A full fuzzy-interval credibility-constrained nonlinear programming approach for irrigation water allocation under uncertainty

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ABSTRACT

To address the water shortage caused by various natural conditions and ineffective irrigation water management in the Zhanghe Irrigation District (ZID) of the Yangtze River basin in China, a full fuzzy-interval credibility-constrained nonlinear programming (FFICNP) model is developed under uncertainty. Derived through incorporating fuzzy credibility constrained programming into the Jensen model optimization framework, FFICNP can not only address intervals (single uncertainty) and fuzzy-interval sets (dual uncertainties) in the model objectives and double-sided constraints, but also reflect nonlinear responsive relationships between the crop yields and irrigation levels by introducing the crop water production functions (CWPFs) under different growth stages. Moreover, an expected-value-based (EVB) approach is introduced to solve the FFICNP model. The FFICNP model is then applied to the case study of irrigation water allocation in the ZID for demonstrating its applicability. Optimal solutions can be generated from the FFICNP model for solving the irrigation water allocation problem under uncertainty. The results indicate that a lower credibility level corresponds to a higher level of system benefits and system efficiency. The system benefits of ZID in a wet year are \[17.72, 24.23\] × 10⁹ CNY when \( \lambda = 1.0 \) and \[17.79, 25.03\] × 10⁹ CNY when \( \lambda = 0.6 \). These findings from the FFICNP model can support in-depth analysis of interrelationships among irrigation water allocation schemes, system benefits, and credibility levels, and thus contribute to the effectiveness of irrigation water management under various inflow levels and complex uncertainties.

1. Introduction

Sustainable and efficient water resources management is a significant concern in many countries, particularly in the backdrop of burgeoning population, urbanization expansion, and global water-shortage crisis (Singh, 2014). In China, the biggest water use sector is agriculture, which consumes around 62.3% of the total water use (Ministry of Water Resources, P.R.O.C., 2017). Agricultural water management thus becomes one of the cores for helping sustainable regional development and ensuring livelihood security. Therefore, many previous researches have targeted to enhance the agricultural water use efficiency and water productivity, especially in arid and semi-arid areas characterized by high evaporation and low rainfall (Li et al., 2015; Niu et al., 2016; Zeng et al., 2017). However, many wet regions of southern China also suffer from serious water shortage crisis caused by various natural conditions and ineffective water management. The Zhanghe Irrigation District (ZID), which is located in the Yangtze River basin of China, is facing the same challenge of water scarcity. It is of great significance to study the rational irrigation water allocation of the ZID as a typical case.

In the process of irrigation water management, crop water production functions (CWPFs) play important roles in reflecting the relationship between the crop yields and crop water requirements (Zhang and Oweis, 1999; Cai et al., 2003; Georgiou and Papamichail, 2008; Tong and Guo, 2013; Li et al., 2016a). As one of the CWPFs, Jensen model (Jensen, 1968) has advantages of not only reflecting the relationship between crop yields and the water applied during different crop growth stages, but also expressing the difference when the same amount of water is supplied following different schedules (Shangguan et al., 2002; Guo et al., 2014a; Sadati et al., 2014; Zhang and Guo, 2016; Yue et al., 2018). However, a general Jensen model cannot tackle fluctuation and variability of input parameters such as hydrological elements, water requirements and market prices, posing an obstacle in practical water-allocation problems. Thereupon, to address the issue, an enhanced Jensen model taking uncertainty techniques into consideration is desired.
Uncertain programming methods, proved to be a kind of effective tools to handle complex inexactness, randomness and vagueness. To address different types of uncertainties, stochastic mathematical programming (SMP), interval parameter programming (IPP) and fuzzy mathematical programming (FMP) have been widely applied to the field of agricultural water management (Li et al., 2010; Lu et al., 2011; Nematian, 2016; Xie et al., 2018; Li et al., 2018; Zhang and Guo, 2018; Ren et al., 2019). Considering the quality of data and computational efficiency, IPP is a better choice to handle uncertain parameters that can be expressed as interval values with known lower and upper bounds, but with unknown membership or distribution functions (Wang et al., 2018). However, in many real world problems, a number of parameters may be more complicated, which can hardly be manipulated as a form of single uncertainty (e.g. interval value, fuzzy sets or stochastic probability distribution). For example, the environmental capacity (e.g. available water) associated with the vagueness of human judgment (Yu et al., 2016) can be expressed as a certain triangular fuzzy number \( \tilde{a} = (a_1, a_2, a_3) \), where \( a_1 \) is the least possible value, \( a_2 \) is the middle value, and \( a_3 \) is the highest possible value. However, it is still difficult to obtain the deterministic and crisp membership function of fuzzy sets in many cases. Moreover, each value (e.g. \( a_1, a_2, a_3 \)) may be specified as a range between the upper bound and the lower bound without knowing distributions. It is necessary to introduce the fuzzy-interval sets (FIS), a type of dual uncertainties (see Fig. 1) to address such a complexity. Previously, Zhang et al. (2018a) introduced the fuzzy-interval membership functions to an irrigation water allocation model to deal with both intervals (single uncertainty) and FIS (dual uncertainties) within a general Jensen model optimization framework for the first time. However, this method cannot tackle intervals and FIS simultaneously existed in the optimization objectives and both sides of model constraints, so that it cannot deal with the complexities associated with the violation of system constraints.

In fact, in real-world water allocation problems of the ZID, dual uncertainties need to be reflected in both objectives and double-sided constraints of the optimization models. To further address this issue, the fuzzy credibility-constrained programming (FCCP) (Liu and Iwamura, 1998) can be introduced to effectively tackle the violation of system constraints. However, the general FCCP models can only handle fuzzy uncertainties in the right-hand side constraints, while imprecise parameters in the left-hand side constraints being presented as interval numbers (Pishvaee et al., 2012; Guo et al., 2013; Zeng et al., 2014). Therefore, this study aims to develop a full fuzzy-interval credibility-constrained nonlinear programming (FFICNP) model for tackling both single uncertainty and dual uncertainties existing in the objective function and double-sided constraints simultaneously. Since the \( \alpha \)-cut method and credibility measure proposed in previous studies (Guo et al., 2014b; Zhang and Guo, 2018) cannot be used for solving the FFICNP model, an expected-value-based (EVB) method based on credibility measure and expected value theory (Heilpern, 1992) is integrated with the IPP solving method. Based on such a novel solving method, the FFICNP model can be transformed into two deterministic ones, consisting of upper bound submodel and lower bound submodel with predetermined credibility levels. Thus, the FFICNP can be solved and generate flexible solutions for irrigation water management.

This study will apply the proposed FFICNP approach to a real-world case study of irrigation water allocation under uncertainty in the ZID for demonstrating its applicability. The framework of study system is shown in Fig. 2. As the first attempt of integrating intervals, fuzzy-interval sets, and FCCP within a general Jensen model optimization framework, the FFICNP model has four distinct advantages: 1) it can accurately reflect the relationship between water demands and crop yields during different growth stages; 2) it can address precipitation variabilities and imprecise water demands, as well as multiple uncertainties expressed as intervals (single uncertainty) and fuzzy-interval sets (dual uncertainties) in both objectives and constraints of the water allocation models; 3) it can help analyze the tradeoffs between economic benefits and risk of violation constraints under different risk-aversion levels; 4) it can generate a set of decision alternatives under different hydrological years and credibility levels to further help local water managers identify the optimal management strategies.

2. Overview of the study area

The Zhanghe Irrigation District (ZID) (Fig. 3) is located in the Yangtze River basin of China, and covers three cities of Hubei Province: Jingmen (JM), Jingzhou (JZ), and Dangyang (DY). This area is one of the most important agriculture bases of grain production in the Province. It features a subtropical continental monsoon climate, with an average annual rainfall of 922.2 mm during the years of 1952–2016. Although the annual rainfall of ZID is high, droughts frequently occur due to the unbalanced rainfall conditions among different months and years (Chen et al., 2016). Approximately 82.6% of the yearly rainfall occurs between April and October. The difference among inter-annual distribution is up to 2.2 times (432.8 mm in 1981 and 1190.6 mm in 1983). When precipitation is insufficient in ZID, irrigation is needed to meet the water requirements of crop growth. The main crops in ZID are rice, winter rape and winter wheat. However, irrigation water is not required for winter rape or winter wheat because their water demand can be fully satisfied by rainfall and groundwater. Thus, rice is the only irrigation water consumer due to its high water demands. In ZID, rice is primarily irrigated by water from general reservoirs, ponds, and the Zhanghe Reservoir. In this paper, the total water volume from general reservoirs and ponds is defined by internal water while water from the Zhanghe Reservoir is called reservoir water. In reality, general reservoirs and ponds are operated independently following decentralized management guided by subarea managers and farmers. They could provide flexible supplies and allow farmers to gain water on-demand. Thus, internal water has been prioritized by subarea managers and farmers to meet crop water demands timely and easily. In addition, when internal water availability is insufficient, the Zhanghe Reservoir acts as a complementary water source to cope with the water shortages during crop growth periods.

The Zhanghe Reservoir, built in a tributary of the Yangtze River, can be operated for multipurpose uses including irrigation, flood control, domestic water supply, industrial use, and hydropower generation. During recent years, the increased water demands of other water users and limited water supplies have exacerbated the agricultural water shortages. Thence, how to optimally allocate the limited irrigation water resources, and thus to improve agricultural water productivity have become a growing urgent issue for the sustainable development of local agriculture. The problem is more compounded by a plenty of uncertainties existing in the agricultural water management systems, including market conditions, precipitation and evapotranspiration characteristics, and stream flows. In addition, the uncertainty of
Fig. 2. The framework of study system.

Fig. 3. The geographical position of study area.
available water supplies may be influenced by extreme weather conditions, thus creating a baffle to the water resources allocation (Hu et al., 2016a, b). Besides, the different preferences of decision makers due to cognitive vagueness, and the various requirements of crop yields along growth stages make it more complicated to obtain appropriate water allocation schemes. Therefore, in response to above issues of the ZID, it is crucial to develop a comprehensive systematic approach of agricultural water resources allocation under uncertainty.

In this study, irrigation water allocation needs to be planned for four growth stages, which include the tillering stage, booting stage, heading stage and milky stage. The data of growth period division, water sensitivity and potential crop production are obtained from the previous field researches (Zhang et al., 2001 and Chen et al., 2017), as presented in Table 1. The crop water requirement under each growth stage is computed according to the method recommended by Allen et al. (1997). The daily reference of crop evapotranspiration is obtained by applying the FAO 56 Penman-Monteith method to the meteorological data (from 1988 to 2017) of Tuanlin meteorological station located at the middle and upper reaches of ZID. The total crop potential evapotranspiration in each growth stage can be calculated by summing up the daily values within that growth stage. The distribution is used to estimate the range of interval parameters, where the 95% confidence level of lower and upper bounds are respectively adopted as the lower and upper values of interval parameters. Furthermore, these subareas are assumed to have same soil properties and same hydrological or meteorological conditions, so that they can also use the same crop water production function.

Rainfall in each period follows a Pearson type III distribution (Fig. 4), which can further be divided into three hydrological levels based on the frequency analysis method (Li et al., 2016b): namely the levels of wet year (P < 25%), normal year (25% ≤ P ≤ 75%), and dry year (P > 75%). Environmental capacity (e.g. available water) is subject to human judgments (Xu and Qin, 2010) and difficult to be defined by its accurate characteristics, which can better be expressed as fuzzy-interval sets (FIS) with fuzzy-interval membership functions. Thus, the rainfall and water supply availability are represented as FIS shown in Table 2 and Table 3, respectively. Crop planting area is also characterized as FIS in both objective function and left-hand constraints because of irregular changes and statistical inaccuracy. Table 4 presents some basic data provided by the Hubei Zhanghe Project Administration Bureau, including crop planting area, market price, cultivation expense, and water use efficiency.

### 3. Mathematical model development

This section is to develop a FFICNP model for the case study of ZID. It emphasizes on (1) how to couple multiple uncertainties associated with credibility constraints within a general nonlinear optimization framework and build a FFICNP model; (2) how to use the developed model to balance the constraint-violation risks and the system benefits under various complexities and uncertainties; (3) how to solve this FFICNP model through the EVB method and generate effective crop water allocation schemes.

#### 3.1. Full fuzzy-interval credibility-constrained nonlinear programming (FFICNP)

In order to accurately reflect the impact of water supply on crop productivity during different growth stages, this study needs to establish a crop water production function (CWPF) for the irrigation water allocation system of ZID. According to Zhang et al. (2001), the Jensen model is considered to be superior to other CWPFs for describing rice growth in Hubei province. To better address the uncertain

### Table 1

<table>
<thead>
<tr>
<th>Period</th>
<th>Growth stage</th>
<th>date</th>
<th>days</th>
<th>Sensitive index</th>
<th>Water requirement (mm)</th>
<th>Potential crop production (kg/ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tillering</td>
<td>5.25-7.11</td>
<td>48</td>
<td>0.659</td>
<td>[216.3, 242.2]</td>
<td>[10500, 12500]</td>
</tr>
<tr>
<td>2</td>
<td>Booting</td>
<td>7.12-7.26</td>
<td>15</td>
<td>0.405</td>
<td>[197.0, 217.8]</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Heading</td>
<td>7.27-8.5</td>
<td>10</td>
<td>0.524</td>
<td>[53.6, 62.9]</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Milky</td>
<td>8.6-8.15</td>
<td>10</td>
<td>0.244</td>
<td>[155.8, 172.2]</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4. Rainfall distributions during different crop stages.
characteristics of the ZID, this study requires the integration of multiple uncertain programming techniques (Veiseh et al., 2018) to deal with parameters expressed as the fuzzy-interval sets (FIS) (e.g. effective precipitation, planting area, and available water supply) or intervals (e.g. crop price, crop yield) in the model objective and double-sided constraints. Besides, this study desires to reflect the complexities associated with the violation of system constraints. Therefore, a FFICNP model can be formulated as follows to tackle all these issues in the irrigation water management of ZID:

\[
\begin{align*}
\text{Max}_{\mathbf{z}} & = \sum_{i=1}^{n} \left( B_i \omega_i \bar{A}_i \bar{Y}_i \prod_{j=1}^{n} \left( \frac{W_j^z + IW_j^z + \bar{EP}_j^z}{ET_j^z} \right)^{\gamma_j} \right) - \sum_{i=1}^{n} \bar{A}_i D \\
\text{subject to} & \sum_{j=1}^{n} \omega_i \bar{A}_i \bar{Y}_i \prod_{j=1}^{n} \left( \frac{W_j^z + IW_j^z + \bar{EP}_j^z}{ET_j^z} \right)^{\gamma_j} \leq \bar{Q}_j^z \eta_j \geq \beta, \forall i \\
\end{align*}
\]

where \( f^z \) is the net system benefit (CNY); \( i \) denotes subarea (\( i = 1, 2, 3 \)); \( j \) denotes growth stage (\( j = 1, 2, 3, 4 \)); \( B_i \) is the crop price (CNY / kg); \( \bar{Y}_i \) is the maximum crop yield under full irrigation (kg / ha); \( \omega_i \) is the decision variable denoting irrigation ratio of subarea \( i \), which refers to the ratio of the irrigated area to the total planted area and varies among \([0, 1] \); \( \bar{A}_i \) represents the FIS of planting area in subarea \( i \) (ha); \( \bar{W}_j^z \) is the decision variable denoting the irrigation reservoir water during growth stage \( j \) (mm); \( \bar{I}_j^z \) is the decision variable denoting the irrigation internal water during growth stage \( j \) (mm); \( \bar{EP}_j^z \) denotes the FIS of effective precipitation during stage \( j \) (mm); \( \bar{ET}_j^z \) is the maximum evapotranspiration during stage \( j \) (mm); \( k_i^z \) is water sensitivity index within stage \( j \); \( D \) represents the planting cost (CNY/ha); \( C_i \) and \( C_j \) delegate the cost of irrigated water from reservoir and ponds, respectively (CNY/m³). The model objective is subjected to several constraints including water availability, irrigation water demand, irrigation ratio, and non-negative constraints.

(1) Reservoir water availability constraint

\[
\begin{align*}
\text{Reservoir water availability constraint} & = \sum_{k=1}^{K} W_{jk}^z = 0, \forall j \\
\text{subject to} & \sum_{j=1}^{n} \omega_i \bar{A}_i \bar{Y}_i \prod_{j=1}^{n} \left( \frac{W_j^z + IW_j^z + \bar{EP}_j^z}{ET_j^z} \right)^{\gamma_j} \leq \bar{Q}_j^z \eta_j \geq \beta, \forall i \\
\end{align*}
\]

where \( \bar{Q}_j^z \) is the FIS of available reservoir water (10⁴ m³); \( \eta_j \) is the efficiency of irrigation water utilization; \( \beta \) is the credibility level of the constraints. When the credibility value of a fuzzy event achieves 1, the fuzzy event will surely happen; when the credibility reaches 0, the event will certainly not occur (Huang, 2006; Zhang et al., 2015). The decision makers (DMs) can choose the preferred credibility levels for the constraints to balance the system benefits and the violation risk.

(2) Internal water availability constraint

\[
\begin{align*}
\text{Internal water availability constraint} & = \sum_{i=1}^{n} \omega_i \bar{A}_i \bar{Y}_i \prod_{j=1}^{n} \left( \frac{W_j^z + IW_j^z + \bar{EP}_j^z}{ET_j^z} \right)^{\gamma_j} \leq \bar{Q}_j^z \eta_j \geq \beta, \forall i \\
\end{align*}
\]

where \( \bar{Q}_j^z \) is the FIS of available internal water (10⁴ m³); \( \bar{Q}_j^z \) is the FIS of available internal water in subarea \( i \) (10⁴ m³).

(3) Irrigation water demand constraint

\[
\begin{align*}
\text{Irrigation water demand constraint} & = \sum_{k=1}^{K} W_{jk}^z = 0, \forall j \\
\text{subject to} & \sum_{j=1}^{n} \omega_i \bar{A}_i \bar{Y}_i \prod_{j=1}^{n} \left( \frac{W_j^z + IW_j^z + \bar{EP}_j^z}{ET_j^z} \right)^{\gamma_j} \leq \bar{Q}_j^z \eta_j \geq \beta, \forall i \\
\end{align*}
\]

where \( \varepsilon \) represents the minimum water demand coefficient.

(4) Irrigation ratio constraint

\[
\begin{align*}
\text{Irrigation ratio constraint} & = \sum_{i=1}^{n} \omega_i \bar{A}_i \bar{Y}_i \prod_{j=1}^{n} \left( \frac{W_j^z + IW_j^z + \bar{EP}_j^z}{ET_j^z} \right)^{\gamma_j} \leq \bar{Q}_j^z \eta_j \geq \beta, \forall i \\
\end{align*}
\]

(5) Nonnegative constraint

\[
W_j^z \geq 0, \forall j
\]
The FFICNP model can deal with multiple uncertainties existing in both the objective function and the constraints in a nonlinear programming framework. Furthermore, the FFICNP model can generate flexible solutions and support in-depth analysis under various scenarios by introducing different credibility levels. A new solving method based on the EVB method in Zhang (2011) and the IPP method is also proposed for solving the FFICNP model, which entails the steps being summarized as follows:

Step 1: After formulating the FFICNP model, transform the FIS in the objective function into their expected values. For example, the expected value (Heilpern, 1992) of objective function into their expected values. For example, the expected summary as follows:

\[ \text{Maxf} = \sum_{i=1}^{l} (B^i \omega_i EV (\bar{A}^i_j) Y_m) \prod_{j=1}^{j} \left( \frac{W^j_f + IW^j_f + EV(\bar{E}^j_f)}{ET^j_f} \right)^{h_j} - \sum_{i=1}^{i} EV (\bar{A}^i_j) D - \sum_{i=1}^{i} \sum_{j=1}^{j} EV (\bar{A}^i_j) (C_1 W^j_j + C_1 IW^j_j) \]  

s.t.

\[ 2(1 - \beta) \sum_{i=1}^{i} \sum_{j=1}^{j} \omega_i A^j_i W^j_f + (2 \beta - 1) \sum_{i=1}^{i} \sum_{j=1}^{j} \omega_i A^j_i IW^j_f \leq 2(1 - \beta) Q^j_j + (2 \beta - 1) Q^j_j \]  

\[ 2(1 - \beta) \sum_{i=1}^{i} \sum_{j=1}^{j} \omega_i A^j_i IW^j_f + (2 \beta - 1) \sum_{i=1}^{i} \sum_{j=1}^{j} \omega_i A^j_i IW^j_f \leq 2(1 - \beta) Q^j_j + (2 \beta - 1) Q^j_j \]  

Step 2: Convert double-sided fuzzy-interval credibility constraints into ordinary interval ones. For example, the calculation formula of

\[ k \sum_{k=1}^{K} W^j_k = 0, jEV (\bar{E}^j_f) \geq ET^j_0 \]  

\[ k \sum_{k=1}^{K} W^j_k + EV (\bar{E}^j_f) \leq ET^j_0, jEV (\bar{E}^j_f) \leq ET^j_0 \]  

\[ 0 \leq \omega^j_i \leq 1, \forall i \]  

\[ W^j_j \geq 0, \forall j \]  

\[ IW^j_j \geq 0, \forall j \]  

Fig. 5. Irrigation ratio in different hydrological years and different credibility levels.

\[ BW^j \geq 0, \forall j \]  

\[ 3.2. \text{Expected-value-based method for solving FFICNP model} \]

Generally, to avoid high risks in the constraints violation, the significant credibility level should be greater than 0.5. Based on Step 1, the FFICNP model can be transformed to a typical interval programming model as follows:

\[ \text{Maxf} = \sum_{i=1}^{i} (B^i \omega_i EV (\bar{A}^i_j) Y_m) \prod_{j=1}^{j} \left( \frac{W^j_f + IW^j_f + EV(\bar{E}^j_f)}{ET^j_f} \right)^{h_j} - \sum_{i=1}^{i} EV (\bar{A}^i_j) D - \sum_{i=1}^{i} \sum_{j=1}^{j} EV (\bar{A}^i_j) (C_1 W^j_j + C_1 IW^j_j) \]  

s.t.

\[ 2(1 - \beta) \sum_{i=1}^{i} \sum_{j=1}^{j} \omega_i A^j_i W^j_f + (2 \beta - 1) \sum_{i=1}^{i} \sum_{j=1}^{j} \omega_i A^j_i IW^j_f \leq 2(1 - \beta) Q^j_j + (2 \beta - 1) Q^j_j \]  

\[ 2(1 - \beta) \sum_{i=1}^{i} \sum_{j=1}^{j} \omega_i A^j_i IW^j_f + (2 \beta - 1) \sum_{i=1}^{i} \sum_{j=1}^{j} \omega_i A^j_i IW^j_f \leq 2(1 - \beta) Q^j_j + (2 \beta - 1) Q^j_j \]  

\[ 0 \leq \omega^j_i \leq 1, \forall i \]  

\[ W^j_j \geq 0, \forall j \]  

\[ IW^j_j \geq 0, \forall j \]  

Step 3: Decompose the interval model into two submodels through the interactive algorithm (Huang et al., 1992; Miao et al., 2014; Xie et al., 2018; Zhang et al., 2018b, 2019). Because the objective function is to maximize \( f^z \), the upper bound submodel should be solved firstly.

Step 4: Solve the upper bound submodel and obtain corresponding alternatives under different given credibility levels.

Step 5: Solve the lower bound submodel based on the solutions obtained from Step 4 to obtain the corresponding solutions.

Step 6: Combine solutions from Steps 4 and 5 to generate the final solutions under each given level.
4. Results analysis

4.1. Optimal irrigation ratio

Flexible water allocation schemes responsive to different levels of water scarcity can be generated from the FFICNP model by adjusting the irrigation ratio. Fig. 5 presents the optimization results of irrigation ratio for each subarea under different scenarios. It can be seen that the irrigation ratio of each subarea decreases to varying degrees with the decreasing inflows. As shown in Fig. 5a, the irrigation ratio of each subarea is 1, which means complete irrigation. It is indicated that available water supply can fully meet the crop water demand in wet year. In normal year, the irrigation ratio in JZ city increases sharply when credibility level declines from 1 to 0.8. It means that the decreasing of available irrigation water has remarkably affected the irrigation ratio when the credibility level is more than 0.8. The optimization results are conservative for guiding actual activities due to the low irrigation ratio. It is important to set proper credibility levels for flexible solution alternatives and meaningful decision support. In dry year, the total irrigation area decreases significantly comparing to that of the other hydrological years. The irrigation ratio of each subarea maintains the steady upward trend along with decreasing credibility level. The results are [0.40, 0.45] for JM, [0.07, 0.10] for JZ, and [0.08, 0.09] for DY respectively. The irrigated ratio of JM City is much larger than the other two subareas. It can be speculated that this may be directly related to the planting area. In this study, it is assumed that the CWPF of rice has no spatial variability in three subareas. Thus, the model is likely to satisfy water requirement in JM city firstly due to its larger planting area than the other two subareas. Even though, the irrigation ratio in JM city is less than half of the total cultivated area. It is necessary to transfer water from other regions to ensure the local food security. Additionally, the large inter-annual differences in inflows have seriously affected the actual agricultural production and socio-economic development. Enhancing water resources reserve and regulation capacity in the irrigation district may buffer the negative impacts caused by extreme droughts.

4.2. Optimal irrigation water allocation

Fig. 6 depicts the optimal results of total irrigation water allocation under different hydrological years and credibility levels. The gap between total water distribution and total water supply varies under different scenarios. It can be seen that internal water supply is almost exhausted in each hydrological year, because the price of internal water is set slightly lower than reservoir water. The available reservoir water is mostly used in normal year and dry year, while it accounts for approximately 70%–100% of the total water supply in wet year. It means that there is plenty of water surplus after meeting the basic needs of crop irrigation in wet year. Additional economic benefits can be achieved if the remaining water is stored or transferred to other water sectors.

Fig. 7 depicts the optimized water allocation, effective rainfall, and crop water demand of each growth stage under a compromised credibility of 0.8. The results are presented as interval values. In JM city, crop water demand in the heading stage can be well met by irrigation and precipitation, followed by the tilling stage and booting stage. It indicates that water shortage in the milk stage has a smaller impact than that in other stages, which coincides with the expectations considering the sensitive index distribution of middle rice. Irrigation water allocated to each growth stage increases with decreasing rainfall and reaches a peak in dry year. The actual water supply to each growth stage decreases firstly and then increases, while the scenario varies from wet year to dry year. It indicates that proper deficit irrigation with large cultivation area is better in normal year; while the crop water demand needs to be satisfied to the greatest extent in dry year. Because the production loss by deficit irrigation can be compensated through large area being irrigated in normal year. However, less than half of the total cultivation area can be irrigated in dry year, resulting in different water allocation policy. Similar to JM city, water scarcity in JZ city and DY city is the largest in the milk stage, followed by the booting stage,
The results also show that less water is allocated to the tilling stage, even though it has the largest water requirement, which is directly related to the rainfall. The rainfall has a grand impact on irrigation water allocation and has a downward trend from the tilling stage to the milky stage in three types of hydrological years. It can also be seen that the tilling stage and heading stage have stronger sensitivity to water scarcity. The production loss due to water scarcity in these two key growth stages is difficult to compensate through more irrigation in the other growth stages. Hence, irrigation water should be given priority to the tilling stage and heading stage. Among that, the heading stage has a shorter growth period and less water requirement than the tilling stage, which is more likely to be satisfied. Therefore, several suggestions can be proposed as follows: 1) rainfall during the early growth period can be an indicator to judge the inflow level of the whole growth period, because it decreases from the tilling stage to the milky stage; 2) crop water requirement during the heading stage (July 27 to August 5) must be satisfied under all scenarios; 3) optimal schemes obtained from the FFICNP model provide better choices to guarantee system benefits when irrigation supply is insufficient. Furthermore, the optimal results can help generate reasonable irrigation water allocation, and mitigate water scarcity caused by large inter-annual differences in inflows.

### 4.3. Net system benefits

Generally, crop water and irrigation area are allocated through solving the FFICNP model, thus the net system benefits can be obtained. Under different hydrological years and credibility levels, the solution alternatives and the corresponding economic benefits are different. The solutions are presented as interval values due to the uncertainty of inputs parameters. Fig. 8 shows the system net benefits along with the change of the hydrological years and credibility levels. The system benefits have a deceasing trend from the wet year to the dry year for both lower bound and upper bound. That is because more water can be supplied to the crops under a larger amount of inflows, which leads to the higher crop production. Also, there is a downward trend from $\lambda = 0.6$ to $\lambda = 1$, which is mainly driven by a tight decision space in credibility constraints. In general, the higher credibility level corresponds to the better constraints satisfaction and the higher confidence level of model results. Thus, the credibility level can be regarded as an indicator of how reliable the optimization results would be.

Additionally, it is obvious that hydrological years have far more influences on the decision-making than credibility levels. When the credibility level reaches 1, the system benefits achieve $[17.72, 24.23] \times 10^9$ CNY in wet year, $[8.43, 11.21] \times 10^9$ CNY in normal year, and $[1.51, 3.99] \times 10^9$ CNY in dry year, respectively. The difference of system benefits between different scenarios is substantial. It indicates that there is a necessity to tackle the water allocation problem under different scenarios. Furthermore, it proves that the amount of irrigation water resources is a vital factor to develop local agricultural economy. More practically, the storage capacity of agricultural water resources in the ZID should be enhanced. Meanwhile, more attention should be paid on how to efficiently allocate limited water resources under low inflows.

### 4.4. System efficiency

The system efficiency, defined as the total economic benefits per unit water supply or water productivity, is an important index to incarnate the optimality of water allocation results. Fig. 9 depicts the system efficiency by subarea under different scenarios. It is found that a higher credibility level corresponds to lower benefits per unit water supply. Take the normal year scenario as an example, when the credibility level increases from 0.6 to 1, the benefits per unit water supply varies from $[2.35, 2.97]$ CNY/m³ to $[2.19, 2.78]$ CNY/m³. It demonstrates that the credibility level has more significant and direct influence on economic benefits than water allocation. As shown in Fig. 9b, the lowest water productivity is in JZ city, which may be driven by the low crop production and the low irrigation ratio. Besides, the results suggest that the credibility level has a slight impact on the water productivity. Decreasing credibility level would improve the water productivity, especially from 0.8 to 0.6. However, under some scenarios, the water productivity reaches the lowest value when $\lambda$ is 0.8. Theoretically, the conservative estimate of water allocation would not only contribute to more net benefits, but also lead to higher system efficiency. The results can help managers choose the appropriate credibility level based on their own preferences. For example, a decision maker would choose a compromised credibility level less than 0.8 for a high system efficiency. Furthermore, the results demonstrate that the FFICNP model is applicable for tackling this water management problem of ZID.

### 5. Discussion

#### 5.1. Comparison with the INP model

To better summarize the characteristics and advantages of the FFICNP model, an interval nonlinear programming (INP) model is introduced into the study for comparison. The objective function of INP model is to maximize the net system economic benefits with the discrete interval parameters without introducing the irrigation ratio and credibility constraints. Fig. 10 depicts the comparison of the net system benefit and water productivity of two models in different scenarios. As shown in Fig. 10a, the differences between the upper bound and lower bound of INP model results are substantial in each hydrological year, which lead to a lower practicality than the FFICNP model results. It also indicates that the FIS (dual uncertainty) would help reflect parameter uncertainties better than the general intervals (single uncertainty). Additionally, the INP model leads to higher economic benefits than the FFICNP model in normal year, which is mainly determined by input parameters. In dry year, the net system benefits obtained from INP
model are negative, which means huge economic loss will be faced by the farmers. However, this situation can be effectively avoided through using the FFICNP model with consideration of irrigation ratio.

Taking the JM city as an example, the water productivity of two models can be seen in Fig. 10b. The water productivity results obtained from the FFICNP model are greater than those of the INP model, which can be interpreted by the large amount of irrigation water resources consumed. Additionally, the INP model can only provide simple interval solutions with lower bound and upper bound, while more flexible solutions can be generated by the FFICNP model under different risk levels of constraints violation. In summary, the developed FFICNP model is superior to an INP model in its applicability and performance on providing flexible and effective schemes for irrigation water allocation in the ZID.

5.2. Effect of input parameters

In order to further explore how the key input parameters would affect the outputs, the following 13 settings with different inputs for three key parameters are tested, which include: (a) four settings of the internal water storage capacities: 0.8IQ, 1.2IQ, 1.6IQ, and 2.0IQ, where IQ denotes the actual internal water storage capacity; (b) four settings of the cultivation areas: 0.8A, 1.2A, 1.6A, and 2.0A, where A is the actual crop cultivation area of three subareas; (c) five settings of the minimum irrigation ratio: 0, 0.1, 0.2, 0.3, and 0.4. All these 13 settings above were simulated by FFICNP model, and the corresponding results under λ = 0.8 are shown in Fig. 11. The results illustrate that three parameters have different effects on output results. As displayed in Fig. 11a, the system economic benefit remains stable in wet year, while it climbs along with increasing water storage capacity in normal year and dry year. On the one hand, it indicates that the internal water storage capacity is completely sufficient and there is no need to expand it in wet year. On the other hand, increasing the water storage capacity is a sensible way to improve food production in normal year and dry year in the ZID. It can be speculated that improving the water supply availability of other sources, including reservoir water and transferring water from other regions, has a similar effect on system economic benefits. As presented in Fig. 11b, the economic benefits can be improved remarkably along with the increased crop cultivation area in wet year. On the contrary, increasing planting areas has a serious negative influence on economic benefits in other hydrological years,
which is caused by the increase of crop planting cost without corresponding income. As shown in Fig. 11c, net system benefits have a little sensitivity to the minimum irrigation ratio. Hence, expanding the planting area is a key factor for economic development in wet year, while reducing the cultivation area is necessary in normal year and dry year. Furthermore, engineering measures, including increasing water storage capacity, transferring water from other regions, and improving the canal water use efficiency, are urgent for improving crop production and system economic benefits when low inflow level occurs.

6. Conclusions

A full fuzzy-interval credibility-constrained nonlinear programming (FFICNP) approach has been developed for supporting irrigation water allocation along crop growth stages under uncertainty. The developed FFICNP model can deal with multiple uncertain parameters characterized as intervals or fuzzy-interval sets in the objective function and double-sided constraints. It can also support in-depth analysis on trade-offs among the solutions’ credibility level and maximum system benefits. Furthermore, a novel solving method based on the EVB and IPP methods is proposed for solving the FFICNP model.

The FFICNP model has been applied to a real-world case study of irrigation water allocation in the Zhanghe irrigation district, Yangtze River basin of China. Three types of hydrological years and five credibility levels have been considered for the case study. The obtained results have provided effective decision-making support for the local irrigation water management. The results indicate that: (1) rainfalls and available water supplies have stronger impacts on crop water allocation and production than the credibility levels; (2) a lower credibility level corresponds to higher levels of net benefits and system efficiency, and abundant optimal water allocation schemes can help decision makers formulate appropriate policies; (3) transferring external water and enhancing water storage capacity are effective measures to address water shortage in terms of shrinking inflows.

This study has formulated a framework to address irrigation water allocation problems under multiple uncertainties. The research has provided a new perspective for precision enhancement of crop water allocation system characterization by introducing multi-fold uncertainty technique into a Jensen model framework. Both the model development and practical application in the ZID have formed the foundation for future research of agricultural water management under uncertainty. The model framework and solving method could also be applied to other similar regions to help develop better water-allocation schemes and improve the effectiveness of agricultural water management. However, the corresponding solving method is worthy of continual exploration in our future studies through taking spatial variability of rainfall into account.

Declaration of Competing Interest

The authors declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

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