \( u = 1, 2, \ldots, s_1 \) and \( x_{opt^-} \) for \( u = s_1 + 1, s_1 + 2, \ldots, t \).

The upper bound sub-model is expressed as:

\[
\begin{aligned}
&\max f^+ = \sum_{a=1}^{n} c_1^+ x_1^+ + \sum_{a=n+1}^{N} c_1^+ x_1^+ \\
&\sum_{a=1}^{n} \left| a_{u1} \right| \text{sign}(a_{u1}) x_1^+ + \sum_{a=n+1}^{N} \left| a_{u1} \right| \text{sign}(a_{u1}) x_1^+ \leq b_1^+ \quad \forall v \\
&x_1^u \geq 0, \ u = 1, 2, \ldots, t \\
&x_1^u \leq x_{opt^+} \quad \forall u = s_1 + 1, s_1 + 2, \ldots, t
\end{aligned}
\]

Similarly, solving the above model, we can obtain \( f_{opt^+}, x_{opt^+} \) for \( u = 1, 2, \ldots, s_1 \) and \( x_{opt^+} \) for \( u = s_1 + 1, s_1 + 2, \ldots, t \). The final results are \( x_{opt} = [x_{opt^-}, x_{opt^+}], f_{opt} = [f_{opt^-}, f_{opt^+}] \).

Step 2: Calculate the maximum and minimum values of each objective of the upper and lower bound sub-models under different credibility levels. Transform the multi-objective model into a single objective model, and use the minimum deviation method to solve it. The transformation method of the objective function is presented as follows:

If the objective function is maximized

\[
f_{opt^+} = \frac{f_{opt^+} - f_{min}}{f_{max} - f_{min}}
\]

If the objective function is minimized

\[
f_{opt^-} = \frac{f_{opt^-} - f_{min}}{f_{max} - f_{min}}
\]

Step 3: Obtain the weight of each objective function. In order to avoid the deviation from the objective reality caused by direct weighting, this study compare and evaluate the importance of each objective by Analytic Hierarchy Process (AHP) (Al-Harbi, 2001), and finally determine the weight of each objective. It is worth mentioning that in order to ensure the discussion of results is carried out at different credibility levels in the same multi-objective model, the weight of each objective function is constant at different credibility levels. The final form of the objective function is:

\[
\begin{aligned}
&\min F^+ = \sum_{n=1}^{N} \omega_n f_{opt^+} \\
&s.t. \sum_{n=1}^{N} \omega_n = 1
\end{aligned}
\]

Step 4: Substitute the data obtained by the step 2 and 3 into Eq. 16 to obtain the results under different credibility levels.

3. Case study

3.1. Study area and problem statement

Huangyang Irrigation District (HID) is located in the middle reaches of Shiyang River Basin (102°33′-102°59′ E, 37°23′-37°46′ N), Wuwei City, Gansu Province. It is about 30 km away from the urban area. It is a typical irrigation district that only uses surface water for irrigation, and its water withdrawal mainly comes from Huangyang River in the north of Qilian Mountains. Fig. 3 shows the geographical location of HID and 29 DMUs, which is divided according to the spatial distribution of canal systems and land use types. HID is temperate arid climate, with little precipitation, high evaporation and long sunshine hours. Crops planted in HID are classified as the first type and the second type crops (Shan et al., 2021). The first type crops are defined because they have relationships between yield and water demand of crops through the measured data, which is generally expressed by crop water production function. The second type crops, due to the lack of measured data, are defined with only relationships between economic water productivity
and water demand. In HID, the first crops include wheat, maize, potato, beet and alfalfa and the second type crops consist of medicinal herbs, crown pear and vegetables. The main grain crops are wheat and maize and the main economic crop is crown pear. Recently, the irrigation district is facing serious water shortage problem because of climate change, intensification of human activities and rapid development of social economy. There are conflicts of water use in various competing water use sectors, the contradiction between water supply and demand for irrigation is very prominent, and the ecological environment is deteriorating gradually. The increase of water consumption in domestic and industrial sectors leads to a reduction of agricultural water and ecological water in study area, which leads to the deterioration of ecological environment ultimately. Therefore, how to plan a water allocation scheme that balances agricultural and ecological water allocation in irrigation district, while considering the SEWR, has become a great challenge for institutional management of irrigation district. When optimizing water allocation for crops and ecological vegetation, the existence of uncertain factors such as water supply, water requirement of crops and vegetation, crop prices and production costs make the allocation scheme more complex. Therefore, to solve the above problems, optimal water allocation using uncertain programming method is necessary.

3.2. Modeling framework

In this study, the SEWR in HID and the EEWP are incorporated into the IMFICNP model, and agricultural water and ecological water at different DMUs are optimized to obtain the sustainable development of ecology and agriculture in HID. The objective functions include the minimum of water loss during irrigation, the maximum of net crop benefits, the maximum of yields of main grain crops, the maximum of ecological vegetation benefits and the balance of irrigation water consumption for each DMU. The model considers uncertainties of crop prices, sowing rates, seed prices, EEWP, water supply and water requirement, these uncertainties are characterized by interval numbers, fuzzy numbers and fuzzy-interval sets. Therefore, the developed model is formulated as follows, and its symbolic definitions are shown in Table 1.

\[
\begin{align*}
\text{min} f^+_i &= \sum_{j=1}^{J} \left( \sum_{k=1}^{K} 10-A_k \cdot AW^+_j + \sum_{k=1}^{K} 10-A_k \cdot EW^+_j \right) \cdot \left( 1 - \eta^{\text{water}} \right) \\
&+ \sum_{j=1}^{J} \left( \sum_{k=1}^{K} 10-A_k \cdot AW^+_j + \sum_{k=1}^{K} 10-A_k \cdot EW^+_j \right) \cdot \left( 1 - \eta^{\text{field}} \right) \\
\text{max} f^*_i &= \text{income}^+_i + \text{income}^*_i \\
\text{income}^+_i &= \sum_{j=1}^{J} \sum_{k=1}^{K} 10-A_k \cdot \tilde{P}_j - N_j \cdot \tilde{C}_j - C^{\text{water}} \cdot 10-A_k \cdot AW^+_j / \eta^{\text{field}} \\
\text{income}^*_i &= \sum_{j=1}^{J} \sum_{k=1}^{K} 10-A_k \cdot \tilde{V}_j
\end{align*}
\]

where

\[
Y^+_j = \beta^+_j \left( \left( AW^+_j + P \right) \right) + Y^+_j \left( \left( AW^+_j + P \right) \right) + \delta^+_i
\]

\[
\text{max} f^*_i = \sum_{j=1}^{J} \sum_{k=1}^{K} Y^+_j
\]

\[
\text{min} f^*_i = \max \left\{ \sum_{j=1}^{J} 10-A_k \cdot AW^+_j + \sum_{k=1}^{K} 10-A_k \cdot EW^+_j \right\} \\
- \min \left\{ \sum_{j=1}^{J} 10-A_k \cdot AW^+_j + \sum_{k=1}^{K} 10-A_k \cdot EW^+_j \right\}
\]

Constraints include water supply constraint, crop water requirement constraint, water requirement of ecological vegetation constraint, yield constraint and non-negative constraint.

(1) Water supply constraint

\[
C_r \left\{ \sum_{j=1}^{J} 10-A_k \cdot AW^+_j + \sum_{k=1}^{K} 10-A_k \cdot EW^+_j \geq \tilde{Q}_r - \eta^{\text{canal}} \right\} \geq \lambda
\]

Fig. 3. Study area.
Table 1
Definitions of symbols in the model.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{i}^{+}$, $f_{j}^{+}$, $f_{s}^{+}$, $f_{a}^{+}$, $f_{b}^{+}$</td>
<td>Water loss during irrigation (m$^3$), net crop benefits (CNY), yield of main grain crops (kg/hm$^2$), ecological vegetation benefits (CNY), balance of irrigation water consumption for each DMU (m$^3$)</td>
</tr>
<tr>
<td>$i$</td>
<td>Index of DMU, a total of 29 DMUs</td>
</tr>
<tr>
<td>$j$</td>
<td>Index of crops, a total of eight crops</td>
</tr>
<tr>
<td>$k$</td>
<td>Index of ecological vegetation, a total of three ecological vegetation</td>
</tr>
<tr>
<td>$A_{g}$</td>
<td>Planting area of crop $j$ in DMU $i$ (hm$^2$)</td>
</tr>
<tr>
<td>$A_{w}$</td>
<td>Irrigation amount of crop $j$ in DMU $i$ (mm)</td>
</tr>
<tr>
<td>$A_{p}$</td>
<td>Planting area of ecological vegetation $k$ in DMU $i$ (hm$^2$)</td>
</tr>
<tr>
<td>$E_{w}$</td>
<td>Irrigation amount of ecological vegetation $k$ in DMU $i$ (mm)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Water utilization coefficient</td>
</tr>
<tr>
<td>$\eta_{\text{field}}$</td>
<td>Canal system water utilization coefficient, field water utilization coefficient</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Water utilization coefficient</td>
</tr>
<tr>
<td>$\delta_{j}$</td>
<td>Net crop benefits of the first and second type crops (CNY)</td>
</tr>
<tr>
<td>$\delta_{j}$</td>
<td>Yield per unit area of crop $j$ (kg/hm$^2$)</td>
</tr>
<tr>
<td>$\delta_{j}$</td>
<td>Market price of crop $j$ (CNY/kg), fuzzy parameter</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Economic water productivity of ecological vegetation</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Economic water productivity of crop $j$ (CNY/m$^3$), fuzzy parameter</td>
</tr>
<tr>
<td>$\beta_{i}$</td>
<td>Quadratic term, the first-order term and the constant term of IQCWPFs</td>
</tr>
<tr>
<td>$P$</td>
<td>Annual effective precipitation (mm)</td>
</tr>
<tr>
<td>$E_{w}$</td>
<td>EEWP of ecological vegetation $k$ (CNY/m$^3$), fuzzy parameter</td>
</tr>
<tr>
<td>$\eta_{\text{max}}$</td>
<td>Maximum and minimum of water supply in DMU $i$ (mm), fuzzy-interval sets</td>
</tr>
<tr>
<td>$M_{g}^{\text{min}}$</td>
<td>Minimum irrigation quota for crop $j$ in DMU $i$ (mm)</td>
</tr>
<tr>
<td>$M_{g}^{\text{max}}$</td>
<td>Maximum irrigation quota for crop $j$ in DMU $i$ (mm)</td>
</tr>
<tr>
<td>$S_{w}^{\text{max}}$</td>
<td>Maximum water requirement of ecological vegetation $k$ in DMU $i$ (mm)</td>
</tr>
<tr>
<td>$S_{w}^{\text{min}}$</td>
<td>Maximum water requirement of ecological vegetation $k$ in DMU $i$ (mm)</td>
</tr>
<tr>
<td>$Y_{j}$</td>
<td>Minimum yield of crop $j$ (kg/hm$^2$)</td>
</tr>
</tbody>
</table>

Table 5 shows the estimated value of SEWR in each DMU in HID. Determine the ecological vegetation area through data of land use types (Gong et al., 2019) (spatial resolution is 10 m), and then use NDVI (spatial resolution is 500 m, time resolution is 16-days), PET (spatial resolution is 500 m, time resolution is 8-days) and ET$_0$ to estimate the SEWR. These data come from NASA (https://ladsweb.modaps.eosdis.nasa.gov/search/) and the National Meteorological Science Data Center (http://data.cma.cn/). ET$_0$ data used for PET fitting in this study are obtained from meteorological stations in Minqin (52681), Wuwei (52679) and Yongchang (52674) in Shiyang River Basin, and ET$_0$ after fitting is verified by data from meteorological station in Wushaoling (52787). The upper and lower bound of constraints of SEWR are obtained by the estimated value multiplying by ratio, which is empirically obtained. Since the SEWR is an estimated value in 2019, which has the characteristics of fuzzy-interval sets, so fuzzy-interval credibility-constrained programming is used to characterize it.

4. Results analysis and discussions

The SEWR and EEWP of HID in the Shiyang River Basin can be calculated according to the methods in sections 2.1 and 2.2. Through inputting these parameters into the model of optimized irrigation water allocation, optimal solutions can be generated based on solution method in section 2.4. In this study, credibility levels are set as $\lambda = 0.5, 0.6, 0.7$ and 0.8, 0.9, 1.0 and weighted factors of each objective are assigned as 0.063, 0.313, 0.250, 0.313 and 0.063, then results of agricultural and ecological water allocation for each DMU in six scenarios with violated constraints are obtained. Therefore, more details regarding results and discussions are shown below.

4.1. Optimal solutions of objective function values and water allocation

Table 2 shows the IQCWPFs, crop market prices, sowing rates and seed prices of the first type crops.

Table 2
IQCWPFs, crop market prices, sowing rates and seed prices of the first type crops.

<table>
<thead>
<tr>
<th>Crops</th>
<th>IQCWPFs</th>
<th>Crop market prices (CNY/kg)</th>
<th>Sowing rates (kg/hm$^2$)</th>
<th>Seed prices (CNY)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td>$Y = [-0.015, -0.014]*AW^2$ + [16.606, 16.929]*AW + [2257.339, 2399.771]</td>
<td>(2.14, 24.837, 5.52)</td>
<td>4.40, 4.60</td>
<td>3416.25, 3483.75</td>
</tr>
<tr>
<td>Maize</td>
<td>$Y = [-0.013, -0.012]*AW^2$ + [19.422, 20.543]*AW + [3246.984, 3279.617]</td>
<td>(2.04, 24.183, 4.55)</td>
<td>2.60</td>
<td>2081.25, 2198.75</td>
</tr>
<tr>
<td>Potato</td>
<td>$Y = [-3.096, -3.065]*AW^2$ + [2430.167, 2548.10]</td>
<td>(1.48, 24.5, 3.50)</td>
<td>2.60</td>
<td>[2081.25, 2198.75]</td>
</tr>
<tr>
<td>Beet</td>
<td>$Y = [-0.087, -0.086]*AW^2$ + [16.162, 16.252]</td>
<td>(0.60, 267.60, 280.98)</td>
<td>0.80, 4.00</td>
<td>[6.94, 8.06]</td>
</tr>
<tr>
<td>Alfalfa</td>
<td>$Y = [-0.133, -0.131]*AW^2$ + [171.775, 175.245]</td>
<td>(2.60, 21.60, 24.00)</td>
<td>4.00</td>
<td>[20.81, 24.19]</td>
</tr>
</tbody>
</table>

3.3. Data collection and processing

The water price, canal system water utilization coefficient and field water utilization coefficient are obtained by public report, they are 0.21 CNY/m$^3$, 0.64 and 0.9 respectively. Table 2 shows the IQCWPFs, crop market prices, sowing rates and seed prices of the first type crops. The IQCWPFs come from literature studies (Yang, 2008; Sun et al., 2009; Li et al., 2019a; Meng et al., 2019). The crop market prices and seed prices are obtained by website (https://www.cnhnb.com/) and news reports. The sowing rates comes from public report. The economic water productivity of the second type crops and EEWP are obtained by statistical yearbooks, as shown in Table 3. The planting area of crops and ecological vegetation are shown in Table 4.
credibility level leads to a smaller objective function value from $\lambda = 0.5$ to $\lambda = 1.0$. In other words, as the constraint-violation risk decreases, the model becomes progressively more conservative and the decision maker is able to obtain less benefits. A high economic payoff accompanied by high risk level while lower economic returns correspond to lower risk level. Also, it should be noted that an abrupt change in the upper bounds of the second and third objective when the credibility level is 0.7. When at this risk level, optimal solution is expressed as a reduction in the water allocation of main food crops (i.e. wheat and maize) and ecological vegetation, which in turn reduces their yield and benefits. More economic returns are obtained by giving more water to other crops (e.g. economic crops), thereby improving system economic returns. Fig. 5 presents optimal solutions of agricultural and ecological water allocation under different credibility levels. It can be clearly seen that a higher credibility level leads to less allocated water to irrigation district. Moreover, as the credibility level increases, the water allocation for agricultural and ecological purposes is decreased sequentially. For example, when $\lambda$ is raised from 0.5 to 1.0, the amount of gross agricultural water allocation is decreased from $[5048.31, 5278.76] \times 10^4$ m$^3$ to $[4568.94, 4809.33] \times 10^4$ m$^3$ and the amount of gross ecological water

Table 3
Economic water productivity of the second type crops and EEWP.

<table>
<thead>
<tr>
<th>Crops and vegetation</th>
<th>Economic water productivity (CNY/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medicinal herbs</td>
<td>(31.02, 44.32, 48.75)</td>
</tr>
<tr>
<td>Crown pear</td>
<td>(5.94, 8.48, 10.18)</td>
</tr>
<tr>
<td>Vegetables</td>
<td>(3.33, 4.16, 6.24)</td>
</tr>
<tr>
<td>EEWP (CNY/m$^3$)</td>
<td></td>
</tr>
<tr>
<td>Forest land</td>
<td>(7.64, 7.96, 8.28)</td>
</tr>
<tr>
<td>Shrubland</td>
<td>(6.85, 6.97, 7.10)</td>
</tr>
</tbody>
</table>

Table 4
Planting area of crops and ecological vegetation (hm$^2$).

<table>
<thead>
<tr>
<th>DMU</th>
<th>Crops</th>
<th>Ecological vegetation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wheat</td>
<td>Maize</td>
</tr>
<tr>
<td>1</td>
<td>123</td>
<td>95</td>
</tr>
<tr>
<td>2</td>
<td>99</td>
<td>77</td>
</tr>
<tr>
<td>3</td>
<td>148</td>
<td>115</td>
</tr>
<tr>
<td>4</td>
<td>77</td>
<td>59</td>
</tr>
<tr>
<td>5</td>
<td>101</td>
<td>78</td>
</tr>
<tr>
<td>6</td>
<td>119</td>
<td>92</td>
</tr>
<tr>
<td>7</td>
<td>153</td>
<td>118</td>
</tr>
<tr>
<td>8</td>
<td>124</td>
<td>96</td>
</tr>
<tr>
<td>9</td>
<td>49</td>
<td>38</td>
</tr>
<tr>
<td>10</td>
<td>115</td>
<td>89</td>
</tr>
<tr>
<td>11</td>
<td>91</td>
<td>70</td>
</tr>
<tr>
<td>12</td>
<td>172</td>
<td>133</td>
</tr>
<tr>
<td>13</td>
<td>142</td>
<td>110</td>
</tr>
<tr>
<td>14</td>
<td>194</td>
<td>150</td>
</tr>
<tr>
<td>15</td>
<td>133</td>
<td>103</td>
</tr>
<tr>
<td>16</td>
<td>218</td>
<td>168</td>
</tr>
<tr>
<td>17</td>
<td>190</td>
<td>147</td>
</tr>
<tr>
<td>18</td>
<td>205</td>
<td>159</td>
</tr>
<tr>
<td>19</td>
<td>257</td>
<td>198</td>
</tr>
<tr>
<td>20</td>
<td>38</td>
<td>30</td>
</tr>
<tr>
<td>21</td>
<td>76</td>
<td>59</td>
</tr>
<tr>
<td>22</td>
<td>342</td>
<td>264</td>
</tr>
<tr>
<td>23</td>
<td>119</td>
<td>92</td>
</tr>
<tr>
<td>24</td>
<td>502</td>
<td>388</td>
</tr>
<tr>
<td>25</td>
<td>246</td>
<td>190</td>
</tr>
<tr>
<td>26</td>
<td>262</td>
<td>202</td>
</tr>
<tr>
<td>27</td>
<td>209</td>
<td>161</td>
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<tr>
<td>28</td>
<td>157</td>
<td>121</td>
</tr>
<tr>
<td>29</td>
<td>405</td>
<td>313</td>
</tr>
<tr>
<td>Total</td>
<td>5064</td>
<td>3913</td>
</tr>
</tbody>
</table>

Table 5
Estimated value of SEWR of DMU in HID.

<table>
<thead>
<tr>
<th>DMU</th>
<th>Average ecological water requirement (10$^4$ m$^3$/km$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Area (km$^2$)</td>
</tr>
<tr>
<td>-----</td>
<td>---------------</td>
</tr>
<tr>
<td>1</td>
<td>17.57</td>
</tr>
<tr>
<td>2</td>
<td>20.49</td>
</tr>
<tr>
<td>3</td>
<td>18.18</td>
</tr>
<tr>
<td>4</td>
<td>15.61</td>
</tr>
<tr>
<td>5</td>
<td>19.74</td>
</tr>
<tr>
<td>6</td>
<td>14.99</td>
</tr>
<tr>
<td>7</td>
<td>19.10</td>
</tr>
<tr>
<td>8</td>
<td>23.97</td>
</tr>
<tr>
<td>9</td>
<td>21.93</td>
</tr>
<tr>
<td>10</td>
<td>27.27</td>
</tr>
<tr>
<td>11</td>
<td>19.01</td>
</tr>
<tr>
<td>12</td>
<td>28.51</td>
</tr>
<tr>
<td>13</td>
<td>30.87</td>
</tr>
<tr>
<td>14</td>
<td>18.24</td>
</tr>
<tr>
<td>15</td>
<td>24.03</td>
</tr>
</tbody>
</table>
allocation is decreased from $[1059.03, 1165.29] \times 10^4 \text{ m}^3$ to $[950.65, 1030.39] \times 10^4 \text{ m}^3$, thereby leading to a reduction in gross total water allocation by $[587.76, 604.33] \times 10^4 \text{ m}^3$. Credibility levels offer scenarios showing water allocation options for different risk levels, which can provide reliable reference for decision makers’ choices under different risk scenarios. Besides, Fig. 5 also shows the net water allocation under different scenarios. Taking $\lambda = 0.8$ as an example, its gross water allocation for irrigation district is $[5754.6, 6085.07] \times 10^4 \text{ m}^3$ while the net water allocation is $[3319.83, 3510.48] \times 10^4 \text{ m}^3$. Nearly 42.3% water losses occur during irrigation due to water evaporation.
Taking the upper bound values under $\lambda = 1$ to illustrate the results in each DMU. The results of lower bound can be made a similar analysis. The study system has the lowest violation risk under $\lambda = 1$ because the constraints are completely satisfied. Figs. 6-9 show the upper bound of optimal solutions for each DMU. The upper bound of allocated amount of agricultural and ecological net water for each DMU are presented in Fig. 6. It can be seen that the total allocated water for 24th DMU is the largest, that is, $342.31 \times 10^4$ m$^3$ and its area is the largest. In addition, although the total water allocation has significant differences among DMUs and the allocated water for DMUs range from $36.24 \times 10^4$ m$^3$ to $342.31 \times 10^4$ m$^3$, the majority of total net allocated water per unit area range from $0.165 \times 10^4$ m$^3$/hm$^2$ to $0.210 \times 10^4$ m$^3$/hm$^2$, indicating that the allocation of water each DMU is relatively balanced. Most of DMUs have higher agricultural water allocation than ecological water allocation (except 20th DMU), which is consistent with the characteristics of the irrigation district. The ecological water allocation in irrigation district is mainly distributed in the periphery of the irrigation area (e.g. 18th, 20th, 24th and 29th DMUs). This is due to the large area of ecological vegetation in these DMUs, especially 18th and 20th DMUs where the irrigation riverbanks are more densely covered with ecological vegetation and therefore have more ecological water allocation. Since the difference in planning area between grain crops and economic crops is large, resulting in a larger difference in water allocation results, thus the results for the two types of crops are presented separately. Figs. 7 and 8 present the upper bound of net water allocation for grain crops and economic crops for each DMU. For grain crops, the water allocation for maize is the highest in each DMU, followed by wheat. This is because although wheat is planted on a large area, maize produces more benefits per unit of water use. However, in this case, optimization model guides decision makers to allocate water to more economical grain crops for better profits. Potatoes are not considered because the planted area is small. Thus, less water is allocated to 20th DMU for crop irrigation. Fig. 9 presents the upper bound of ecological vegetation net water allocation for each DMU. Although the EEWAP of forest land and shrubland are higher than those of grassland, the larger area of grassland allows it to receive a larger water allocation. It is worth considering that irrigation districts usually focus on the expansion of arable land area at the expense of the value generated by the ecological environment, which leads to ecological degradation. Such an adverse impact is difficult to repair and compensate in a short while. Therefore, it is suggested that more ecological area should be added to irrigation districts, especially the construction of ecological forests, to maintain ecological sustainability.

### 4.2. Crop yields and system economic benefits

Fig. 10 shows yield and system economic benefits of the first type crops. As the credibility level rises, decision space for water constraints becomes increasingly tight, and crop yields are negatively affected, which results in lower economic benefits. It is obvious that crop yield and economic benefits present similar trends. There is a significant decrease in wheat yields and benefits between $\lambda = 0.5$ to $\lambda = 0.7$, and no further change when $\lambda = 0.8$ to $\lambda = 1.0$, indicating that wheat yields and benefits are at the low limit of what the system can sustain. However, the results of maize show different variations from wheat. The lower bounds of yield and benefits of maize keep unchanged when $\lambda = 0.5$–0.8, while the upper bounds are unchanged at $\lambda = 0.7$–0.8, which can also indicate that the upper and lower bounds of interval model behave differently. In addition, potato yields and benefits are not sensitive to changes in $\lambda$ and therefore show a constant trend in Fig. 10. The upper bound of beet changes abruptly at $\lambda = 0.7$ due to the complexity of mathematical model and the collected data is obtained from multiple sources. Although optimal solution of the upper bound sub-model when $\lambda = 0.7$ has an abrupt compared with solutions under other credibility levels, system results are still optimal under this credibility level. When $\lambda = 0.7$, optimal solution is expressed as a reduction in the water allocation of maize and medicinal herbs, which in turn reduces their yield and benefits. More economic returns are obtained by giving more water to beet, thereby improving system economic returns. Fig. 10 also shows that the most significant yield reduction as the amount of water decreased, indicating that it is quite sensitive to changes in irrigation water. The upper bounds of yield and benefit of alfalfa decreased at $\lambda = 0.5$–0.7 and stay unchanged in all other cases, showing that the small area planted with alfalfa has relatively small impact on the whole system. Fig. 11 shows economic benefits of the second type crops. Since medicinal herbs have a high economic water productivity, they reflect a clear trend of decreasing benefits when $\lambda$ rises, while economic benefits of crown pear and vegetables are essentially unchanged in several scenarios.

### Table 6

Optimal solutions for water allocation of crops and ecological vegetation under different credibility levels ($10^4$ m$^3$).

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Wheat</th>
<th>Maize</th>
<th>Potato</th>
<th>Beet</th>
<th>Alfalfa</th>
<th>Medicinal herbs</th>
<th>Crown pear</th>
<th>Vegetables</th>
<th>Ecological vegetation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>[973.25, 1024.57]</td>
<td>[1190.90, 1229.75]</td>
<td>[183.17, 186.73]</td>
<td>[380.71, 391.64]</td>
<td>[43.62, 46.34]</td>
<td>[20.45, 22.53]</td>
<td>[57.24, 59.94]</td>
<td>[12.53, 13.79]</td>
<td>Forest land</td>
</tr>
<tr>
<td>0.6</td>
<td>[979.03, 997.18]</td>
<td>[1190.90, 1228.28]</td>
<td>[183.11, 186.73]</td>
<td>[380.71, 391.64]</td>
<td>[43.62, 46.34]</td>
<td>[20.45, 22.53]</td>
<td>[57.24, 59.94]</td>
<td>[12.53, 13.79]</td>
<td>Grassland</td>
</tr>
<tr>
<td>0.7</td>
<td>[879.88, 935.93]</td>
<td>[1190.90, 1228.28]</td>
<td>[183.04, 213.72]</td>
<td>[43.62, 46.34]</td>
<td>[20.45, 22.53]</td>
<td>[57.24, 59.94]</td>
<td>[12.53, 13.79]</td>
<td>[12.53, 13.79]</td>
<td>Shrubland</td>
</tr>
<tr>
<td>0.8</td>
<td>[886.81, 935.93]</td>
<td>[1190.90, 1228.28]</td>
<td>[183.04, 213.72]</td>
<td>[43.62, 46.34]</td>
<td>[20.45, 22.53]</td>
<td>[57.24, 59.94]</td>
<td>[12.53, 13.79]</td>
<td>[12.53, 13.79]</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>[886.81, 935.93]</td>
<td>[1190.90, 1228.28]</td>
<td>[183.04, 213.72]</td>
<td>[43.62, 46.34]</td>
<td>[20.45, 22.53]</td>
<td>[57.24, 59.94]</td>
<td>[12.53, 13.79]</td>
<td>[12.53, 13.79]</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>[886.81, 935.93]</td>
<td>[1190.90, 1228.28]</td>
<td>[183.04, 213.72]</td>
<td>[43.62, 46.34]</td>
<td>[20.45, 22.53]</td>
<td>[57.24, 59.94]</td>
<td>[12.53, 13.79]</td>
<td>[12.53, 13.79]</td>
<td></td>
</tr>
</tbody>
</table>

The data in the table are presented as ranges, where the lower and upper bounds are given in square brackets. The optimal solutions are expressed as a reduction in the water allocation of each DMU is relatively balanced. Most of DMUs have higher agricultural water allocation than ecological water allocation (except 20th DMU), which is consistent with the characteristics of the irrigation district. The ecological water allocation in irrigation district is mainly distributed in the periphery of the irrigation area (e.g. 18th, 20th, 24th and 29th DMUs). This is due to the large area of ecological vegetation in these DMUs, especially 18th and 20th DMUs where the irrigation riverbanks are more densely covered with ecological vegetation and therefore have more ecological water allocation. Since the difference in planning area between grain crops and economic crops is large, resulting in a larger difference in water allocation results, thus the results for the two types of crops are presented separately. Figs. 7 and 8 present the upper bound of net water allocation for grain crops and economic crops for each DMU. For grain crops, the water allocation for maize is the highest in each DMU, followed by wheat. This is because although wheat is planted on a large area, maize produces more benefits per unit of water use. However, in this case, optimization model guides decision makers to allocate water to more economical grain crops for better profits. Potatoes are not considered because the planted area is small. Thus, less water is allocated to 20th DMU for crop irrigation.
4.3. Comparison of models

To demonstrate the effectiveness and advantages of the developed multi-objective optimization model in addressing water allocation problems in irrigation districts, four optimization models are formulated for comparison in this section based on interval objective models. Table 7 shows the objective functions, credibility levels and their constraints of four models. After solving the above models separately, and

Fig. 6. The upper bound of allocated amount of agricultural and ecological net water for each DMU.

Fig. 7. The upper bound of allocation of grain crop net water for each DMU.
substituting the model results as input parameters into each objective of IMFICNP model, different objective values are obtained as shown in Fig. 12. It can be clearly seen that using the results of Model 1 as input parameters, the net crop benefits (Ob2) is the highest, reaching $2.27 \times 10^8$ CNY, and using the results of Model 2 as input parameters, the ecological vegetation benefits (Ob4) is the highest, reaching $3.86 \times 10^6$ CNY. However, Model 1 and 2 perform poorly in other objectives, which is the defect that the single-objective model cannot take
other factors into account. Compared with Model 1 and 2, the net crop benefits (Ob2) and ecological vegetation benefits (Ob4) obtained by Model 3 and 4 are lower, which are $[2.19, 2.64] \times 10^8$ CNY and $[2.89, 3.20] \times 10^6$ CNY for Model 3 and $[1.64, 2.61] \times 10^8$ CNY and $[3.52, 4.23] \times 10^6$ CNY for Model 4 respectively. But they have considered both agricultural and ecological water use, and a set of optimal allocation schemes have been formed in balance and coordination. The IMFICNP model adds considerations to irrigation loss and water distribution balance based on Models 3 and 4, which further reflects the advantages of the multi-objective model to coordinate various objects.

Table 7: Objective functions, credibility levels and constraints of the four models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Objective function</th>
<th>Credibility level</th>
<th>Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>$f_2$</td>
<td>$\lambda$</td>
<td>Consistent with the IMFICNP model</td>
</tr>
<tr>
<td>Model 2</td>
<td>$f_4$</td>
<td>$\lambda$</td>
<td></td>
</tr>
<tr>
<td>Model 3</td>
<td>$f_2$, $f_3$, $\lambda$</td>
<td>$\lambda = 1$</td>
<td></td>
</tr>
<tr>
<td>Model 4</td>
<td>$f_2$, $f_3$, $f_4$</td>
<td>$\lambda$</td>
<td></td>
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making optimal allocation scheme more in line with the actual situation of the irrigation district and the decision makers' actual demand. In contrast, the advantages of the model proposed in this study are obvious when dealing with the actual water allocation problem in irrigation districts.

In addition, Fig. 13 shows the water allocation of the above models. The ecological water in Model 1 and 3 only account for 13.7–14.2% and 12.9–13.5% of the total amount of water allocation. Both focus on the agricultural water allocation but neglect the ecological water allocation. Such allocation schemes may lead to the deterioration of ecological environment of the irrigation district if the decision makers use these two schemes, which is detrimental to the sustainable development of the regional ecology. In contrast, the ecological water in Model 2 accounts for 21.1–21.4% of the total water allocation, it pays too much attention to the ecological water allocation, but neglects the agricultural water allocation in irrigation district. Due to the lack of agricultural water, it may cause a reduction in crop yields and unable to guarantee the farmers' incomes. Although Model 4 considers the agricultural and ecological water allocation, when the interval value is at the lower bound, the total water allocation is only $4460.75 \times 10^4 \text{ m}^3$. The water supplied by irrigation district is not fully used, which may cause two consequences: one is the waste of resources caused by water resources residual (Guo et al., 2019); the other is the decline of yield and economic benefits caused by insufficient water demand. Finally, compared with the other four models, the IMFICNP model not only guarantees supply of ecological water, but also satisfies the demand for agricultural water as much as possible, which is beneficial to the efficient development of agriculture and the sustainable development of ecology in irrigation district.

5. Conclusion

An interval multi-objective fuzzy-interval credibility-constrained nonlinear programming (IMFICNP) model combined with the SEWR estimation has been developed for optimal water allocation of agricultural and ecological water in irrigation districts under uncertainty. The developed IMFICNP model can handle the conflicts in objective functions under single uncertainty and dual-uncertainties, such as interval parameters, fuzzy parameters and interval-fuzzy sets. It can also formulate optimal water allocation schemes for agricultural irrigation and ecological sustainability in DMUs under credibility levels. The estimation of SEWR is proposed to provide a basis for ecological water allocation. In addition, the IQCWPFs are introduced for expressing nonlinear relationships between crop yield and irrigation amount and the EEW is used to express the ratio of the output value to the water consumption of ecological vegetation.

The IMFICNP model has been applied to a real-world case study of agricultural and ecological water allocation in HID, Shiyang River Basin of Northwest China. Eight crops and three types of vegetation have been considered in case study. The results indicate that: (1) with the increase of credibility levels, both the objective values and allocated water amount will decrease, which result in the decline of crop yields and economic benefits; (2) crops and ecological vegetation with small planting area are not sensitive to the change of credibility levels; (3) optimal water allocation schemes in DMUs are mainly based on the planting area, but there are few exceptions that allocation schemes depend on the economic value of crops and ecological vegetation; (4) the IMFICNP model is more suitable for real-world situations than other models with the same constraints when tackle the agricultural and ecological water allocation problems. The results can provide theoretical basis and scientific guidance for balancing agricultural and ecological management in irrigation district. This study has established a model combined with a new estimation method of SEWR to solve the problem of agricultural and ecological water allocation in irrigation districts under multiple uncertainties. The model considering SEWR can also be applied to other similar irrigation districts to provide better water allocation schemes and ensure the sustainability of agricultural and ecological. However, by refining the water allocation process of ecological vegetation, the corresponding models and methods are worthy of continued exploration in future study.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
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References


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