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An Interval-based Fuzzy Chance-constrained Irrigation Water Allocation model with double-sided fuzziness

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ABSTRACT

This study presents an Interval-based Fuzzy Chance-constrained Irrigation Water Allocation (IFCIWA) model with double-sided fuzziness for supporting irrigation water management. It is derived from incorporating double-sided chance-constrained programming (DFCCP) into an interval parameter programming (IPP) framework. The model integrates interval linear crop water production functions into its general framework for irrigation water allocation. Moreover, it can deal with uncertainties presented as discrete intervals and fuzziness. It can also allow violation of system constraints with double-sided fuzziness, where each confidence level consists of two reliability scenarios (i.e. minimum and maximum reliability scenarios). To demonstrate its applicability, the model is then applied to a case study in the middle reaches of the Heihe River Basin, northwest China. Therefore, optimal solutions have been generated for irrigation water allocation under uncertainty. The results indicate that planning under a lower confidence level and a minimum reliability scenario can provide maximized system benefits. System benefits under the high water level are [2.659, 7.913] $\times$ 10^9 Yuan when $\alpha = 0$, [2.650, 7.822] $\times$ 10^9 Yuan when $\alpha = 0.5$ and [2.642, 7.734] $\times$ 10^9 Yuan when $\alpha = 1.0$ under the minimum reliability scenario. Furthermore, the results can support in-depth analysis of interrelationships among system benefits, confidence levels, reliability levels and risk levels. These results can effectively provide decision-support for managers identifying desired irrigation water allocation plans in study area.

1. Introduction

Nowadays, there is a growing awareness of the necessity to effectively alleviate the contradiction between the increasing demands for agricultural production and the shortages of agricultural water supply from a global perspective, which has a profound effect on arid areas already dominated by irrigated agriculture (Elliott et al., 2014; Kang et al., 2017). In fact, irrigation water consumption accounts for nearly 90% of the total water availability in arid areas of northwestern China (Li et al., 2016a). Moreover, unscientific and unreasonable irrigation water management can also directly cause environmental and ecological degradations and natural resources shortages problems. Therefore, it is indeed necessary to improve irrigation water management and optimize irrigation water allocation, which will ensure the sustainable development of agricultural production (Lu et al., 2016).

Optimizing irrigation water allocation, in the technical sense, implies how much water should be allocated to different subareas under certain goals (Zeng et al., 2010). Therefore, various mathematical methods have been developed for irrigation planning and management to identify optimal solutions (Singh and Panda, 2012), including traditional methods including linear programming (Bartolini et al., 2007), nonlinear programming (Cai et al., 2001), dynamic programming (Shang and Mao, 2006), and artificial intelligence search methods like genetic algorithms (Arabi et al., 2006; Safavi and Esmikhani, 2013) and simulated annealing (Brown et al., 2010; Pérez-Sánchez et al., 2018). These techniques have made significant contributions to the development of irrigation water management. However, the above methods may have limitations in response to uncertainties (e.g. stochastic, fuzzy and interval variables/parameters) existing in irrigation water management problems. Practically, an irrigation system typically covers a multitude of aspects associated with resources capacity, economic development and environmental impact (Xu and Qin, 2010), leading to uncertain factors such as water availability, irrigation water demand, market price, and crop yields. Such inherent uncertainties may cause intensified difficulties in the decision making of practical applications.

Therefore, a series of inexact mathematical programming methods

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including interval mathematical programming (IMP), stochastic mathematical programming (SMP) and fuzzy mathematical programming (FMP) have been developed for generating effective decision solutions under uncertainty. Generally, SMP can provide more explicit decision solutions while it may be impeded by its rigorous data requirements, complicated probability analysis and time-consuming computational burden (Xu and Qin, 2010; Tan et al., 2011). Conversely, IMP, based on interval analysis using interval parameters or variables, is more convenient for dealing with uncertain information with known ranges but unknown distributions. More practically, as a basic tool for irrigation planning and management, crop water production functions (CWPFs) can be empirically obtained by the fitting results based on field experimental data. These data, including actual evapotranspiration (ET) and crop yields, are easily influenced by the measurement methods, observation error and calculation methods, leading to imprecise and uncertain specifications of CWPFs. Therefore, interval CWPFs can be considered as a better choice to quantitatively describe the relationship between ET and crop yield in practical problems (Tong and Guo, 2013; Li et al., 2016b). Accordingly, IMP is capable of handling such a problem of integration interval CWPFs and other interval information into its optimization framework. For example, Li et al. (2016b) developed an interval linear fractional irrigation water allocation model by integrating interval CWPFs into the model's framework. However, it does not allow violation of system constraints and may be infeasible when the right-hand side coefficients of constraints are highly uncertain (Huang et al., 1992). Furthermore, some parameters are subject to human judgments, and the linguistic terms of "approximately equal" and "approximately satisfactory" are more acceptable in decision making (Zeng et al., 2010). It is desired that optimization methods be developed to further address above-mentioned problems.

Therefore, fuzzy chance-constrained programming (FCCP), as an improved FMP method, can be introduced to effectively tackle fuzzy uncertainties and violation of system constraints. The fuzzy constraints can be transformed into deterministic ones at predetermined confidence levels, which has a lower computational burden and provides more flexible solutions. There are two types of FCCP model from the literature review, including chance-constrained programming with fuzzy parameters (Liu and Iwamura, 1998) and chance-constrained programming with DFP (distribution with fuzzy probability) parameters (Iskander, 2005; Guo and Huang, 2009; Guo et al., 2014). For example, Guo and Huang (2009) developed a two-stage fuzzy chance-constrained programming approach for water resources management under dual uncertainties and applied it to a hypothetical case. Zhang and Guo (2018) developed a fuzzy linear fractional programming model with double-sided fuzziness for irrigation water management. Although the FCCP model with DFP parameters can reflect the dual-uncertainty feature (i.e. probabilistic and possibilistic information), it is difficult to acquire the dual-uncertain information and further popularize the model in practical application. Moreover, it can only address the fuzzy uncertainties in the right-hand side constraints, while those in the left-hand side constraints are presented as interval numbers. This will lead to the potential to miss some valuable uncertain information. In practice, fuzzy uncertainties may exist in both sides of constraints of the model. Thus, double-sided FCCP (DFCCP) is introduced to address the above complexities. Additionally, few studies of the DFCCP model for irrigation water management have been conducted.

Therefore, this study aims at integrating the advantages of the above outlined methods. An Interval-based Fuzzy Chance-constrained Irrigation Water Allocation (IFCIWA) model with double-sided fuzziness is developed for supporting irrigation water management. It is derived from incorporating double-sided chance-constrained programming (DFCCP) into an interval parameter programming (IPP) framework. Moreover, it integrates the interval CWPFs into its optimization framework. The objective of the developed model is to optimize irrigation water allocation to different crops in different subareas, achieving maximum system benefits. It is able to handle uncertainties that are presented as intervals and fuzzy sets arising from the subjective and objective variability of the irrigation systems. To demonstrate its applicability, it will be applied to a case study in the middle reaches of the Heihe River Basin in northwest China to optimize irrigation water allocation under uncertainty. Thus, more flexible and tractable solutions can be generated under different scenarios. These optimal solutions can provide decision support for managers to make final decisions for irrigation water allocation. Fig. 1 graphically illustrates the framework of the study.

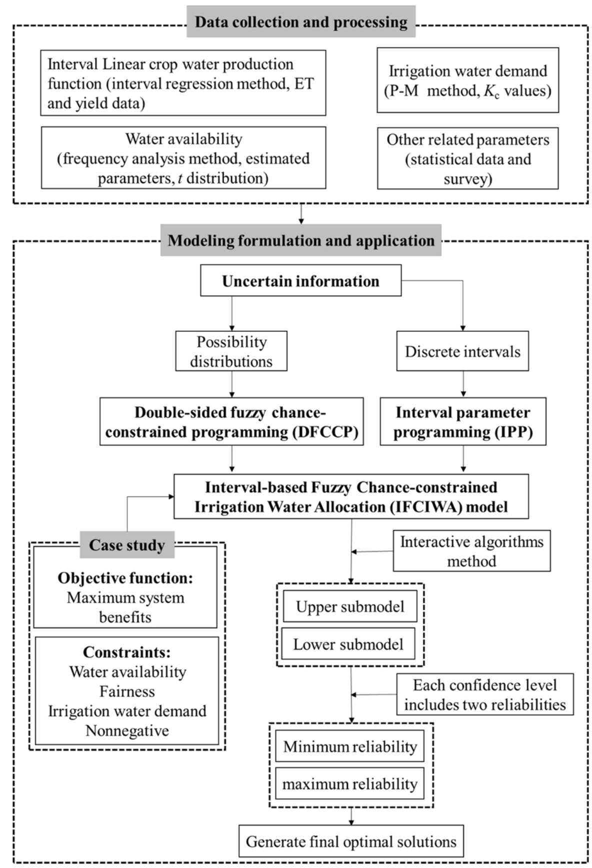

Fig. 1. The general framework of the study.

2. Methodology

In this section, three aspects of methodology including interval linear crop water production functions, Interval-based Fuzzy Chance-constrained Irrigation Water Allocation (IFCIWA) model with double-sided fuzziness and the corresponding solution method will be presented. Therefore, the further details can be outlined as follows.

2.1. Interval linear crop water production functions (ILCWPFs)

In this study, the linear CWPFs (i.e. $Y = aET + b$, where Y is the harvested crop yield, kg/ha; ET is the actual evapotranspiration, m^3/ha; and a, b are empirical coefficients mathematically determined by fitting the field experimental data) are chosen to quantitatively express the relationship between crop yield and ET (Li et al., 2016b). Due to the actual conditions, the collected field experimental data is easily influenced by measurement methods and observation errors, causing uncertainties in determining CWPFs. Using deterministic CWPFs may thereby have difficulties in dealing with these uncertainties. Thus, interval linear crop water production functions (ILCWPFs) are introduced to better reflect the above concerns. Then, an interval regression method is used to calculate ILCWPFs for different crops (Tanaka and

Lee, 1998; Tong and Guo, 2013). It can be written as follows:

$$Y(x) = A_0 + A_1x_1 + \cdots + A_nx_n = Ax \tag{1a}$$

where $x = (1, x_1, \cdots, x_n)^t$ is a real input vector; $A = (A_0, A_1, \cdots, A_n)$ is an interval coefficient vector, and $Y(x)$ is the corresponding interval output. An interval coefficient A_i is denoted as $A_i = (a_i, c_i)$ where a_i is a center value and c_i is a radius value. Now, an interval regression analysis method based on quadratic programming is adopted because it provides more diverse spread coefficients than linear programming. It also considers the central tendency of least squares and possibilistic characteristics of fuzzy regression. The objective of the interval regression based on quadratic programming is to minimize the sum of $\sum_{j=1}^{p}(y_j - a^Tx_j)^2$ and $\sum_{j=1}^{p} c^T|x_j||x_j|^T c$ by giving different weights to obtain expected interval coefficients $A_i = (a_i, c_i)$, $i = 1, \cdots, n$. Therefore, it is presented as follows:

$$\text{Min} \quad J = k_1\sum_{j=1}^{p}(y_j - a^Tx_j)^2 + k_2\sum_{j=1}^{p} c^T|x_j||x_j|^T c \tag{1b}$$

where the output y_j should be included in the output $Y(x_j)$, indicating that these outputs should be satisfied ($y_i \in Y(x_i)$, $j = 1, 2, \cdots, p$). Thus, the objective function (Eq. 1b) should be subjected to the following constraints:

$$a^Tx_j + c^T|x_j| \geq y_j, j = 1, \cdots, p \tag{1c}$$

$$a^Tx_j - c^T|x_j| \leq y_j, j = 1, \cdots, p \tag{1d}$$

$$c_i \geq 0, i = 1, \cdots, n \tag{1e}$$

where $|x_j| = (1, |x_{j1}|, \cdots, |x_{jn}|)^T$, $a = (a_0, \cdots, a_n)^T$, $c = (c_0, \cdots, c_n)^T$, k_1 and k_2 are the given weight coefficients. The larger values of k_1: k_2, the more central tendency will appear, meaning that the interval regression results approximately tend to be the results obtained from the least squares regression method. The regression results may be different with the ratio of k_1 and k_2 changes. Therefore, the corresponding results of ILCWPFs can be obtained, which are the basis of the following optimization irrigation water allocation model.

2.2. Interval-based Fuzzy Chance-constrained Irrigation Water Allocation (IFCIWA) model with double-sided fuzziness

To address interval parameters/variables and ILCWPFs in irrigation water management problems, interval parameter programming (IPP) can be used to solve such a problem. Moreover, when the parameters of both left-hand and right-hand sides in the constraints are fuzzy sets that can be expressed as possibility distributions, and the violation of system constraints exists in the optimization model, the double-sided fuzzy chance-constrained programming (DFCCP) method can be adopted (Fiedler et al., 2006). Therefore, an Interval-based Fuzzy Chance-constrained Irrigation Water Allocation (IFCIWA) model with double-sided fuzziness is formulated to allocate irrigation water to different crops and subareas. The system objective is then presented as follows:

$$\max \quad f^{\pm} = \sum_{i=1}^{3}\sum_{j=1}^{3}(NB_{ij}^{\pm} - CP_{ij}^{\pm})A_{ij}^{\pm}[a_j^{\pm}(SW_{ij}^{\pm} + GW_{ij}^{\pm} + P_{e,i}^{\pm}) + b_j^{\pm}] - \sum_{i=1}^{3}\sum_{j=1}^{3}A_{ij}^{\pm}(CS_i^{\pm}SW_{ij}^{\pm}/\eta_s + CG_i^{\pm}GW_{ij}^{\pm}/\eta_g) \tag{2a}$$

where $f^{\pm}$ is system objective that is to maximize system benefits from the agricultural production, 10^9 Yuan; $f^{\pm}$ is an interval variable, and the '+' and '−' represent the upper and lower bounds of an interval parameter or variable. i is subarea ($i = 1, 2, 3$); j is type of crop ($j = 1, 2, 3$); $NB_{ij}^{\pm}$ is the price of crop j in subarea i (Yuan/kg); $CP_{ij}^{\pm}$ is the cost of crop production for crop j in subarea i, including all the costs such as seed, fertilizer, pesticides, machinery, harvesting and other costs (Yuan/kg); $A_{ij}^{\pm}$ is the irrigated area of crop j in subarea i (ha); $a_j^{\pm}$ and $b_j^{\pm}$ are the empirical coefficients of the ILCWPFs for crop j; $SW_{ij}^{\pm}$ and $GW_{ij}^{\pm}$ are the decision variables denoting the amount of irrigated surface water and groundwater for crop j in subarea i (m^3/ha); $P_{e,i}^{\pm}$ is the effective precipitation of subarea i (m^3/ha); $CS_{ij}^{\pm}$ and $CG_{ij}^{\pm}$ are the cost of surface water and groundwater use in subarea i (Yuan/m^3); η_s and η_g are the comprehensive irrigation water use coefficients of surface water and groundwater.

The system objective, i.e. Eq. (2a), is subjected to the following system constraints.

(1) Surface water availability constraint

$$Pos\left\{\tilde{\lambda}_s, \tilde{Q}_s \,\middle|\, \sum_{i=1}^{3}\sum_{j=1}^{3}(1 + \tilde{\lambda}_s)A_{ij}^{\pm}SW_{ij}^{\pm} \leq \tilde{Q}_s\eta_s\beta_s\right\} \geq \alpha \tag{2b}$$

(2) Groundwater availability constraint

$$Pos\left\{\tilde{\lambda}_g, \tilde{Q}_g \,\middle|\, \sum_{i=1}^{3}\sum_{j=1}^{3}(1 + \tilde{\lambda}_g)A_{ij}^{\pm}GW_{ij}^{\pm} \leq \tilde{Q}_g\eta_g\beta_g\right\} \geq \alpha \tag{2c}$$

where constraints (2b) and (2c) are the water availability constraints. $\tilde{\lambda}_s$ and $\tilde{\lambda}_g$ are the rates of surface water and groundwater loss during water conveyance that are presented as fuzzy sets. $\tilde{Q}_s$ and $\tilde{Q}_g$ are the surface water and groundwater availabilities, which are also expressed as fuzzy sets (10^6 m^3). β_s and β_g are the proportion of surface water and groundwater used for agricultural irrigation. In this study, $\beta_s = 0.9$ and $\beta_g = 0.9$ in the above constraints (Li et al., 2016a). α denotes α -cut confidence level and it describes the fuzzy degree of membership level.

(3) Fairness constraint

$$\frac{\sum_{i_1}^{I}\sum_{i_2}^{I}|(SW_{i_1j}^{\pm} + GW_{i_1j}^{\pm} + P_{e,i_1j}^{\pm}) - (SW_{i_2j}^{\pm} + GW_{i_2j}^{\pm} + P_{e,i_2j}^{\pm})|}{2n\sum_{i=1}^{I}(SW_{ij}^{\pm} + GW_{ij}^{\pm} + P_{e,ij}^{\pm})} \leq G_0, \forall j \quad and \quad i_1, i_2 \in I \tag{2d}$$

where i_1 and i_2 are the two subareas of the three subareas independently. n, the number of total subareas; G_0 is the Gini coefficient, which is generally used to measure economic inequality. This constraint can be interpreted as the ratio between the sum of water shortages of each two subareas and total water demands is not greater than an acceptable gap.

Understandably, for some subareas with higher water use efficiency and net benefit will be allocated more water, this may not be fair to farmers with lower water use efficiency. Therefore, to achieve a balance to some degree, the fairness constraint is considered by introducing the Gini coefficient (Yang et al., 2015). Theoretically, it ranges from 0 to 1 and higher value represents more unequal distribution. Less than 0.2 denotes the absolute average; 0.2–0.3 denotes relatively average; 0.3–0.4 denotes relatively reasonable; 0.4–0.5 represents relatively bigger gap and more than 0.5 can be regarded as disparity (Sun, 2013). Absolute average indicates that each farmer is distributed exactly the same amount of money; relatively average indicates that each farmer is distributed almost the same amount of money and a relatively reasonable refers to the existence of an acceptable gap. A value of $G_0 = 0.4$ was selected for fairness constraints because the international warning level is 0.4 (Zhang and Xu, 2011).

(4) Irrigation water demand constraints

$$ET_{\min,ij}^{\pm} \leq SW_{ij}^{\pm} + GW_{ij}^{\pm} + P_{e,ij}^{\pm} \leq ET_{\max,ij}^{\pm}, \quad \forall i, j \tag{2e}$$

$ET_{\min,ij}^{\pm}$ and $ET_{\max,ij}^{\pm}$ are the minimum and maximum irrigation water requirements of crop j in subarea i (m^3/ha).

(5) Nonnegative constraints

$$SW_{ij}^{\pm} \geq 0, \quad GW_{ij}^{\pm} \geq 0, \quad \forall i, j \tag{2f}$$

$SW_{ij}^{\pm}$ and $GW_{ij}^{\pm}$ are the decision variables, which should be positive.

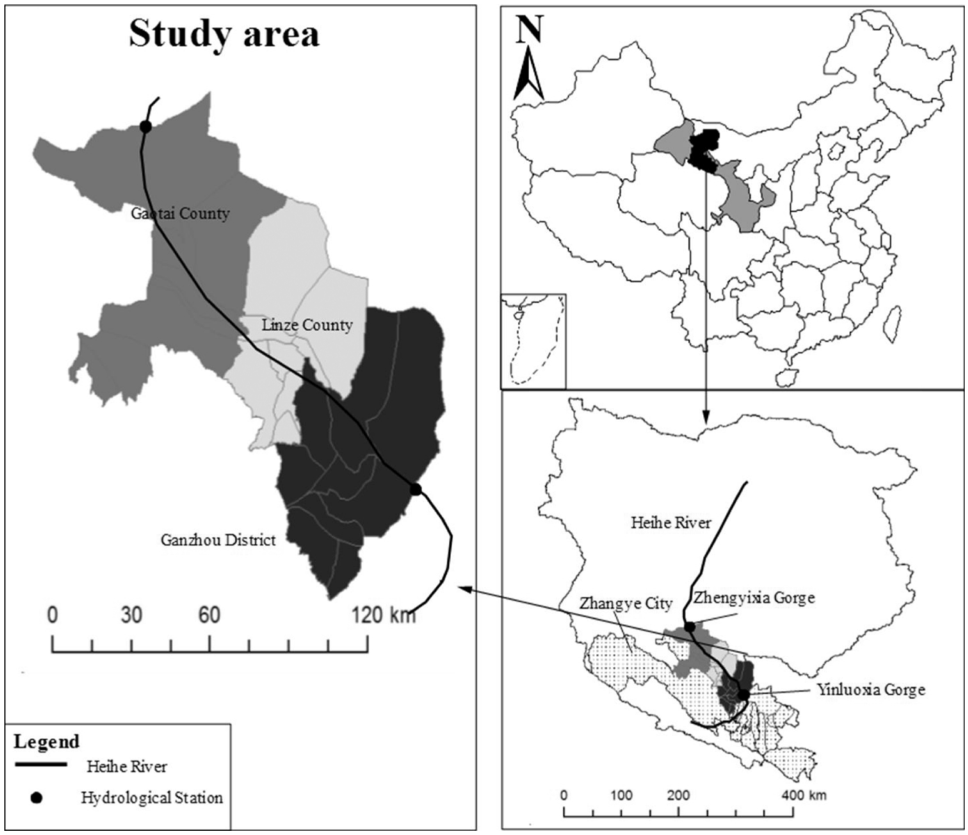

Fig. 2. The study area.

2.3. Solution method

To solve the IFCIWA model, it should be transformed into deterministic submodels. Therefore, the solution method consists of two parts, i.e. solution methods of IPP and DFCCP, respectively.

First, it should be transformed into the upper and the lower bounds submodels based on the interactive algorithms method proposed by Huang et al. (1992). Because the objective function is maximum system benefits, the submodel corresponding to f^+ (i.e. upper bound submodel) should be first formulated and solved. Subsequently, the submodel corresponding to f^- (i.e. lower bound submodel) is formulated and solved based on the solutions of the upper bound submodel.

Second, for the upper or lower submodels, according to Fiedler et al. (2006), the violation of constraints with double-sided fuzziness in the submodel at each confidence level consists of two reliability scenarios (i.e. the minimum reliability and maximum reliability). They are defined as follows:

$$Pos\{\tilde{r}_{ij} \lessapprox^{\max} \tilde{v}_{ij}\} = \inf\{\max(1-\mu_{\tilde{r}_{ij}}(x_{ij}), 1-\mu_{\tilde{v}_{ij}}(y_{ij}))\,|\,x_{ij}, y_{ij} \in \Re, x_{ij} \le y_{ij}\} \ge \alpha \Leftrightarrow r_{ij}^{R}(1-\alpha) \le v_{ij}^{L}(1-\alpha) \tag{3a}$$

$$Pos\{\tilde{r}_{ij} \lessapprox^{\min} \tilde{v}_{ij}\} = \sup\{\min(\mu_{\tilde{r}_{ij}}(x_{ij}), \mu_{\tilde{v}_{ij}}(y_{ij}))\,|\,x_{ij}, y_{ij} \in \Re, x_{ij} \le y_{ij}\} \ge \alpha \Leftrightarrow r_{ij}^{L}(\alpha) \le v_{ij}^{R}(\alpha) \tag{3b}$$

where $\tilde{r}_{ij} \lessapprox^{\max} \tilde{v}_{ij}$ denotes that the equations $\tilde{r}_{ij} \lessapprox \tilde{v}_{ij}$ are satisfied at the maximum reliability while $\tilde{r}_{ij} \lessapprox^{\min} \tilde{v}_{ij}$ denotes that the equations $\tilde{r}_{ij} \lessapprox \tilde{v}_{ij}$ are satisfied at the minimum reliability. $\mu_{\tilde{r}_{ij}}(x_{ij})$ and $\mu_{\tilde{v}_{ij}}(y_{ij})$ are the fuzzy membership functions of the variables x_{ij} and y_{ij}. $r_{ij}^{L}(\alpha)$ is defined as the minimum values of all possible values at α -cut level, that is, $r_{ij}^{L}(\alpha) = \inf\{R|R = \mu^{-1}(\alpha)\}$. Likewise, $v_{ij}^{R}(\alpha)$ is defined as maximum values of all possible values at α -cut level, that is, $v_{ij}^{R}(\alpha) = \sup\{V|V = \mu^{-1}(\alpha)\}$. μ^{-1} is the inverse function of μ.

According to Eqs. (3a) and (3b), upper and lower bounds submodels at each confidence level can be transformed into two crisp equivalents accordingly. By giving different confidence levels, the final solutions of each deterministic submodel can be generated. The detailed procedure of the solution method is summarized as follows:

Step 1: Acquire the parameters of model required in terms of interval boundaries and fuzzy sets (fuzzy membership function) under different water levels.

Step 2: Formulate the IFCIWA model.

Step 3: Reformulate the developed model into two submodels through the interactive algorithms method. Based on the solution method of the IPP, the upper bound of submodel (f^+) is first solved.

Step 4: Transform the submodel corresponding to f^+ into two crisp equivalents (i.e. the minimum and the maximum reliability scenarios) based on the solution method of DFCCP.

Step 5: Give different confidence levels, optimal solutions corresponding to f^+ can be completely obtained.

Step 6: Solve the other submodel by following Steps 4 and 5 and optimal solutions corresponding to f^- can be completely obtained.

Step 7: Combine solutions from Steps 5 and 6, and the final solutions of $f_{opt}^{\pm} = [f_{opt}^{-}, f_{opt}^{+}]$, $SW_{ij,opt}^{\pm} = [SW_{ij,opt}^{-}, SW_{ij,opt}^{+}]$ and $GW_{ij,opt}^{\pm} = [GW_{ij,opt}^{-}, GW_{ij,opt}^{+}]$ can be generated under different confidence levels and water levels.

3. Case study

3.1. Study area

The Heihe River Basin is the second largest arid inland in northwest

China. It is located in the middle of the Hexi Corridor, lying between latitudes 38° and 42 °N, and longitudes 98° and 101° 30′ E. It has three parts, i.e. upstream, midstream and downstream areas. The midstream area was selected as the study area (see Fig. 2), mainly including three administrative regions, i.e. Ganzhou district (GZ), Linze county (LZ) and Gaotai county (GT). The annual average temperature is about 7.6 °C and annual average precipitation is 195 mm (90% of all precipitation occurs from March to September) and annual average potential evapotranspiration is 1710 mm. Main crops such as spring wheat, maize and economic crops are planted and cultivated. Economic crops includes cotton, fruits and vegetables that can bring higher economic benefits than grain crops. The growing period of these crops in the study area ranges from March to October. Moreover, agricultural water consumption in the midstream area utilizes approximately 90% of total water consumptions (Li et al., 2016a). Crops are mainly irrigated by surface water through densely distributed canal networks, and traditional irrigation methods such as flood and furrow methods are generally adopted by farmers. Groundwater will be extracted to compensate for the deficiency of surface water because of its seasonal variations (Jiang et al., 2016). Although nearly all the main canals have been lined, a portion of irrigation water is still lost during water conveyance due to multiple levels of the canal system. Therefore, the comprehensive irrigation water use coefficients of surface water and groundwater are 0.52 and 0.60 according to Statistical data of Zhangye City from 2002 to 2015.

How to reasonably manage irrigation water and effectively optimize the limited irrigation water allocation is a critical issue for the sustainable development of agriculture. Moreover, various uncertain factors exist in agricultural systems such as water availability, irrigation water demands and market conditions. These uncertainties may inevitably influence irrigation water management problems. Therefore, inexact optimization models are desirable to support irrigation water management under uncertainty.

3.2. Data collection and processing

To solve the IFCIWA model, input parameters are required. Basically, information regarding crop data, environmental capabilities, and hydrological conditions is collected and calculated. Each detailed component can be explained as follows.

3.2.1. Interval linear crop water production functions

Table 1 presents the ILCWPFs for each study crop. Spring wheat, maize and economic crops were chosen as study crops. Based on the Section 2.1, an interval regression method was adopted to obtain the desired ILCWPFs for different crops. The field experimental data originated from Li (2005) and the regression results refer to Li (2017). To make the regression results tend towards the central tendency, the ratio of $k_1 = 1$ and $k_2 = 0.0001$ was used to determine the specifications of ILCWPFs. However, for spring wheat, the ratio of $k_1 = 0.0001$ and $k_2 = 1$ was used to minimize the sum of squared spreads because the constant term coefficients of regression result at the ratio of $k_1 = 1$ and $k_2 = 0.0001$ are deterministic (Li et al., 2016b). Additionally, it is assumed that the ILCWPFs are the same in different subareas attributed to their similar soil physical properties (soil types), crops, hydrological and meteorological conditions.

Table 1
Interval linear crop water production functions (Y, kg/ ha; ET, m^3/ha).

Crop	Interval linear CWPFs
Spring wheat	$Y = (3071.59,\ 753.41) + (0.6932,\ 0.0568)ET$
Maize	$Y = (-1188.98,\ 1486.45) + (1.6453,\ 0.0159)ET$
Economic crops	$Y = (-52388.43,\ 3125.19) + (20.99,\ 0)ET$

Note: CWPFs denote crop water production functions. $Y = (\gamma,\ r\gamma) + (b,\ rb)ET$, Y is crop yield (kg/ha); ET is actual evapotranspiration (m^3/ha) ; γ, b denote coefficients of constant term, first order term of CWPFs, respectively; $r\gamma$, rb denote the radius of constant term, first order term of CWPFs, respectively.

Table 2
Water availability under different water levels (10^6 m^3).

Water level	Surface water availability			Groundwater availability		
	Lower value	Middle value	Upper value	Lower value	Middle value	Upper value
Low	457	542.5	628	305	317	329
Medium	719	751.5	784	254	270	286
High	855	950	1045	205	216	227

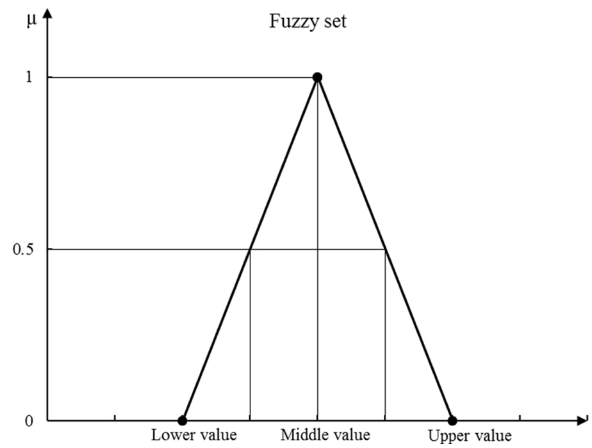

Fig. 3. Symmetric triangular fuzzy number.

3.2.2. Water availability

Table 2 presents surface water and groundwater availabilities. Both surface water and groundwater availabilities are presented as symmetric triangular fuzzy numbers under three water levels, thereby requiring its middle, upper and lower values (see Fig. 3). For surface water availability, water supply for the midstream area is directly delivered from the main stream of the Heihe River. Thus, the category of three inflow levels is based on the frequency analysis method (Li et al., 2016a), that is, low level ($P > 75\%$), medium level ($25\% \le P \le 75\%$) and high level ($P < 25\%$). The estimation of the range of availability under each inflow level is based on t distribution method due to its unknown standard deviation of the sample. The 95% confidence level was chosen to obtain confidence intervals. Therefore, a triangular fuzzy number with the lower, middle and upper values can be obtained under each water level.

Groundwater availability can be obtained from historical records of groundwater use. It is also presented as a symmetric triangular fuzzy number with the division of three water levels. Accordingly, the middle and lower values are estimated from the mean and the minimum values of the statistical data from 2002 to 2015. Moreover, the effective precipitation under three water levels (see Table 3) was obtained from the observation data of meteorological stations in typical hydrological years.

3.2.3. Crop irrigation water demand

Tables 3 and 4 present the irrigation water demands and K_c values of the crops. Crop irrigation water demand is obtained: $ET_c = K_c \times ET_0$, where ET_c is the actual crop evapotranspiration (mm/day), K_c is the crop coefficient and ET_0 is the reference crop evapotranspiration (mm/ day). Daily reference evapotranspiration is calculated based on the FAO 56 Penman-Monteith method where the basic meteorological data from Zhangye (No. 52562), Linze (No. 52557) and Gaotai (No. 52546)

Table 3
Maximum evapotranspiration ET_{max} and effective precipitation (m^3/ha).

Subarea	Spring wheat			Maize			Economic crops			Effective precipitation		
	High	Medium	Low	High	Medium	Low	High	Medium	Low	High	Medium	Low
GZ	[5064.39, 6851.82]	[4920.23, 6656.78]	[4673.64, 6323.16]	[5226.65, 7071.35]	[5106.04, 6908.17]	[4939.69, 6683.11]	[4754.82, 6432.99]	[4638.88, 6276.13]	[4450.01, 6020.60]	[1065.05, 1440.95]	[1037.85, 1404.15]	[976.65, 1321.35]
LZ	[5508.65, 7452.87]	[5347.91, 7235.41]	[5085.72, 6880.68]	[5578.35, 7547.17]	[5451.87, 7376.05]	[5275.17, 7136.99]	[5237.84, 7086.48]	[5112.61, 6917.07]	[4909.92, 6642.84]	[958.55, 1296.86]	[934.07, 1263.74]	[878.99, 1189.22]
GT	[5804.82, 7853.58]	[5633.04, 7621.17]	[5360.44, 7252.36]	[5812.81, 7864.39]	[5682.42, 7687.98]	[5498.82, 7439.58]	[5559.85, 7522.15]	[5428.44, 7344.36]	[5216.54, 7057.67]	[862.69, 1167.17]	[840.66, 1137.36]	[791.09, 1070.29]

Table 4
K_c values for different crops.

Month	$K_c$values for different crops		
	Spring wheat	Maize	Economic crops
March	–	–	[0.34, 0.35]
April	0.22	[0.22, 0.23]	[0.31, 0.35]
May	[0.69, 1.02]	[0.23, 0.33]	[0.63, 0.71]
June	[1.16, 1.35]	[0.56, 1.03]	[0.95, 1.01]
July	[0.74, 1.11]	1.20	[0.99, 1.20]
August	–	[1.09, 1.20]	[0.64, 0.77]
September	–	[0.54, 0.82]	[0.48, 0.62]

meteorological stations located in midstream area during 1967–2009 (Yin et al., 2012). Thus, total crop irrigation water demand for the whole growth stage can be obtained by the sum of daily values of each growth stage in typical hydrological years.

3.2.4. Other related parameters

Table 5 presents other related parameters associated with the IFCIWA model. Crop planting area, crop price and water use cost are obtained from statistical data, local government reports and websites. Especially, the rate of water loss during water conveyance is considered as a symmetric triangular fuzzy number in the left-hand side constraints. The rates of water loss during surface water and groundwater conveyance are (0.20, 0.25, 0.30) and (0.10, 0.15, 0.20), respectively.

4. Results analysis and discussion

To examine the effects of α -cut confidence levels and water levels, eleven confidence levels (i.e. $\alpha = 0, 0.1, \cdots, 1.0$) and three water levels (i.e. high, medium and low levels) were applied to the IFCIWA model. Each confidence level can be separated into two reliability scenarios, i.e. minimum and maximum reliability scenarios, while each scenario can further be transformed into upper and lower bounds of submodels. Therefore, there are a total of 132 submodels that need to be solved and then 66 sets of optimal interval solutions can be generated for supporting irrigation water management. As the IFCIWA model combines the DFCCP method into the IPP optimization framework, it thus potentially possesses the advantages of the two methods. Because the DFCCP method addresses violation of system constraints with double-sided fuzziness and thus generates the corresponding solutions at different confidence levels, choosing the proper confidence level becomes more important for decision-making. Theoretically, a confidence level means the corresponding satisfaction degree of fuzzy uncertainty, which can basically reflect managers' subjective attitudes and directly affect the results. Moreover, the obtained results contain a combination of deterministic and interval information, which can reflect different forms of uncertainties and characteristics of model. More details about results analysis and discussion are provided in the following three sections.

4.1. Optimal solutions of irrigation water allocation

Table 6 presents optimal solutions of irrigation water allocation at different confidence levels and low water level. With the confidence level ranging from 0 to 1.0 at intervals of 0.1, the results of LZ and GT remain unchanged, which is seemingly insensitive to the variation of confidence levels. This can be caused by the crop planting area of the LZ and GT being less than GZ so that the confidence level has an insignificant influence on their optimal solutions. Besides, in the LZ and GT, the results show that crops are totally irrigated by surface water without using groundwater, indicating groundwater will be protected for ensuring local groundwater table and ecological security. For example, the allocated irrigation surface water is 1907.1 m^3/ha for spring wheat, 2022.4 m^3/ha for maize and [4030.9, 5453.6] m^3/ha for economic crops in LZ under the minimum reliability scenario, and the results obtained are the same as the above-mentioned under the maximum reliability scenario. Meanwhile, optimal solutions of irrigation surface water allocation are 2193.3 m^3/ha for spring wheat, [4707.7, 6369.9] m^3/ha for maize and [4425.4, 5987.4] m^3/ha for economic crops in GT under minimum and maximum reliability scenarios. Obviously, the fuzzy parameters of both water availability and the rate of water loss during water conveyance have no effect on the system variations in LZ and GT subareas. However, the results from the model developed in GZ indicate that confidence levels have slight influences on the output results. For example, when confidence level is increased from 0 to 1.0 under the minimum reliability scenario, the allocated surface water for spring wheat will be slightly decreased from 691.1

Table 5
Basic data of different crops in the study area.

Subarea	Crop planting area (ha)			Average crop price (Yuan/kg)			Crop production costs (Yuan/kg)			Surface water cost (Yuan/m^3)	Groundwater cost (Yuan/m^3)
	Spring wheat	Maize	Economic crops	Spring wheat	Maize	Economic crops	Spring wheat	Maize	Economic crops		
GZ	[1894, 2563]	[41679, 56390]	[4739, 6412]	[2.24, 2.46]	[2.32, 2.52]	[4.68, 5.56]	[0.40, 0.50]	[0.40, 0.50]	[0.50, 0.70]	[0.13, 0.20]	[0.30, 0.50]
LZ	[1090, 1475]	[8809, 11918]	[1309, 1771]	[1.98, 2.22]	[2.14, 2.62]	[4.54, 5.14]	[0.40, 0.50]	[0.40, 0.50]	[0.50, 0.70]	[0.20, 0.22]	[0.40, 0.60]
GT	[1666, 2254]	[9951, 13463]	[3706, 5014]	[2.20, 2.34]	[2.46, 2.58]	[5.02, 5.30]	[0.40, 0.50]	[0.40, 0.50]	[0.50, 0.70]	[0.18, 0.33]	[0.50, 0.70]

Table 6
Optimal solutions of irrigation water allocation at different confidence levels and low water level (m^3/ha).

Subarea	α-cut	Minimum reliability scenario						Maximum reliability scenario					
		Spring wheat*	Maize*	Economic crops*	Spring wheat**	Maize**	Economic crops**	Spring wheat*	Maize*	Economic crops*	Spring wheat**	Maize**	Economic crops**
GZ	α = 0	691.1	1386.2	1200.8	833	2428.5	[2272.6, 3498.5]	672	680.5	893.8	852	2168.3	[2579.6, 3805.5]
	α = 0.1	689.8	1312.7	1171.1	834.2	2401.7	[2302.2, 3528.1]	668.1	613.9	857.9	856	2142.7	[2615.5, 3841.4]
	α = 0.2	688.5	1239.9	1141.5	835.6	2375.2	[2331.9, 3557.8]	663.2	548.1	819.6	860.9	2116.9	[2653.7, 3879.6]
	α = 0.3	687.1	1167.6	1111.8	837	2348.8	[2361.6, 3587.4]	657	483.3	778.4	867	2090.9	[2695.0, 3920.9]
	α = 0.4	685.7	1096	1082.8	838.4	2322.7	[2390.6, 3616.5]	649.1	419.6	733.3	875	2064.6	[2740.1, 3966.0]
	α = 0.5	684.1	1025	1053.3	839.9	2296.7	[2420.1, 3646.0]	638.6	357.1	683.2	885.5	2037.8	[2790.1, 4016.0]
	α = 0.6	682.4	954.7	1023.2	841.7	2270.9	[2450.2, 3676.1]	624.3	296	626.9	899.8	2010.2	[2846.5, 4072.4]
	α = 0.7	680.3	885.1	992.3	843.7	2245.1	[2481.0, 3706.9]	604.3	236.7	562.2	919.7	1981.7	[2911.1, 4137.0]
	α = 0.8	678	816.2	960.6	846.1	2219.5	[2512.8, 3738.6]	575.1	179.5	486.6	949	1951.6	[2986.7, 4212.6]
	α = 0.9	675.3	748	927.9	848.8	2193.9	[2545.5, 3771.4]	530.4	125.1	397.1	993.6	1919.4	[3076.2, 4302.1]
	α = 1.0	672	680.5	893.8	852	2168.3	[2579.6, 3805.5]	462.9	74.1	292.3	1061.1	1884.7	[3181.1, 4407.0]
LZ	α = 0 ~ 1.0	1907.1	2022.4	[4030.9, 5453.6]	0	0	0	1907.1	2022.4	[4030.9, 5453.6]	0	0	0
GT	α = 0 ~ 1.0	2193.3	[4707.7, 6369.9]	[4425.4, 5987.4]	0	0	0	2193.3	[4707.7, 6369.3]	[4425.4, 5987.4]	0	0	0

Note: the superscript '*' and '**' mean the allocated surface water and groundwater, respectively.

m^3/ha to 672.0 m^3/ha while the allocated groundwater will be gradually increased from 833.0 m^3/ha to 852.0 m^3/ha. Moreover, for the maize, the obtained results have the same falling trends of both the allocated surface water and groundwater. Furthermore, the solutions of surface water for economic crops are decreased from 1200.8 m^3/ha to 893.8 m^3/ha but the solutions of groundwater present an opposite trend (i.e. from [2272.6, 3498.5] m^3/ha to [2579.6, 3805.5] m^3/ha). The results indicate that different confidence levels are associated with different optimal solutions, which can help managers distinguish desired solutions based on their subjective preferences and choices. In fact, the interaction between decision variables can reflect the uncertainty and complexity of agricultural systems. Therefore, it is important for managers to figure out the uncertainties and interactions of variables and make a final decision.

By contrast, the results under the maximum reliability scenario show similar changing trend as the confidence level increases. In other words, the distinctive difference between minimum and maximum reliability scenarios is merely the amount of allocated irrigation surface water and groundwater. The reason is that, based on the DFCCP algorithm, the surface water and groundwater availabilities under the maximum reliability scenario are less than the minimum reliability scenario. Therefore, planning under the minimum reliability scenario of the system objective leads to higher system benefits due to the increased water availabilities, which represents the optimistic attitudes of managers although they will face higher system-failure risk levels. However, planning under the maximum reliability scenario is related to lower water availabilities and lower system benefits, which can provide more reliable results because of conservative attitudes.

To investigate the effects of different water levels on the variation of results, Table 7 presents optimal solutions of surface water and groundwater allocation under three water levels and $\alpha = 0.5$. The differences among three water levels are their water availabilities, irrigation water demands and effective precipitation. Among them, the total water availabilities under the high water level are greater than medium and low water levels at the same confidence level. Moreover, under a higher water level, surface water availabilities will increase but groundwater availabilities will decrease. Therefore, when the system objective is to maximize system benefits, more surface water will be allocated to different crops and subareas under the higher water level (e.g. high level > medium level > low level) while the allocated groundwater presents an opposite trend. It can be explained in that water surface water cost is less than groundwater cost (i.e. $CS_i^{\pm} < CG_i^{\pm}$), thus surface water should be allocated first to satisfy irrigation water demand. This promising phenomenon shows that less groundwater will be extracted for agricultural irrigation to alleviate the dropping groundwater table due to previous over-exploitation of groundwater. Furthermore, the total amount of allocated surface water and groundwater for three crops in GZ under the minimum reliability scenario are not less than that under the maximum reliability scenario. Taking medium water level as an example, the total amount of allocated irrigation is 1591.4/1591.4 m^3/ha for spring wheat, [4068.2, 4070.1]/3423.5 m^3/ha for maize and [3601.0, 4872.0]/[3601.0, 4872.0] m^3/ha for economic crops under the minimum/maximum reliability scenarios. From the perspective of the DFCCP algorithm rule, the fuzzy chance-constraints in the model can be transformed into two crisp equivalents under minimum and maximum reliabilities. Therefore, the above results can provide more choices for managers based on their preferences and help them analyze tradeoffs between system variations and reliability levels.

4.2. System economic benefits

Figs. 4a, b, 5 a, b, and 6 a, b compare system economic benefits resulting from the IFCIWA model at varying confidence levels under the high, medium and low water levels, respectively. As shown in Fig. 4a, system benefits under the high water level are [2.659, 7.913] × 10^9

Table 7
Optimal solutions of irrigation water allocation under three water levels and $\alpha = 0.5$ (m^3/ha).

Subarea	Minimum reliability scenario						Maximum reliability scenario					
	Spring wheat*	Maize*	Economic crops*	Spring wheat**	Maize**	Economic crops**	Spring wheat*	Maize*	Economic crops*	Spring wheat**	Maize**	Economic crops**
	Under the high water level											
GZ	860.5	3506.6	2413.9	781.9	[655.0, 1556.8]	[1275.9, 2578.1]	897	2620.3	2424.6	745.4	1389.8	[1265.1, 2567.4]
LZ	2056.9	2099.4	[4279.3, 5789.6]	0	0	0	2056.9	2099.4	[4279.3, 5789.6]	0	0	0
GT	2366.9	[4950.1, 6697.2]	[4697.2, 6355.0]	0	0	0	2366.9	[4950.1, 6697.2]	[4697.2, 6355.0]	0	0	0
	Under the medium water level											
GZ	740.1	2117.9	1569.4	839.9	[1950.2, 1952.2]	[2031.6, 3302.6]	741.6	1710.1	1500.9	849.8	1713.4	[2100.1, 3371.1]
LZ	1992.2	2055.5	[4178.5, 5653.3]	0	0	0	1992.2	2055.5	[4178.5, 5653.3]	0	0	0
GT	2292.2	[4841.8, 6550.6]	[4587.8, 6207.0]	0	0	0	2292.2	[4841.8, 6550.6]	[4587.8, 6207.0]	0	0	0
	Under the low water level											
GZ	684.1	1025	1053.3	851.3	2296.7	[2420.1, 3646.0]	638.6	357.1	683.2	885.5	2037.8	[2790.1, 4016.0]
LZ	1907.1	2022.4	[4030.9, 5453.6]	0	0	0	1907.1	2022.4	[4030.9, 5453.6]	0	0	0
GT	2193.3	[4707.7, 6369.9]	[4425.4, 5987.4]	0	0	0	2193.3	[4707.7, 6369.3]	[4425.4, 5987.4]	0	0	0

Note: the superscript '*' and '**' mean the allocated surface water and groundwater, respectively.

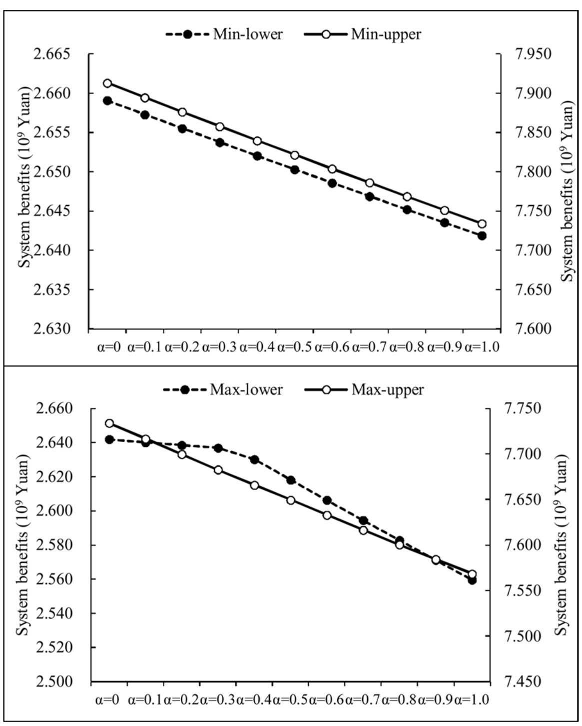

Fig. 4. a, b. System benefits resulting from the IFCIWA model at varying confidence levels under the high water level.
Note: Min-lower and Min-upper denote the lower and the upper bounds submodels of the minimum reliability scenario; Max-lower and Max-upper denote the lower and the upper bounds submodels of the maximum reliability scenario.

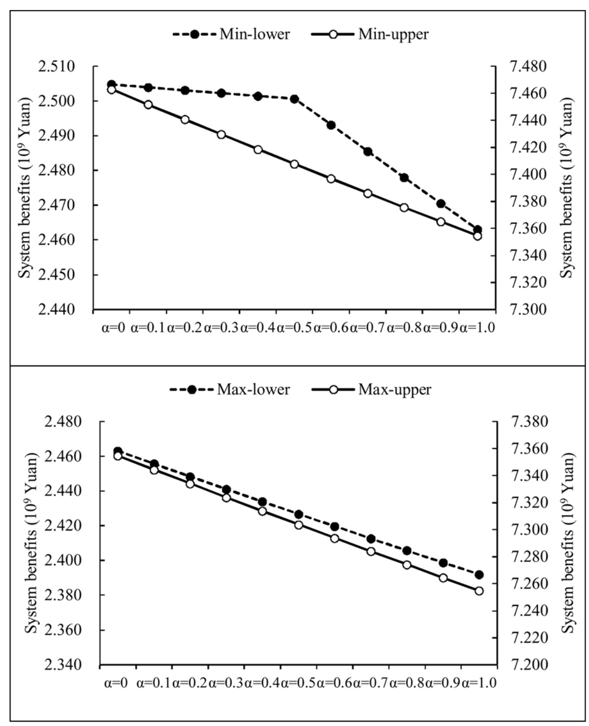

Fig. 5. a, b. System benefits resulting from the IFCIWA model at varying confidence levels under the medium water level.

Yuan when $\alpha = 0$, [2.650, 7.822] $\times 10^9$ Yuan when $\alpha = 0.5$ and [2.642, 7.734] $\times 10^9$ Yuan when $\alpha = 1.0$ under the minimum reliability scenario. The results show that a higher confidence level will bring lower system benefits and higher reliability level. Generally, an increased confidence level means much higher satisfaction degree level of constraints, thereby causing a lower possibility that the constraints can be violated. This theoretically leads to an increased strictness for the system constraints and finally results in a narrow decision space (e.g., decreased water availability and increased rate of water loss). Therefore, the confidence level is related to a manager's risk preferences and thus an acceptable and proper risk is critical to generate optimal solutions for decision making.

According to Fiedler et al (2006), each confidence level consists of two reliability scenarios, namely, the confidence level under minimum and maximum reliabilities. Similarly, from Fig. 4b, system benefits

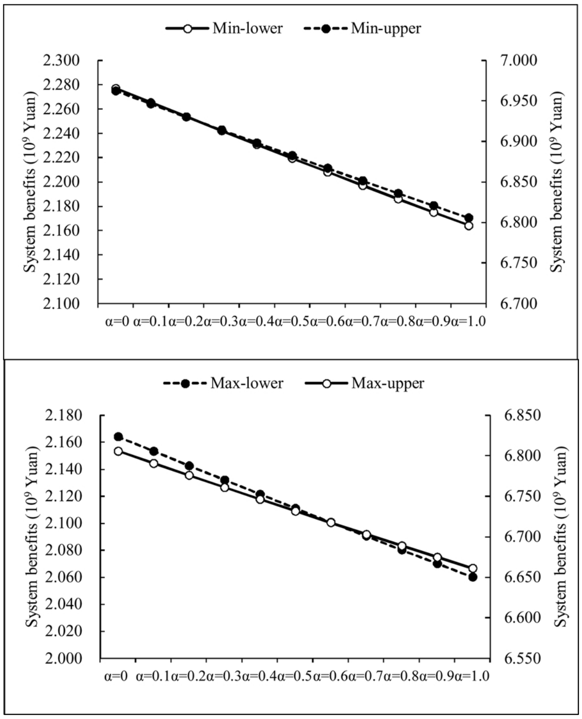

Fig. 6. a, b. System benefits resulting from the IFCIWA model at varying confidence levels under the low water level.

under the maximum reliability scenario can also be seen. For example, system benefits are [2.642, 7.734] × 10^9 Yuan when $\alpha = 0$, [2.618, 7.650] × 10^9 Yuan when $\alpha = 0.5$ and [2.560, 7.568] × 10^9 Yuan when $\alpha = 1.0$. Obviously, the latter leads to slightly lower system benefits than the former under a series of confidence levels. Such a difference can be explained by the reason that the former represents an optimistic strategy for decision making but the latter are associated with higher system reliability levels due to pessimistic preferences. Understandably, as shown in Fig. 4a, b, planning under a lower confidence level and a minimum reliability scenario can lead to maximize system benefits. Conversely, planning under a higher confidence level and maximum reliability scenario will bring minimized system benefits. Therefore, the above results can support in-depth analysis of the interrelationships among system benefits, confidence levels, reliability levels and risk levels.

In terms of three water levels, the results from Figs. 4a, b to 6a, b indicate that system benefits under the high water level are greater than medium and low water levels. For example, under the minimum reliability scenario when confidence level is increased, system benefits resulting from the high, medium and low water levels are [2.659, 7.913] × 10^9 Yuan, [2.505, 7.463] × 10^9 Yuan and [2.277, 6.963] × 10^9 Yuan ($\alpha = 0$), [2.650, 7.822] × 10^9 Yuan, [2.501, 7.408] × 10^9 Yuan and [2.219, 6.883] × 10^9 Yuan ($\alpha = 0.5$), [2.642, 7.734] × 10^9 Yuan, [2.463, 7.355] × 10^9 Yuan, and [2.164, 6.806] × 10^9 Yuan ($\alpha = 1.0$), respectively. The results also show a tradeoff between system benefits and water levels. Therefore, under advantageous system conditions with higher total water availabilities, system benefits may be higher. However, under demanding conditions due to less total water availability, system benefits may be decreased.

4.3. Discussion

Because of the uncertainties in agricultural irrigation systems, decision variables in the form of intervals are more reasonable because they provide more decision-making options to managers. Moreover, α -cut confidence level is also introduced to quantitatively describe some parameters that are expressed as fuzzy numbers and violation of fuzzy constraints due to their own fuzzy attributes. Therefore, managers can base decisions on these constraints to ultimately generate different types of results by selecting or adjusting within the range of feasible solutions. These decision-making processes require that managers, in combination with their understanding of socio-economic and environmental conditions in the study area, and the preferences of systemic risk levels and returns, should obtain the final plans.

Without accounting for the uncertainties in the system, the IFCIWA model can be rewritten into a deterministic linear programming model. That is, taking mean values of interval parameters and middle values of fuzzy parameters as inputs of the IFCIWA model, then solutions can be obtained by adopting and solving this model (see Table 8). It can be seen that the results are not as flexible as the results obtained by the IFCIWA model because the deterministic model can only provide a set of solutions for managers. For example, under medium water level, only one solution can be generated for irrigation water allocation to each crop in each subarea. Moreover, system benefits obtained under medium water level is 4.753 × 10^9 Yuan, which is a deterministic value between the fluctuant values of 2.392 × 10^9 Yuan (lowest value) and 7.463 × 10^9 Yuan (highest value) (see Fig. 5a, b). However, the developed model enables to provide more results (e.g., 44 sets of solutions under a certain water level) based on the system variations and managers preferences and attitudes. This undoubtedly provides managers with more information when deciding on irrigation water management. For example, planning under a lower confidence level and minimum reliability scenario will achieve higher system benefits but at the same time bear a higher risk level. In contrast, lower system benefits will be obtained under a decreased system-failure risk level. In summary, the developed model is superior to a deterministic model in its broader applicability. It can also be used to better handle tradeoffs among the

Table 8
Optimal solutions of the deterministic model under three water levels.

Subarea	Spring wheat*	Maize*	Economic crops*	Spring wheat**	Maize**	Economic crops**	System benefits (10^9 Yuan)
	Under the high water level (m^3/ha)						
GZ	3511.9	3159.7	3284.7	1193.2	1736.3	1056.2	5.003
LZ	5353.1	5435.1	5034.5	0	0	0	
GT	5814.3	5823.7	5526.1	0	0.0	0	
	Under the medium water level (m^3/ha)						
GZ	649.3	2576.5	1420.7	734.5	2209.6	2665.6	4.753
LZ	1732.3	4134.1	4915.9	0	0	0	
GT	1993.2	5696.2	5397.4	0	0	0	
	Under the low water level (m^3/ha)						
GZ	625.1	1961.6	1223.6	700.2	2700.8	3012.9	4.322
LZ	1658.3	1758.6	4742.3	0	0	0	
GT	1907.2	3717.4	5206.4	0	0	0	

Note: the superscript '*' and '**' mean the allocated surface water and groundwater, respectively.

economy, the environment, and system reliability, and to further provide more flexible and effective solutions for irrigation water allocation.

5. Conclusions

In this study, an Interval-based Fuzzy Chance-constrained Irrigation Water Allocation (IFCIWA) model with double-sided fuzziness was developed for irrigation water management. This model incorporates interval parameter programming (IPP) and double-sided fuzzy chance-constrained programming (DFCCP) into an irrigation water management framework. Interval linear crop water production functions (ILCWPFs) are introduced as the basis of irrigation planning, which can be obtained by the interval regression method. Therefore, the model can address interval and fuzzy uncertainties. It allows violation of double-sided constraints at predetermined confidence levels and enables transformation of fuzzy chance-constraints into two crisp equivalents under minimum and maximum reliability scenarios, respectively.

A case study of irrigation water management was provided to demonstrate the applicability of the proposed model. More flexible and effective solutions can be generated at each confidence level and water level for supporting irrigation water allocation. These results are useful for identifying desired solutions with maximized system benefits. Moreover, the solutions obtained can support in-depth analysis of interrelationships among system benefits, confidence levels, reliability levels and risk levels.

This study developed an irrigation water allocation model associated with interval and fuzzy inputs as well as ILCWPFs. The flexible and feasible solutions suggest that it is also applicable to other resource management and environmental problems, such as water quality management, air quality management and energy system management. For potential improvements, techniques of quadratic crop water production functions, nonlinear programming and stochastic mathematical programming can be incorporated to improve upon the modeling framework.

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