Integrated IMO-TSP and AHP Method for Regional Water Allocation under Uncertainty

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Abstract: This paper developed an interval multiobjective two-stage stochastic programming (IMO-TSP) model. The model integrated interval two-stage stochastic programming (ITSP), interval linear multiobjective programming (IMOP), and the analytic hierarchy process (AHP) method. The IMO-TSP model could not only address the conflicts of multiple objectives under uncertainty but could also obtain the optimal allocation results based on the water managers' experience, knowledge, and local policies. In addition, a solution method was proposed to solve the IMO-TSP model by integrating the Zimmermann's fuzzy method and the typical ITSP solution method. The developed model was applied to a real-world case for supporting the allocation of limited water resources to various water-use sectors. From the results analysis, the single objective model with a certain indicator can obtain the maximum value of that objective but can hardly take other significant factors into consideration. Conversely, the IMO-TSP model can effectively balance the three objectives of maximizing the system net benefit, minimizing the total COD discharge, and minimizing the water deficit by obtaining the best coordinated satisfaction. The obtained coordination satisfaction by the IMO-TSP model was [0.74, 0.83] in this study, which indicates that the results obtained by the developed model can allocate limited water resources to various water users with a high degree of coordination among the three objectives under different flow levels. The developed model can help the water managers of arid regions to develop the desired and reasonable water resources management policies under uncertainty. **DOI: 10.1061/(ASCE)WR.1943-5452.0000933.**

Author keywords: Water resources management; Interval two-stage programming; Interval multiobjective programming; Analytic hierarchy process method; Uncertainty.

Introduction

Due to the rapid socioeconomic development and continuous population growth, the conflict between a decreasing water supply and an increasing water demand has become increasingly severe in many countries, which poses critical challenges in the sustainable development of agriculture, industry, tertiary industry, and ecology (Li et al. 2015a; Wang et al. 2016; Ren et al. 2016). Optimization models have been considered useful by water resources managers for allocating water resources in more efficient and environmentally benign ways. Currently, due to the complexity of water allocation in the real world, more advanced optimization models are desired for solving the complex practical problems.

In water management problems, managers need to face complex uncertainties in the water system. Over the past decades, many research studies have focused on the development of optimization models that deal with the uncertainties (Mulvey and Ruszczyński 1995; Huang and Loucks 2000; Guo et al. 2008; Tan et al. 2011; Li and Guo 2015; Zhou et al. 2015, 2016; Fan et al. 2017). Among these approaches, two-stage stochastic programming (TSP) is an effective approach for addressing the uncertainties expressed as random variables with known probability distributions (Wang et al. 2016).Two-stage stochastic programming (TSP) refers to a tradeoff between predefined strategies and the associated adaptive adjustments (Li and Huang 2008; Cui et al. 2015). Because the uncertainties in many practical problems exist as ambiguous intervals, interval parameters are introduced to the stochastic programming framework to facilitate communication of the uncertainties to the optimization process, resulting in ITSP (Huang and Loucks 2000). However, the existing models based on ITSP can hardly obtain satisfactory water-allocation plans when the water managers attempt to achieve the coordinated development of multiple objectives. Multiobjective models were widely used in water resources management (Cheng and Chau 2002; Ren et al. 2016), and few studies integrated ITSP into a multiobjective programming (IMOP) framework for obtaining optimal water-allocation plans. Thus, in this study, an interval multiobjective two-stage stochastic programming (IMO-TSP) model was developed on the basis of ITSP to address the conflicting objectives under uncertainty. There were also many algorithms for solving the multiobjective problems, such as the typical multiobjective solving algorithms and evolutionary algorithms (Jeganathan 2011; Taormina and Chau 2015), but none of them have been proved effectively in solving the IMO-TSP model when the ITSP was introduced into the IMOP. Therefore, an integrated algorithm based on the solution of the ITSP and IMOP is proposed to solve the developed model.

Water allocation systems are complicated and require the consideration of many factors. Among the variety of factors, the

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managers' experience, knowledge, and local policies have been neglected in the formulation of the optimization models. The analytic hierarchy process (AHP) method introduced by Saaty (1980) can be used to solve that problem by breaking a water management problem into hierarchies. The AHP method has been widely used in water resources management, and it was proved effective in tackling management problems with multiple criteria (Kim et al. 2007; Yu et al. 2009; Yang et al. 2013; Dai et al. 2016). However, the managers' subjective judgments played a dominant role in the results of the AHP method in these studies, which might have led to deviations of the AHP management solution from objective reality. The method that put AHP into an IMO-TSP model to quantify the managers' subjective judgments has scarcely been found in the existing research. Therefore, this study attempted to integrate the IMO-TSP and the AHP method to realistically reflect the real-world problems and to obtain consistent optimal results.

In general, the intent of this study was to develop an IMO-TSP model that integrated ITSP and the AHP method for supporting water resources management under uncertainty. The IMO-TSP model can not only address the conflicts of multiple objectives under the uncertainties expressed as interval and random parameters but can also obtain an optimal allocation result based on the water managers' experience, knowledge, and local policies. An integrated algorithm was proposed to solve the developed model. To demonstrate the applicability of the proposed model and to handle the practical water-allocation problems, the developed model was applied to the middle reaches of the Heihe River basin in the Gansu province of China.

Methodology

Model Framework

Fig. 1 shows the technical framework of the proposed methodology. As illustrated by this figure, the methodology consists of three main methods, the ITSP optimization model, the AHP method, and the interval multiobjective optimization model.

All regional water managers face the problem of how to effectively allocate the limited water to the various water sectors. In the process of developing a water resources allocation plan, the water managers are required not only to consider the decision-making impacts and risks but also to make full use of their experience, knowledge, and local policies. All the sectors need to know how much water they can expect so they can arrange their production activities and plan their development (Li et al. 2006; Wang et al. 2016). Thus, ITSP and the AHP method are integrated in an interval multiobjective programming model to help water managers more reasonably allocate the limited water.

Model Development

In the process of water allocation, the water resources management problems with random variables could be handled with TSP (Huang and Loucks 2000). Then, the interval two-stage stochastic programming (ITSP) was developed to deal with the interval parameters on the basis of TSP (Mo et al. 2015; Guo et al. 2008; Li and Huang 2008). An ITSP for a water-allocation problem with a maximized planting benefit objective can be formulated

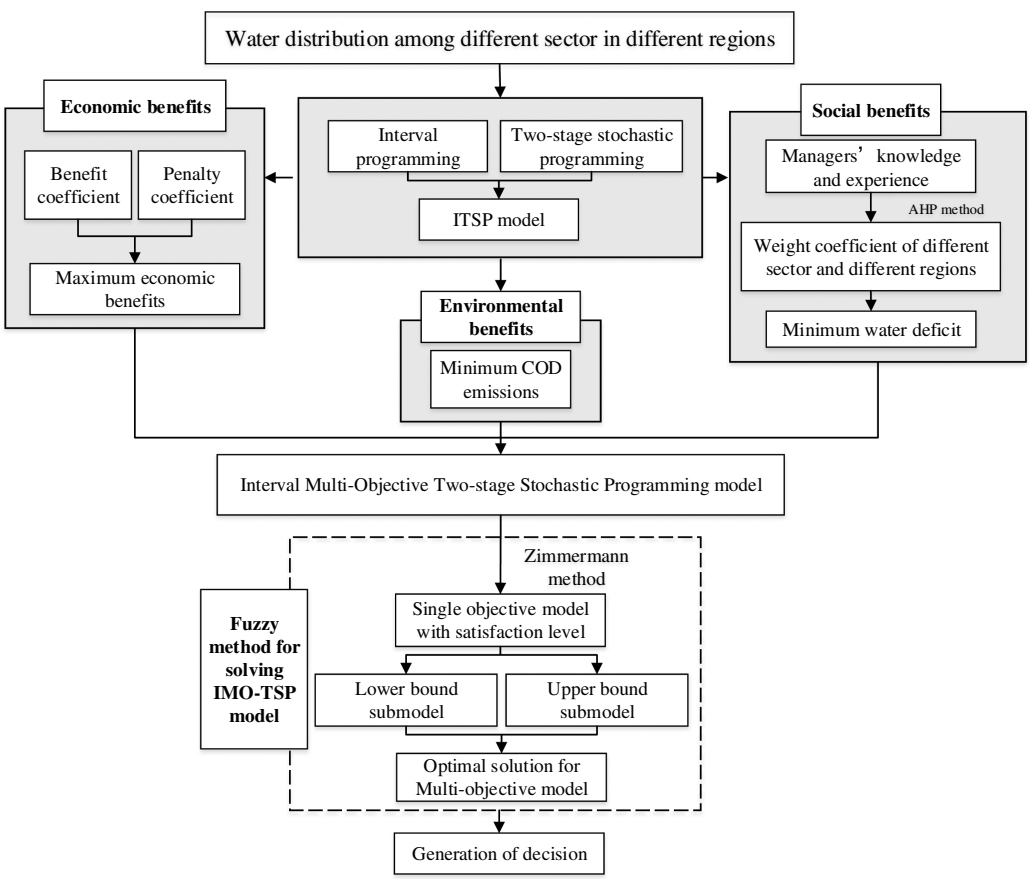

Fig. 1. Framework of model development

$$Max f^{\pm} = \sum_{t=1}^{T}\sum_{i=1}^{I} C_{ti}^{\pm} W_{ti}^{\pm} - \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} p_k L_{ti}^{\pm} S_{tik}^{\pm} \quad \forall\ t, i, k \tag{1a}$$

subject to

$$\sum_{t=1}^{T}\sum_{i=1}^{I} W_{ti}^{\pm} - S_{tik}^{\pm} \le Q_k^{\pm} \quad \forall\ t, i, k \tag{1b}$$

$$W_{ti\max}^{\pm} \ge W_{ti}^{\pm} \ge W_{ti\min}^{\pm} - S_{tik}^{\pm} \ge 0 \quad \forall\ t, i, k \tag{1c}$$

$$W_{ti}^{\pm} \ge S_{tik}^{\pm} \ge 0 \quad \forall\ t, i, k \tag{1d}$$

where $f^{\pm}$ = total net benefit (in a monetary currency, such as Chinese yuan); t = region that requires water allocation; i = index of water sectors ($i = 1, 2, 3, 4$, where $i = 1$ represents the agriculture sector, $i = 2$ denotes the industrial sector, $i = 3$ denotes the tertiary industry sector, and $i = 4$ denotes the ecology sector); k represents hydrological years ($k = 1, 2, 3$, where $k = 1$ denotes a low-flow year, $k = 2$ denotes a medium-flow year, and $k = 3$ denotes a high flow year); $C_{ti}^{\pm}$ = net benefit to sector i per cubic meter of water allocated in region t (yuan/m^3); $W_{ti}^{\pm}$ = first-stage decision variable that denotes the allocation target for water that is promised to sector i in region t (m^3); $L_{ti}^{\pm}$ = loss to sector i per cubic meter of water not delivered in region t, $L_{ti}^{\pm} > C_{ti}^{\pm}$ (yuan/m^3); $S_{tik}^{\pm}$ = second-stage decision variable that is the shortage of water to sector i when the seasonal flow is $Q_k^{\pm}$ with probability p_k in region t (m^3); and $W_{ti\max}^{\pm}$ and $W_{ti\min}^{\pm}$ = maximum and minimum allowable allocation amounts, respectively, for sector i in region t (m^3). The superscript $\pm$ represents an interval with lower and upper bounds, and $+$ and $-$ are the upper and lower bounds, respectively, of the corresponding parameters; for example, the upper and lower bounds of the second-stage decision variable $S_{tik}^{\pm}$ can be expressed S_{tik}^{+} and S_{tik}^{-}, respectively.

Although ITSP is effective in tackling the uncertainties in water resources management, it takes only a single objective into consideration and thus neglects other issues that need to be considered. The aim of the reasonable allocation of water is to promote the coordinated development among the economy, society and the environment. Such a decision-making problem is usually a multiobjective problem (Han et al. 2011; Tan et al. 2017). In addition, the managers' knowledge and experience should play an important role in the decision making. The AHP method was proposed to address such a problem (Zhou and Huang 2007; Srdjevic and Medeiros 2008; Minatour et al. 2013). By integrating ITSP, the AHP method, and interval multiobjective programming (IMOP) in a general framework, an IMO-TSP model incorporating ITSP and the AHP method for water resources allocation could be formulated

$$Max\ f_1^{\pm} = \sum_{t=1}^{T}\sum_{i=1}^{I} C_{ti}^{\pm} W_{ti}^{\pm} - \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} p_k L_{ti}^{\pm} S_{tik}^{\pm} \quad \forall\ t, i, k \tag{2a}$$

Economic objective: maximized net economic benefit

$$Min\ f_2^{\pm} = \sum_{t=1}^{T}\sum_{i=1}^{I} d_i \alpha_i W_{ti}^{\pm} - \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} d_i \alpha_i p_k S_{tik}^{\pm} \quad \forall\ t, i, k \tag{2b}$$

Ecological objective: minimized chemical oxygen demand discharge

$$Min\ f_3^{\pm} = \sum_{t=1}^{T}\sum_{i=1}^{I} \lambda_{ti}(T_{ti} - W_{ti}^{\pm}) + \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} \lambda_{ti} p_k S_{tik}^{\pm} \quad \forall\ t, i, k \tag{2c}$$

Social objective: minimized water deficit

$$\text{subject to} \sum_{t=1}^{T}\sum_{i=1}^{I} W_{ti}^{\pm} - S_{tik}^{\pm} \le Q_k^{\pm} \quad \forall\ t, i, k \tag{2d}$$

$$W_{ti,\min} \le W_{ti}^{\pm} \le W_{ti,\max} \quad \forall\ t, i \tag{2e}$$

$$W_{ti}^{\pm} \ge S_{tik}^{\pm} \ge 0 \quad \forall\ t, i, k \tag{2f}$$

where $f_1^{\pm}$ = total net benefit (in a monetary currency, such as Chinese yuan); $f_2^{\pm}$ = chemical oxygen demand (COD) discharged from various water sectors (g); $f_3^{\pm}$ = total water deficit under the importance weights by the AHP method; T_{ti} = water supply target of the water-use sector i in region t (m^3); λ_{ti} = importance weights calculated by the AHP method of sector i in region t; d_i = primary pollutant concentration per unit wastewater discharge of sector i; and α_i = sewage discharge coefficient (SDC) of sector i.

The Appendix shows how the AHP method obtains the importance weights of every water-use sector in the regional water management. The existing methods for solving interval multiobjective linear programming or ITSP cannot be used to solve this integrated model. To solve this model, a solution method was proposed for solving the IMO-TSP model by integrating Zimmermann fuzzy method (Zimmermann 1978; Guo and Li 2008; Lence et al. 2017) and the typical ITSP (Huang and Loucks 2000), with which it might be easier to obtain results than with the evolutionary algorithms.

Every interval set $a^{\pm}$ could be converted using the equation $a_{ti}^{\pm} = a_{ti}^{-} + \Delta a_{ti} r_{ti}$, where $\Delta a_{ti} = a_{ti}^{+} - a_{ti}^{-}$ and $r_{ti} \in [0, 1]$. The first-step variable could be converted in the same way. z_{ti} are the decision variables used to identify an optimized set of $W_{ti}^{\pm}$. The objective f_1 in Eq. (2a) can be transformed

$$\begin{aligned} Max\ f_1^{\pm} = &\sum_{t=1}^{T}\sum_{i=1}^{I} (C_{ti}^{-} + a_{ti}\Delta C_{ti})(W_{ti}^{-} + z_{ti}\Delta W_{ti}) \\ &- \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} p_k (L_{ti}^{-} + e_{tik}\Delta L_{ti}) S_{tik}^{\pm} \quad \forall\ t, i, k \end{aligned} \tag{3}$$

Let the objective function transform into a minimized one; Eq. (3) can be converted

$$\begin{aligned} Min\ f_1^{\pm} = &\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} p_k (L_{ti}^{-} S_{tik}^{\pm} + n_{tik}\Delta L_{ti} S_{tik}^{\pm}) - \sum_{t=1}^{T}\sum_{i=1}^{I} (C_{ti}^{-} W_{ti}^{-} \\ &+ a_{ti}\Delta C_{ti} W_{ti}^{-} + z_{ti} C_{ti}^{-} \Delta W_{ti} + c_{ti}\Delta C_{ti}\Delta W_{ti}) \quad \forall\ t, i, k \end{aligned} \tag{4}$$

where $c_{ti} = a_{ti} z_{ti}$ and $n_{tik} = e_{tik} S_{tik}^{\pm}$.

Eqs. (2a)–(2f) can be rewritten

$$\begin{aligned} Min\ f_1^{\pm} = &\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} p_k (L_{ti}^{-} S_{tik}^{\pm} + n_{tik}\Delta L_{ti} S_{tik}^{\pm}) - \sum_{t=1}^{T}\sum_{i=1}^{I} (C_{ti}^{-} W_{ti}^{-} \\ &+ a_{ti}\Delta C_{ti} W_{ti}^{-} + z_{ti} C_{ti}^{-} \Delta W_{ti} + c_{ti}\Delta C_{ti}\Delta W_{ti}) \quad \forall\ t, i, k \end{aligned} \tag{5a}$$

$$Min\ f_2^{\pm} = \sum_{t=1}^{T}\sum_{i=1}^{I} d_i\alpha_i(W_{ti}^{-} + z_{ti}\Delta W_{ti}) - \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} d_i\alpha_i p_k S_{tik}^{\pm} \quad \forall\ t,i,k \tag{5b}$$

$$Min\ f_3^{\pm} = \sum_{t=1}^{T}\sum_{i=1}^{I} \lambda_{ti}(T_{ti} - W_{ti}^{-} - z_{ti}\Delta W_{ti}) + \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} \lambda_{ti} p_k S_{tik}^{\pm} \quad \forall\ t,i,k \tag{5c}$$

subject to

$$\sum_{t=1}^{T}\sum_{i=1}^{I}(W_{ti}^{-} + z_{ti}\Delta W_{ti}) - S_{tik}^{\pm} \le Q_k^{-} + m_k\Delta Q_k \quad \forall\ t,i,k \tag{5d}$$

$$W_{ti,\min} \le W_{ti}^{-} + z_{ti}\Delta W_{ti} \le W_{ti,\max} \quad \forall\ t,i \tag{5e}$$

$$W_{ti}^{-} + z_{ti}\Delta W_{ti} \ge S_{tik}^{\pm} \ge 0 \quad \forall\ t,i,k \tag{5f}$$

$$a_{ti}, \qquad c_{ti}, \qquad z_{ti}, \qquad m_k \in [0,1] \tag{5g}$$

$$n_{tik} \in [0, S_{tik}^{\pm}] \tag{5h}$$

The Min operator in Zimmermann's method (Guo and Li 2008) was used to solve the model. Each objective function needed to be calculated separately to obtain the best value of the maximum objective f_h^{+} and the minimum objective f_h^{-} $(h = 1,2,3)$, where $h = 1$ denotes objective f_1; $h = 2$ represents objective f_2; and $h = 3$ is objective f_3. The membership functions needed to be defined for each goal

$$\eta_h(z,S) = \begin{cases} 1 & f_h(z,S) < f_h^{-} \\ \dfrac{f_h^{+} - f_h(z,S)}{f_h^{+} - f_h^{-}} & f_h^{-} \le f_h(z,S) < f_h^{+} \\ 0 & f_h^{+} \le f_h(z,S) \end{cases} \tag{6}$$

With the help of these membership functions, IMO-TSP could be converted into linear programming with the single objective coordination satisfaction

$$\text{Max } \eta^{\pm} \tag{7a}$$

subject to

$$\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} p_k L_{ti}^{\pm} S_{tik}^{\pm} - \sum_{t=1}^{T}\sum_{i=1}^{I} C_{ti}^{\pm}(W_{ti}^{-} + z_{ti}\Delta W_{ti}) + \eta^{\pm}(ob_1^{+} - ob_1^{-}) \le ob_1^{+} \quad \forall\ t,i,k \tag{7b}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{I} d_i\alpha_i(W_{ti}^{-} + z_{ti}\Delta W_{ti}) - \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} d_i\alpha_i p_k S_{tik}^{\pm} + \eta^{\pm}(ob_2^{+} - ob_2^{-}) \le ob_2^{+} \quad \forall\ t,i,k \tag{7c}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{I} \lambda_{ti}(T_{ti} - W_{ti}^{-} - z_{ti}\Delta W_{ti}) + \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} \lambda_{ti} p_k S_{tik}^{\pm} + \eta^{\pm}(ob_3^{+} - ob_3^{-}) \le ob_3^{+} \quad \forall\ t,i,k \tag{7d}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{I} W_{ti}^{-} + z_{ti}\Delta W_{ti} - S_{tik}^{\pm} \le Q_k^{\pm} \quad \forall\ t,i,k \tag{7e}$$

$$W_{ti,\min} \le W_{ti}^{-} + z_{ti}\Delta W_{ti} \le W_{ti,\max} \quad \forall\ t,i \tag{7f}$$

$$W_{ti}^{-} + z_{ti}\Delta W_{ti} \ge S_{tik}^{\pm} \ge 0 \quad \forall\ t,i,k \tag{7g}$$

$$z_{ti} \in [0,1] \tag{7h}$$

Because the objective was to maximize the coordination satisfaction, the submodel corresponding to η^{+} needed to be solved first. Eqs. (7a)–(7h) could be decomposed into two submodels and solved sequentially.

The upper bound value of the objective function

$$\text{Max } \eta^{+} \tag{8a}$$

subject to

$$\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} p_k L_{ti}^{-} S_{tik}^{+} - \sum_{t=1}^{T}\sum_{i=1}^{I} C_{ti}^{+}(W_{ti}^{-} + z_{ti}\Delta W_{ti}) + \eta^{+}(ob_1^{+} - ob_1^{-}) \le ob_1^{+} \quad \forall\ t,i,k \tag{8b}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{I} d_i\alpha_i(W_{ti}^{-} + z_{ti}\Delta W_{ti}) - \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} d_i\alpha_i p_k S_{tik}^{+} + \eta^{+}(ob_2^{+} - ob_2^{-}) \le ob_2^{+} \quad \forall\ t,i,k \tag{8c}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{I} \lambda_{ti}(T_{ti} - W_{ti}^{-} - z_{ti}\Delta W_{ti}) + \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} \lambda_{ti} p_k S_{tik}^{+} + \eta^{+}(ob_3^{+} - ob_3^{-}) \le ob_3^{+} \quad \forall\ t,i,k \tag{8d}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{I} W_{ti}^{-} + z_{ti}\Delta W_{ti} - S_{tik}^{+} \le Q_k^{-} \quad \forall\ t,i,k \tag{8e}$$

$$W_{ti,\min} \le W_{ti}^{-} + z_{ti}\Delta W_{ti} \le W_{ti,\max} \quad \forall\ t,i \tag{8f}$$

$$W_{ti}^{-} + z_{ti}\Delta W_{ti} \ge S_{tik}^{+} \ge 0 \quad \forall\ t,i,k \tag{8g}$$

$$z_{ti} \in [0,1] \tag{8h}$$

where z_{ti} and S_{tik}^{+} = decision variables. Let $z_{ti\,opt}$, $S_{tik\,opt}^{+}$, and η_{opt}^{+} be the solutions of the submodel Eqs. (8a)–(8h). The optimized first-stage variables can be obtained $W_{ti\,opt} = W_{ti}^{-} + \Delta W_{ti} \cdot z_{ti\,opt}$. Based on these solutions, the lower bound value of Eqs. (8a)–(8h) could be formulated

$$\text{Max } \eta^{-} \tag{9a}$$

subject to

$$\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} p_k L_{ti}^{+} S_{tik}^{-} - \sum_{t=1}^{T}\sum_{i=1}^{I} C_{ti}^{-} W_{ti\,opt}^{\pm} + \eta^{-}(ob_1^{+} - ob_1^{-}) \le ob_1^{+} \quad \forall\ t,i,k \tag{9b}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{I} d_i\alpha_i W_{ti\,opt} - \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I} d_i\alpha_i p_k S_{tik}^{-} + \eta^{-}(ob_2^{+} - ob_2^{-}) \le ob_2^{+} \quad \forall\ t,i,k \tag{9c}$$

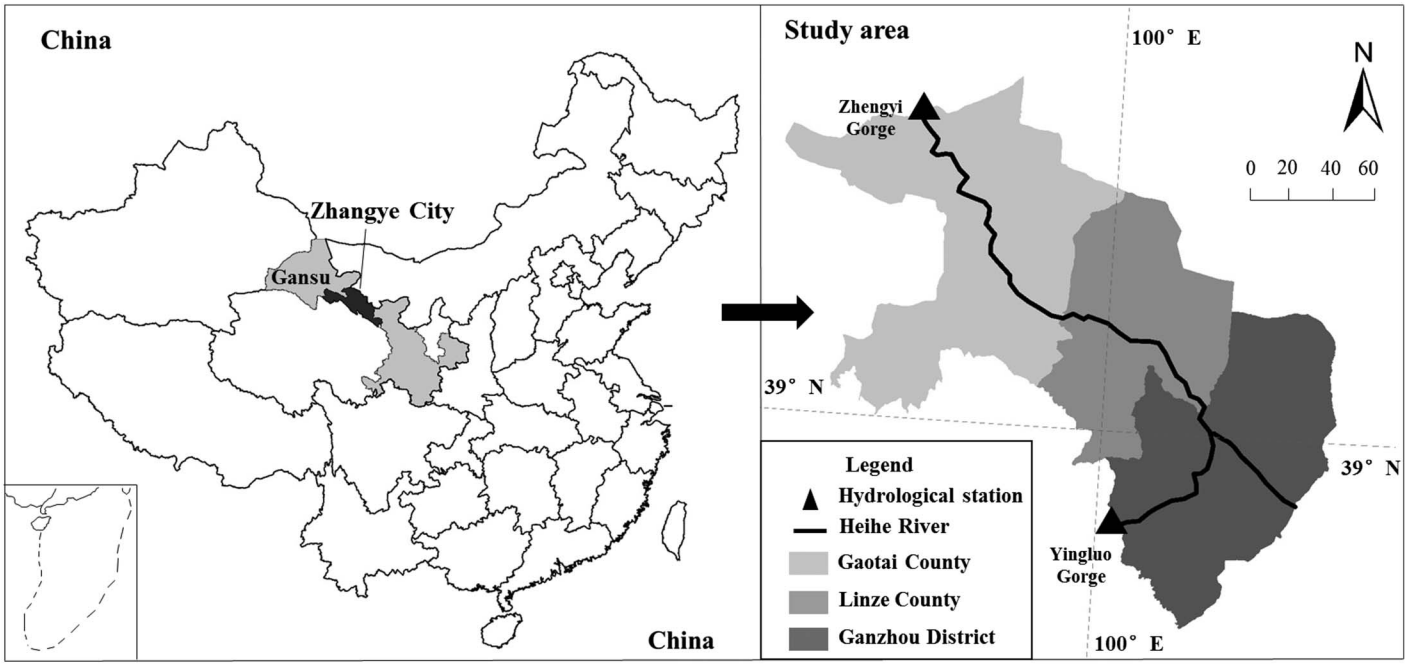

Fig. 2. Geographical location of study area

Table 1. Available Water Resources under Different Flow Levels

Flow level	Available water Q_k	Probability p_k
Low flow level ($k = 1$)	[12.27, 13.73]	0.259
Medium flow level ($k = 2$)	[13.84, 14.55]	0.482
High flow level ($k = 3$)	[14.38, 16.24]	0.259

$$\sum_{t=1}^{T}\sum_{i=1}^{I}\lambda_{ti}(T_{ti}-W_{ti\,opt})+\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{I}\lambda_{ti}p_kS_{tik}^{-}+\eta^{-}(ob_3^{+}-ob_3^{-})\le ob_3^{+}\quad\forall\ t,i,k \tag{9d}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{I}W_{ti\,opt}-S_{tik}^{-}\le Q_k^{-}\quad\forall\ t,i,k \tag{9e}$$

$$S_{tik\,opt}^{+}\ge S_{tik}^{-}\ge 0\quad\forall\ t,i,k \tag{9f}$$

where S_{tik}^{-} = decision variable. The solutions of $S_{tik\,opt}^{-}$ and η_{opt}^{-} could be obtained from the submodel Eqs. (9a)–(9f). Thus, the optimal solutions of Eqs. (7a)–(7h) could be obtained

$$\eta_{opt}^{\pm}=[\eta_{opt}^{-},\eta_{opt}^{+}] \tag{10}$$

$$W_{ti\,opt}=W_{ti}^{-}+\Delta W_{ti}\cdot z_{ti\,opt} \tag{11}$$

$$S_{tik\,opt}^{\pm}=[S_{tik\,opt}^{-},S_{tik\,opt}^{+}] \tag{12}$$

Thus, the optimized water-allocation schemes

$$WA_{ti\,opt}^{\pm}=[W_{ti\,opt}-S_{tik\,opt}^{+},W_{ti\,opt}-S_{tik\,opt}^{-}] \tag{13}$$

where $WA_{ti\,opt}^{\pm}$ = optimal water allocation.

The solution algorithm of the IMO-TSP model can be summarized as the following steps:

1. Formulate the IMO-TSP model;
2. Acquire the model parameters, such as the economic parameter, the runoff probability parameter, and the importance weights, by the AHP process;

Table 2. Water Allocation Target, Related Economic Data and Others Associated

Water user sectors	Subarea	SDC α_i	COD concentration (g/m^3) d_i	Water distribution target (10^8 m^3) T_{ti}	Minimum water requirement (10^8 m^3) $W_{ti,\min}$	Benefit per unit (yuan/m^3) $C_{ti}^{\pm}$	Penalty coefficient (yuan/m^3) $L_{ti}^{\pm}$
Agriculture ($i = 1$)	GZ	0.1	60	7.97	3.51	[0.31, 0.34]	[0.37, 0.41]
	LZ			4.05	2.00	[0.23, 0.26]	[0.28, 0.31]
	GT			5.17	2.58	[0.30, 0.33]	[0.36, 0.40]
Industry ($i = 2$)	GZ	0.5	100	0.22	0.10	[9.71, 10.79]	[14.57, 16.19]
	LZ			0.06	0.03	[9.71, 10.79]	[14.57,16.19]
	GT			0.07	0.03	[9.71, 10.79]	[14.57, 16.19]
Tertiary industry ($i = 3$)	GZ	0.7	230	0.28	0.13	[1.26, 1.40]	[1.89, 2.10]
	LZ			0.05	0.02	[1.13, 1.25]	[1.69, 1.88]
	GT			0.07	0.03	[1.30, 1.45]	[1.96, 2.18]
Ecology ($i = 4$)	GZ	0.8	30	0.29	0.13	[0.64, 0.72]	[0.78, 0.86]
	LZ			0.64	0.30	[0.64, 0.72]	[0.78, 0.86]
	GT			0.21	0.10	[0.64, 0.72]	[0.78, 0.86]

Table 3. Weight Calculation Results by AHP

Water user sector	Subarea	Effectiveness 0.23	Fairness 0.12	Sustainability 0.65	Importance weight by AHP
Agriculture ($i = 1$)	GZ	0.29	0.29	0.12	0.18
	LZ	0.14	0.14	0.01	0.09
	GT	0.09	0.09	0.03	0.10
Industry ($i = 2$)	GZ	0.02	0.02	0.23	0.06
	LZ	0.12	0.12	0.07	0.03
	GT	0.04	0.04	0.02	0.04
Tertiary industry ($i = 3$)	GZ	0.03	0.03	0.03	0.05
	LZ	0.02	0.02	0.16	0.03
	GT	0.13	0.13	0.09	0.04
Ecology ($i = 4$)	GZ	0.07	0.07	0.02	0.16
	LZ	0.04	0.04	0.05	0.11
	GT	0.02	0.02	0.16	0.11

3. Transform the IMO-TSP model into an interval linear programming (ILP) model with a single objective, by the Zimmermann method;
4. Decompose the ILP model into two submodels with coordination satisfaction;
5. Solve the submodel corresponding to the upper bound of the coordination satisfaction, and then solve the other model; and
6. Combine the solutions from the two submodels, thus obtaining the final solution of the IMO-TSP model.

Case Study

Study Area

The research area (Fig. 2) is located in the middle reaches of the Heihe River basin (between 98°00′E and 101°30′E, 38°00′N and 42°30′) in the Gansu province of China, including the Ganzhou district, Linze county, and Gaotai county (Li and Guo 2015). The middle reaches consume the majority of water resources, as that section has more than 80% of the artificial oasis, 92% of the population, 83% of the gross domestic product (GDP), and 95% of the farmland of the entire Heihe River basin (Li et al. 2015b). It has an arid and semiarid climate with low rainfall and strong evaporation, and thus it is intrinsically deficient in water resources. Along with the rapid population growth and socioeconomic development, water conflicts in the different water-use sectors of the middle reaches have become an acute problem (Zhang et al. 2017). The increasing water consumption has resulted in the reduction of available water resources in the lower reaches, which has caused some ecological problems such as soil erosion and sandstorms and further threatened the living conditions of the local residents. Consequently, the optimal allocation of water resources in the middle reaches is necessary and significant for both the middle reaches and the lower reaches, to ensure the ecological balance and promote the sustainable development of the Heihe River basin. The amount of available water resources presents a strong random feature, and the data collected may be imprecise. In order to address the problems above, this paper will optimize the water allocation between the different water-use sectors of the middle reaches under uncertainty and demonstrates the effectiveness and applicability of the developed IMO-TSP model. The results of this study will help the local decision makers to make reasonable water-allocation decisions for the middle reaches of the Heihe River basin.

Four water-use sectors including agriculture ($i = 1$), industry ($i = 2$), tertiary industry ($i = 3$), and ecology ($i = 4$) in the Ganzhou (GZ) district ($t = 1$), Linze (LZ) county ($t = 2$), and Gaotai (GT) county ($t = 3$) are taken into consideration in the water-allocation optimization efforts. Table 1 provides the annual availability of the water resources under different flow levels, which were obtained by the statistics. Table 2 provides the water-allocation targets, economic data, and other associated information, which were obtained by field research and statistics.

Results Analysis and Discussion

Three aspects, including the effectiveness, fairness, and sustainability, were considered in the calculation of the importance weights of the water-use sectors by the AHP method. Table 3 presents the calculated importance weights of the water-use sectors. The consistency ratios (CR) of the effectiveness, fairness, and sustainability were 0.03, 0.04, and 0.06, respectively, none of which exceeded 0.1; they demonstrated that the judgment matrix was consistent. In addition, the CR of the index layer was 0.003, which was also less than 0.1. Therefore, the calculated weights proved to be reasonable. The data of Table 3 illustrates that the importance weights of the agricultural and ecological sectors were significantly higher than the others, which reflects the managers' opinions on the development priorities.

Optimal water-allocation results were obtained by the developed IMO-TSP model. Fig. 3 shows the optimal water-allocation results obtained by the economic objective model (OB1), the ecologic objective model (OB2), the social objective model (OB3), and the IMO-TSP model under low (L), medium (M) and high (H) flow levels. The obtained coordination satisfaction was $\eta_{opt}^{\pm} = [0.74, 0.83]$. Fig. 3 shows clearly that the developed IMO-TSP model coordinated the three different objective functions with a high level of satisfaction in this study.

According to the optimal results from the IMO-TSP model, the water needs of the industrial sector are fully met because the benefits of industry are far higher than the other sectors in the three regions. In addition, the water allocated to the ecological sector is highly resilient. It is noteworthy that interval results can be easily found in the agricultural sector. This might mean that the agricultural water use could be reduced, and the only way to increase the ecological water use is to decrease the agricultural water use. As shown in Fig. 3, the IMO-TSP model results coordinated the contradictions among the water-allocation plans obtained by OB1, OB2, and OB3, which avoided unreasonably optimal results. Fig. 4 shows the total benefits, total COD discharge, and total water deficit of each model under the three flow levels, respectively.

As shown in Fig. 4, the single objective model with a certain indicator can obtain the maximum value of that objective but can hardly take other significant factors into consideration. Conversely, the IMO-TSP model can effectively balance the three objectives of maximizing the system net benefit, minimizing the total COD

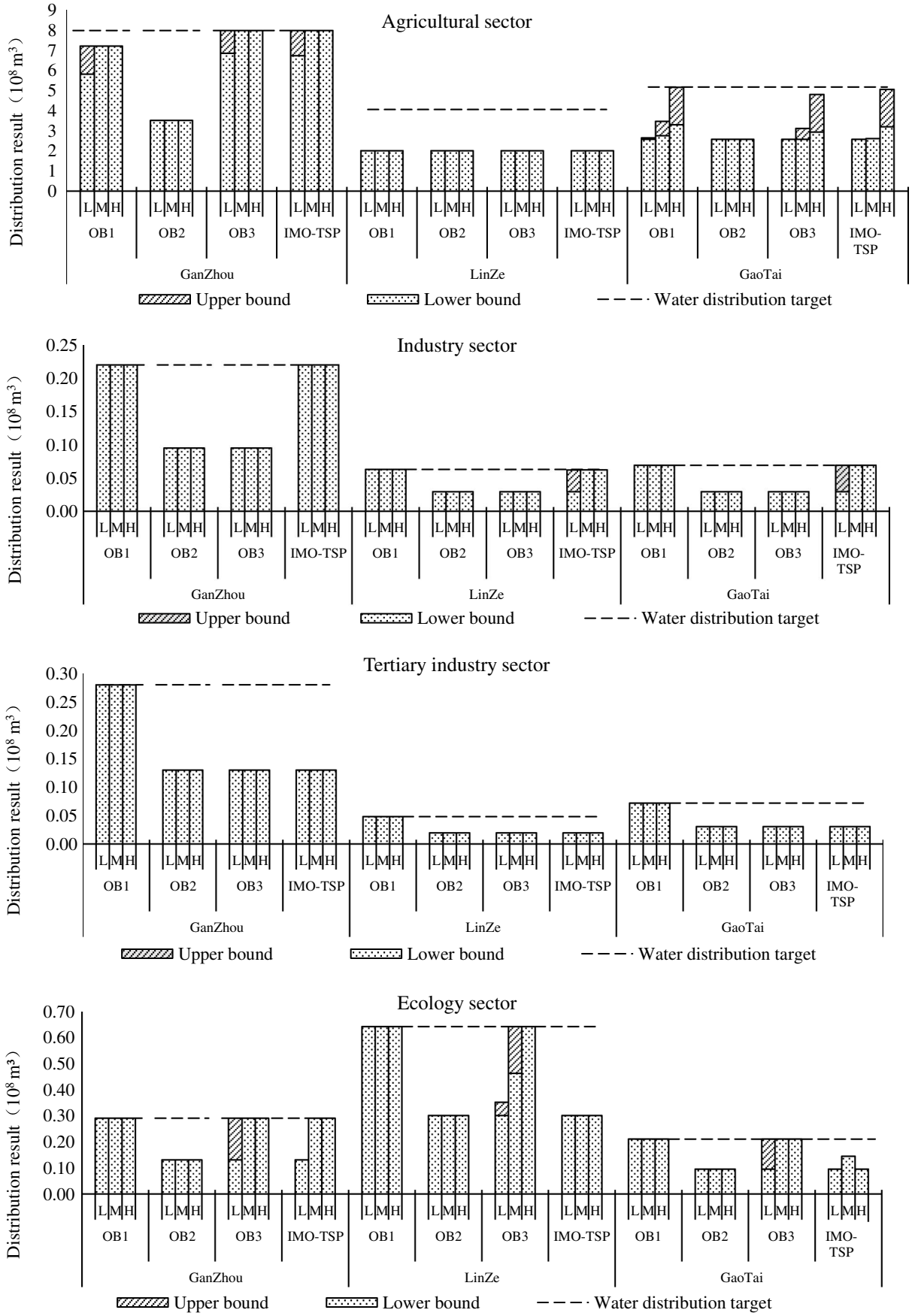

Fig. 3. Results of the each single objective model and IMO-TSP model in different regions

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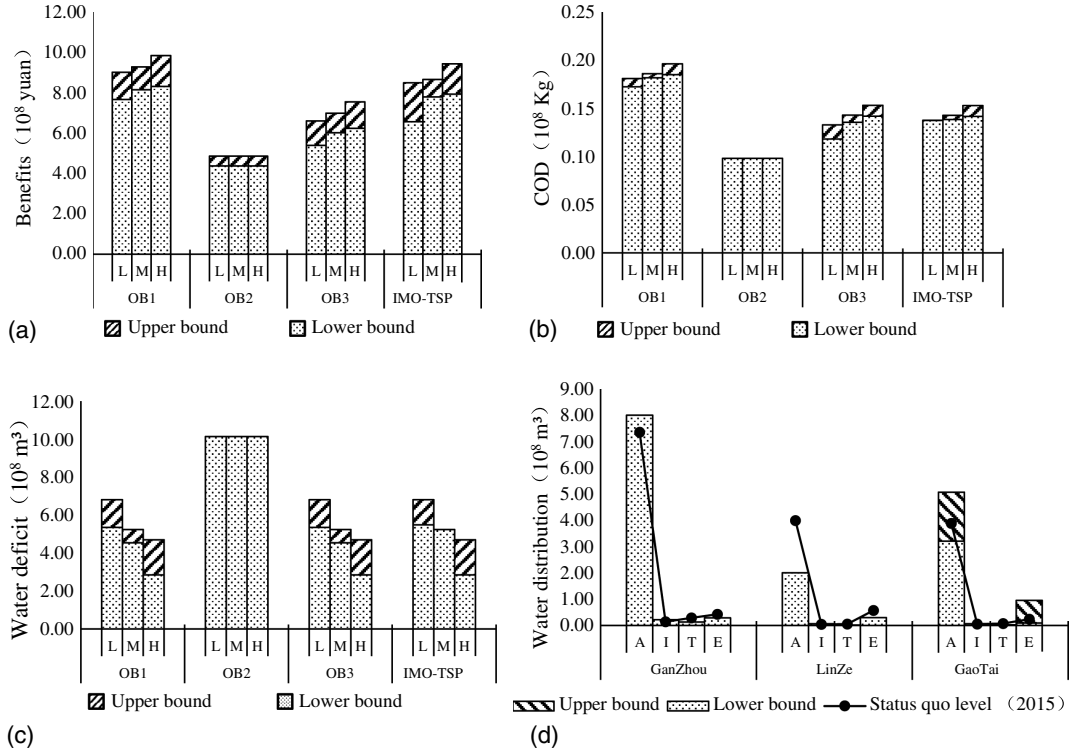

Fig. 4. Comparisons of the models in the three flow levels in the (a) total benefits, (b) total COD emissions, and (c) total water deficit; in (d), the comparison between actual water allocation in 2015 and IMO-TSP model results in the sectors agriculture (A), industry (I), tertiary industry (T), and ecology (E)

discharge, and minimizing the water deficit by obtaining the best coordinated satisfaction. As a result, the indicators calculated by the developed model were close to the optimal value from the single objective models, as shown in Figs. 4(a–c). To illustrate the practicality of the developed IMO-TSP model, the actual water-allocation plans in 2015 (close to the high flow level) was compared with the optimization results of the IMO-TSP model [Fig. 4(d)]. Fig. 4(d) shows that the IMO-TSP model would increase the amount of water allocated to the agricultural sector in the Ganzhou district, whereas the water allocated to the agricultural sector in the other regions would be decreased. The reason for this is that the agriculture in the Ganzhou district is far more developed than in the other regions, which brings with it a high water efficiency and the associated benefits. Moreover, the water saved in the agriculture sectors could be transferred to the ecological sectors according to the optimal results, which is of great significance for

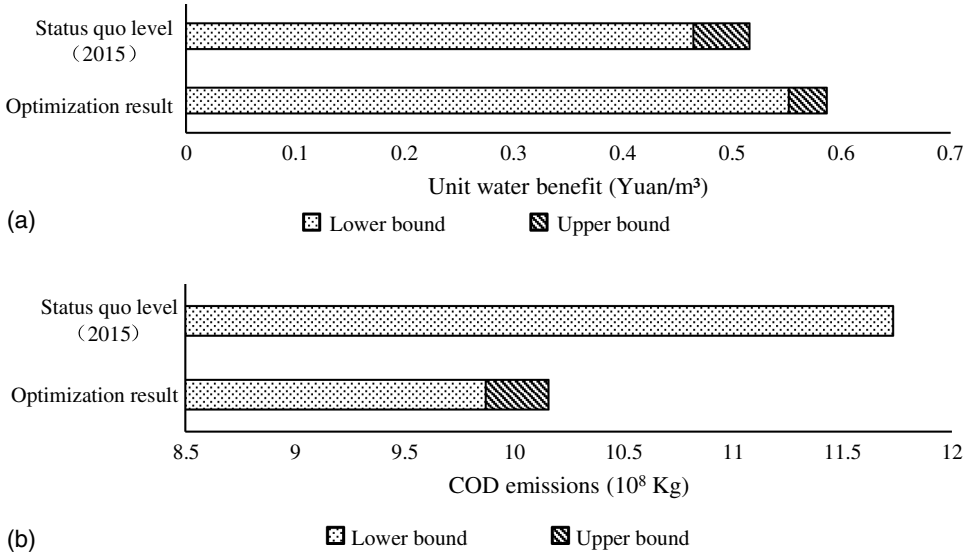

Fig. 5. Comparisons between actual water allocation in status quo level (2015) and IMO-TSP model result in high flow level: (a) benefits per unit of water; (b) total COD emissions

restoring the local ecology. Fig. 5 shows a comparison of the benefits per unit of water and the total COD discharge.

As the comparison in Fig. 5 shows, the water-allocation policy obtained by the IMO-TSP model not only improves the water efficiency of the entire region but also sharply reduces the COD discharge under similar flow levels, which is beneficial to the sustainable development of the region. These comparisons manifest that the optimal results from the developed IMO-TSP model for regional water allocation is meaningful for the local water managers in planning the water-allocation policies.

The ecoenvironment in the middle reaches of the Heihe River basin needs to be improved, because not only can the Heihe oasis protect the local living environment, but it is an important barrier against the sandstorms in northwest China. Therefore, the local water managers have implemented agriculture water reduction policies to ensure the local ecological water, which coincides with the IMO-TSP model results. Moreover, the IMO-TSP model provides the local water managers with specific adjustment allocation suggestions under different flow levels and contributes to the sustainable development of the local economy, society, and the environment. It can link objective and subjective factors in water resources management by integrating the IMO-TSP model and the AHP method, obtain the appropriate water-allocation policies, and effectively communicate the system uncertainties to reflect the real-world conditions. These efforts made by IMO-TSP obviously can obtain more reasonable water-allocation results than the existing methods and are helpful for the local managers in the middle reaches of the Heihe River basin. The obtained water-allocation schemes can serve as a reference for the regional water resources allocation practices, especially for the arid and semi-arid areas.

Conclusions

An interval multiobjective (IMO-TSP) model integrating two-stage stochastic programming and the AHP method was developed in this study. The IMO-TSP model has advantages in (1) handling multiple objectives, including economical, ecological, and social benefits; (2) tackling multiple uncertainties such as random flow levels and interval parameters; (3) allowing water managers to practice appropriate water-allocation management according to their experience, knowledge and local water policies through the AHP method; and (4) providing a number of alternatives under the different probabilities of flow-level occurrences to help the water managers analyze the trade-offs of the system benefits and associated risks.

The developed IMO-TSP model was applied to the middle reaches of the Heihe River basin to plan the regional water-allocation systems, which helped the water managers obtain reasonable water-allocation schemes. This study allocated limited water resources to different water users by coordinating three objectives. The results show that the IMO-TSP model has good applicability and could promote sustainable development in arid regions.

This study attempts to incorporate the AHP method and ITSP into an interval multiobjective model for supporting scientific water management. The framework of the developed model could be applied not only to water resources management but also to other resource management systems. However, the IMO-TSP model can hardly handle detailed information such as the precise relationship functions between the water supply and its benefits, the different sources of water, and water conversion between the different water-use sectors in the region. These should be taken into account in future work.

Appendix. Calculation of the Importance Weight by AHP Method

A typical AHP has the following basic steps (Saaty 2000): (1) break down the problem and build the hierarchy; (2) construct pairwise comparison matrices; (3) calculate the relative weights of the comparison elements by a judgment matrix; (4) calculate the weight of each element; (5) and calculate the consistency index by using the eigenvalue to ensure the result passes the consistency test.

When the AHP is used in the regional water management between N schemes, the hierarchy can be built as shown in Fig. 6. On this basis, a pairwise comparison judgment matrix can be established easily with the help of a pairwise comparison scale for AHP preferences (Saaty 1990) (Table 4). After all the pairwise comparisons have been made, the consistency of the judgment matrix needs to be determined by using the eigenvalue, $\lambda_{\max}$, to calculate the consistency index (CI). The CI can be described as follows (Al-Harbi 2001): $CI = (\lambda_{\max} - n)/(n-1)$, where N is the matrix size. The judgment consistency can be checked by taking the consistency ratio (CR) of the CI with the appropriate value, where the CR can be calculated by $CR = CI/RI$; and RI denotes the mean random consistency index (RI). Table 5 presents parts of values of the RI with different matrix sizes (Hong et al. 2002). The CR is acceptable if it does not exceed 0.10. If it is greater than that, the judgment matrix is inconsistent. To obtain a consistent matrix,

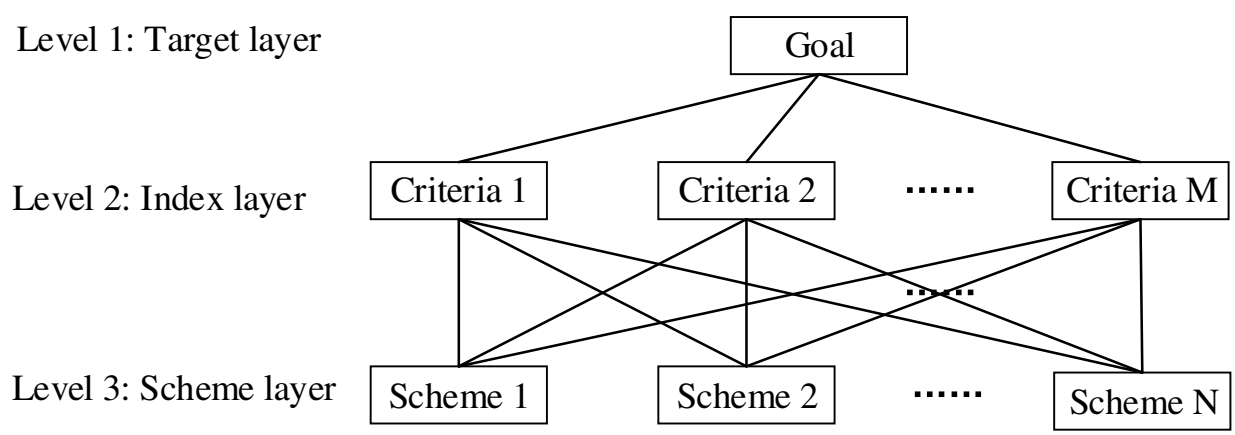

Fig. 6. Hierarchy of AHP method

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Table 4. Pairwise Comparison Scale for AHP Preferences

Numerical rating	Definition
1	Equally important
2	Equally to moderately important
3	Moderately important
4	Moderately to strongly important
5	Strongly important
6	Strongly to very strongly important
7	Very strong important
8	Very strongly to extremely important
9	Extremely important

Table 5. Parts of Values of Mean Random Consistency Index

Size of matrix	Random consistency
1	0
2	0
3	0.52
4	0.89
5	1.12
6	1.26
7	1.36
8	1.41
9	1.46
10	1.49
11	1.52
12	1.54

judgments should be reviewed and improved. Finally, the M judgment matrix with consistency characteristics can be used to derive the weight of each criterion to the target.

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