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FLFP: A fuzzy linear fractional programming approach with double-sided fuzziness for optimal irrigation water allocation

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ABSTRACT

In this study, a fuzzy linear fractional programming (FLFP) approach with double-sided fuzziness is developed for optimal irrigation water allocation under uncertainty. The FLFP model can be derived from incorporating double-sided fuzzy chance-constrained programming (DFCCP) into linear fractional programming (LFP) optimization framework. The developed model can deal with uncertainty presented as fuzziness in both right-hand and left-hand sides of constraints. Moreover, it has advantages in: (1) addressing two objectives directly without considering subjective factors, (2) effectively reflecting economic water productivity between total system economic benefit and total irrigation water use, (3) introducing the concept of confidence levels of fuzzy constraints-satisfaction under both the minimum and maximum reliabilities to generate more flexible solutions and (4) facilitating in-depth analysis of interrelationships among economic water productivity, system benefits and varying confidence levels. The model is applied to a case study of irrigation water allocation in the middle reaches of Heihe River Basin, northwest China. The optimal irrigation water allocation solutions from the FLFP model can be obtained. These results can provide decision-support when deciding on selecting reasonable irrigation water resources management and agricultural production.

1. Introduction

Nowadays, it is commonly believed that growing population and diminishing water availability in China are jointly affect the contradiction between water supply and water demand, especially in arid or semiarid regions dominated by agriculture. Moreover, irrigated agriculture is the biggest consumer attributing to water use, consuming 70% of total water use in the world (Kang et al., 2017). Irrigation water management naturally becomes worldwide concerns for agricultural production and livelihood security with limited water resources (Storm et al., 2011; Parsinejad et al., 2013). Therefore, how to improve agricultural water use efficiency or agricultural water productivity and further effectively allocate irrigation water resources is expected to ensure food security and promote sustainable development of agriculture.

To address the above concerns, many optimization methods or models have been developed for optimal irrigation water allocation (Dudley et al., 1971; Rao et al., 1990; Mainuddin et al., 1997; Shangguan et al., 2002; Noory et al., 2012; Yang et al., 2015; Zhang et al., 2017a). However, among these studies, the objectives of model almost focus on the system benefits and few studies optimize the agricultural water use efficiency. Agricultural water use efficiency is defined as the production per unit of irrigation application (Playán and Mateos, 2006), which is also known as agricultural water productivity reflecting the system efficiency. More importantly, in many developing countries, irrigation water management suffers from lower water use efficiency and the irrigation water resources are not utilized efficiently (Li et al., 2016a). In water limited areas where water is the most limiting factor, to obtain higher water productivity may be economically more profitable for the farmer and it is more likely to be considered rather than higher crop yield or economic benefits. This problem can be quantitatively described by an optimization model using linear fractional programming (LFP). The LFP model is capable of facilitating the analysis of system efficiency through transforming a bi-objective (e.g. cost and volume, output and input) problem into a ratio one, especially for addressing the issue with better achievements per unit of inputs (Lara and Stancu-Minasian, 1999; Gomez et al., 2006; Guo et al., 2014; Zhang and Guo, 2017a). However, the LFP model has difficulty in tackling uncertainties in irrigation water allocation problems.

Uncertainties existing in irrigation water management systems should be considered strategically. Generally, the irrigation

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water management basically consists of various parameters, such as hydrological elements, economic parameters and other related parameters, which are not easily quantified and defined in the model (Regulwar and Gurav 2011; Elliott et al., 2014; Zhang et al., 2017b). There are many inexact optimization methods mainly including stochastic mathematical programming (SMP), fuzzy mathematical programming (FMP) and interval linear programming (ILP) were developed to tackle uncertainties (Huang, 1996; Shastri and Diwekar, 2006; Kerachian and Karamouz, 2007; Guo et al., 2010; Li et al., 2013; Liu et al., 2014; Li et al., 2015; Nematian, 2016; Wang et al., 2016). In detail, SMP is generally used to deal with random uncertainties (i.e. hydrological parameters) in the irrigation water allocation problems. However, the data requirement to specify parameter with probability distribution functions (PDFs) and intensive computational burden may result in difficulties in its practical applications (Li et al., 2016a). ILP can deal with all uncertain parameters expressed as interval values without considering distribution information. However, it is incapable of tackling problems with violation of system constraints (Huang, 1996). In practical, some parameters exhibit features of vagueness and imprecision, they are estimated empirically and subject to human judgments, and can better be expressed by fuzzy membership functions (Xu and Qin, 2010). Moreover, FMP has a lower data requirements but reflects more flexible information in practical application because the related membership functions are more easily defined than PDFs (Xu et al., 2010; Wan and Li, 2015). Besides, when the violation of system constraints exists in the optimization model, the chance-constrained programming with fuzzy parameters needs to be introduced into the general optimization framework. Therefore, chance-constrained programming (CCP) was extended from stochastic (PDFs) to fuzzy (possibility distribution) environments (Liu and Iwamura, 1998). Similar to chance-constrained programming with random uncertainties models, fuzzy chance-constrained programming (FCCP) also requires that the fuzzy constraints be transformed into deterministic ones at predetermined confidence levels, which has a lower computational burden and generates more flexible solutions. Therefore, FCCP is an improved FMP method and it's effective for tackling fuzzy uncertainties and violation of system constraints (Xu and Qin, 2010). However, in many practical applications, fuzzy uncertainties may exist in both sides of constraints within the model. Thus, double-sided FCCP (DFCCP) method is a better choice for addressing these complexities. Nevertheless, DFCCP method cannot deal with ratio optimization problems. In addition, there are few studies on DFCCP method for irrigation water allocation under uncertainty.

Therefore, this study aims to develop a fuzzy linear fractional programming (FLFP) approach with double-sided fuzziness for optimal irrigation water allocation under uncertainty. Technique of DFCCP will be incorporated into LFP optimization framework. Therefore, the developed model can deal with uncertainty presented as fuzziness in both left-hand side and right-hand side components of constraints. Moreover, it has advantages in: (1) addressing two objectives simultaneously without considering subjective factors, (2) effectively reflecting economic water productivity, (3) introducing the concept of confidence levels of fuzzy constraints-satisfaction under both the minimum and maximum reliabilities to generate more flexible solutions and (4) facilitating in-depth analysis of interrelationships among economic water productivity, system benefits and varying confidence levels. The model is applied to a case study to demonstrate its applicability. The study area is a major agricultural production base in China while it currently faces increasingly severe water shortages problems. The objective is to maximize the economic water productivity (i.e. to obtain maximum system benefits with minimum irrigation water allocation) to allocate the limited irrigation water to different crops and different subareas. The optimal solutions of irrigation water allocation can be generated under each confidence level, which can be used to facilitate the analysis for the various irrigation water management schemes under complex uncertainties. The framework of this study is illustrated in (Fig. 1).

2. Formulation of the FLFP approach

Consider an agricultural irrigation system in a region where multiple crops need to be irrigated in different irrigation districts. The managers are responsible for making irrigation plans for these competing water users. Generally, a higher agricultural water productivity is a greater concern under limited water conditions for creating a sustainable agriculture system. Such a problem can be effectively addressed by linear fractional programming (LFP) model (Charnes and Cooper, 1963; Zhu and Huang, 2011). Moreover, when both left-hand and right-hand side parameters in the constraints are of fuzzy features and can be presents as possibility distributions, and the violation of system constraints exists in the optimization model, thus the double-sided fuzzy chance-constrained programming (DFCCP) method can be adopted (Fiedler et al., 2006; Xu et al., 2012). For example, in this study, parameters associated with the water availability and the rate of water loss during water conveyance are expressed as fuzzy sets. Therefore, in response to above concerns, a fuzzy linear fractional programming (FLFP) approach with double-sided fuzziness is developed by integrating the DFCCP method into LFP framework. Consequently, based on FLFP approach, the study problem for irrigation water allocation to different crops in different subarea can be formulated as follows:

System objective:

$$\max f=\frac{\displaystyle\sum_{i=1}^{12}\sum_{j=1}^{3}\left(NB_{ij}-CP_{ij}\right)A_{ij}\left[a_j\left(SW_{ij}+GW_{ij}+P_{e,i}\right)+b_j\right]-\sum_{i=1}^{12}\sum_{j=1}^{3}A_{ij}\left[CS_iSW_{ij}/\eta_s+CG_iGW_{ij}/\eta_g\right]}{\displaystyle\sum_{i=1}^{12}\sum_{j=1}^{3}\left(SW_{ij}/\eta_s+GW_{ij}/\eta_g\right)} \tag{1a}$$

where f is the objective function reflecting economic water productivity (10^6 Yuan/m^3); i denotes irrigation district ($i=1, 2, \cdots, 12$); j denotes type of crop ($j=1, 2, 3$); NB_{ij} is the price of crop j in irrigation district i (Yuan/kg); CP_{ij} is the cost of crop production of crop j in irrigation district i, which includes all other costs such as seed, fertilizer, pesticides, machinery, harvesting, marketing, drying and unexpected costs. This cost is independent of the amount of applied irrigation water (Yuan/kg); A_{ij} is the irrigated area of crop j in irrigation district i (ha); a_j and b_j are the coefficients of linear crop water production function of crop j; SW_{ij} and GW_{ij} are the decision variables, which means that the allocated surface water and groundwater (m^3/ha); $P_{e,i}$ is the effective precipitation of irrigation district i (m^3/ha); CS_i, CG_i are the cost of surface water and groundwater use in irrigation district i (Yuan/m^3); η_s and η_g are the irrigation water use coefficients of surface water and ground water respectively.

Constraints

The objective is subjected to a series of constraints in water availability, irrigation water demand, fairness, and other related concerns. These constraints can determine the reasonable decision space of decision variables.

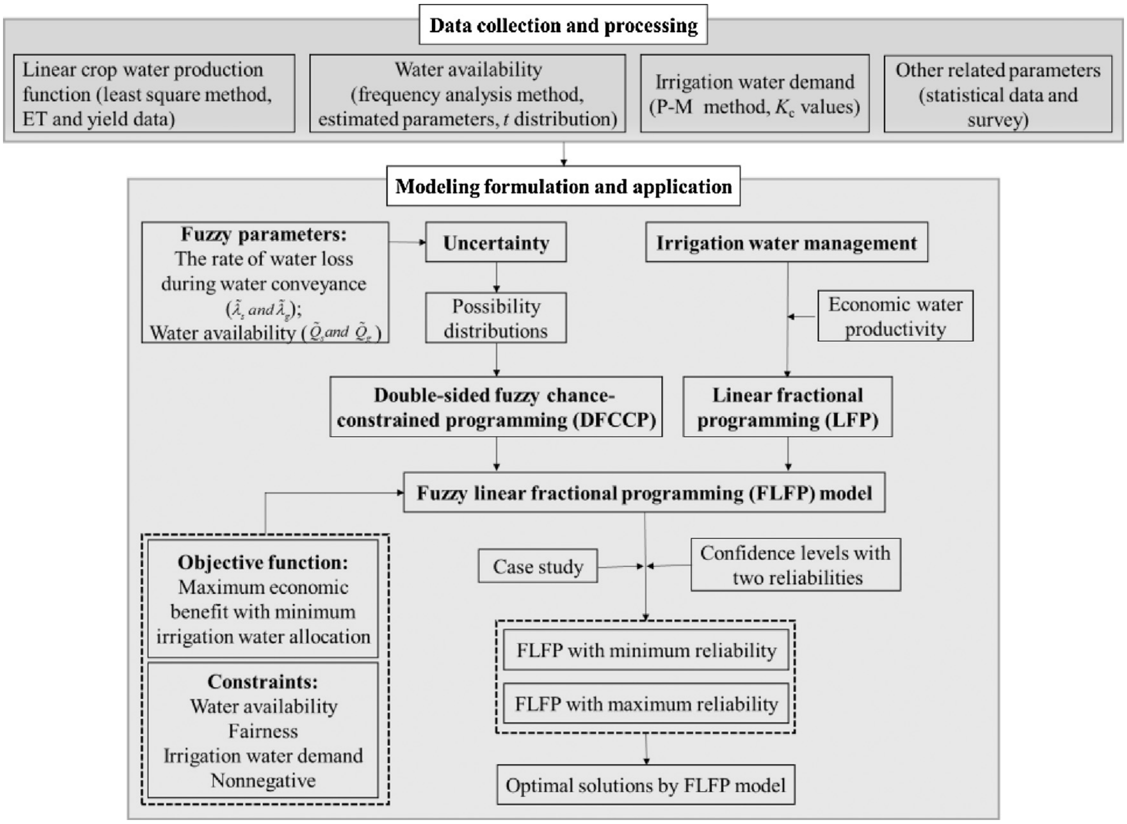

Fig. 1. The general framework of this study.

(1) Surface water availability constraint

$$Pos\left\{\tilde{\lambda}_s, \tilde{Q}_s | \sum_{i=1}^{12}\sum_{j=1}^{3}\left(1+\tilde{\lambda}_s\right)A_{ij}SW_{ij} \le \tilde{Q}_s\eta_s\beta_s\right\} \ge \alpha \tag{1b}$$

(2) Groundwater availability constraint

$$Pos\left\{\tilde{\lambda}_g, \tilde{Q}_g | \sum_{i=1}^{12}\sum_{j=1}^{3}\left(1+\tilde{\lambda}_g\right)A_{ij}GW_{ij} \le \tilde{Q}_g\eta_g\beta_g\right\} \ge \alpha \tag{1c}$$

where constraints (1b) and (1c) are the water availability constraints. $\tilde{\lambda}_s$ and $\tilde{\lambda}_g$ are the rate of surface water and groundwater loss during water conveyance exhibiting features of vagueness and imprecision. $\tilde{Q}_s$ and $\tilde{Q}_g$ are the surface water and groundwater availability ($10^6\,m^3$). They can be expressed by fuzzy membership functions. η_s and η_g are the irrigation water use coefficients of surface water and groundwater. β_s and β_g are the proportion of surface water and groundwater used for agricultural irrigation. In this study, let $\beta_s = 0.9$ and $\beta_g = 0.9$ in above constrains. α-cut confidence level is used to describe the fuzzy degree of membership level and quantify the fuzzy parameter. Fig. 2 presents the triangular membership function and α-cut interval with lower and upper bounds for each α-cut level.

(3) Fairness constraint

$$\frac{\sum_{i_1}^{I}\sum_{i_2}^{I}\left|\left(SW_{i_1j}+GW_{i_1j}+P_{e,i_1}\right)-\left(SW_{i_2j}+GW_{i_2j}+P_{e,i_2}\right)\right|}{2n\sum_{i=1}^{I}\left(SW_{ij}+GW_{ij}+P_{e,i}\right)} \le G_0,\ \forall j \text{ and } i_1, i_2 \in I \tag{1d}$$

where i_1 and i_2 are the two subareas of the 12 IDs independently. n, the number of total IDs; G_0, the Gini coefficient. This constraint means that the ratio between the sum of water shortages of each two subareas and total water demands is less than or equal to an acceptable gap (Zhang and Xu, 2011).

For some districts with higher water use efficiency and net benefit will be allocated with more water, this may not fair to farmers with lower water use efficiency. Therefore, the Gini coefficient is introduced when considering the fairness constraint (Yang et al., 2015). Let $G_0 = 0.4$ in fairness constraint because the international warning level is 0.4.

(4) Irrigation water demand constraint

$$ET_{\min,ij} \le SW_{ij} + GW_{ij} + P_{e,i} \le ET_{\max,ij},\ \forall i, j \tag{1e}$$

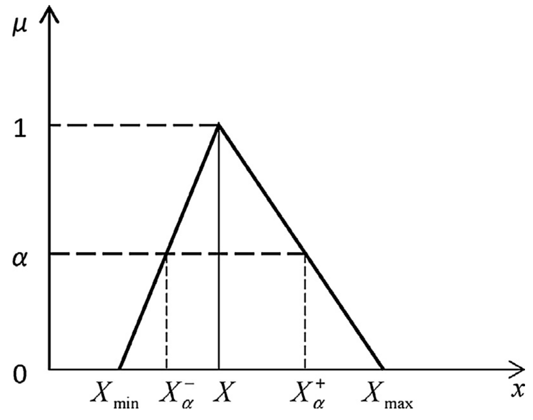

Fig. 2. Triangular membership function and α-cut interval with lower and upper bounds.

$ET_{\min,ij}$ and $ET_{\max,ij}$ are the minimum and maximum crop water requirement of crop j in irrigation district i (m^3/ha).

(5) Nonnegative constraint

$$SW_{ij} \geq 0, GW_{ij} \geq 0, \forall i, j \tag{1f}$$

SW_{ij} and GW_{ij} are the decision variables, which should be positive.

According to Fiedler et al. (2006), the violation of constraints in the DFCCP method at each predefined confidence level consists of two reliability scenarios, namely, the minimum reliability and the maximum reliability. Therefore, they can be presented as follows:

$$Pos\left\{\tilde{r}_{ij} \lesssim^{\min} \tilde{v}_{ij}\right\} = \sup\left\{\min\left(\mu_{\tilde{r}_{ij}}\left(x_{ij}\right), \mu_{\tilde{v}_{ij}}\left(y_{ij}\right)\right) | x_{ij}, y_{ij} \in \Re, x_{ij} \leq y_{ij}\right\} \geq \alpha \Leftrightarrow r_{ij}^{L}(\alpha) \leq v_{ij}^{R}(\alpha) \tag{2a}$$

$$Pos\left\{\tilde{r}_{ij} \lesssim^{\max} \tilde{v}_{ij}\right\} = \inf\left\{\max\left(1-\mu_{\tilde{r}_{ij}}\left(x_{ij}\right), 1-\mu_{\tilde{v}_{ij}}\left(y_{ij}\right)\right) | x_{ij}, y_{ij} \in \Re, x_{ij} \leq y_{ij}\right\} \geq \alpha \Leftrightarrow r_{ij}^{R}(1-\alpha) \leq v_{ij}^{L}(1-\alpha) \tag{2b}$$

where $\tilde{r}_{ij} \lesssim^{\min} \tilde{v}_{ij}$ means that the equations $\tilde{r}_{ij} \lesssim \tilde{v}_{ij}$ should be satisfied at the minimum reliability while $\tilde{r}_{ij} \lesssim^{\max} \tilde{v}_{ij}$ means that the equations $\tilde{r}_{ij} \lesssim \tilde{v}_{ij}$ should be satisfied at the maximum reliability. $\mu_{\tilde{r}_{ij}}\left(x_{ij}\right)$, $\mu_{\tilde{v}_{ij}}\left(y_{ij}\right)$ are the fuzzy membership functions of the variables x_{ij} and y_{ij}. The term $r_{ij}^{L}(\alpha)$ is defined as the minimum values of all possible values at α-cut level, namely, $r_{ij}^{L}(\alpha) = \inf\left\{R | R = \mu^{-1}(\alpha)\right\}$. Similarly, $v_{ij}^{R}(\alpha)$ is defined as maximum values of all possible values at α-cut level, namely, $v_{ij}^{R}(\alpha) = \sup\left\{V | V = \mu^{-1}(\alpha)\right\}$. μ^{-1} is the inverse function of μ.

Based on Eqs. (2a) and (2b), the FLFP approach can be transformed into two crisp equivalents accordingly. Therefore, these two deterministic submodels can be presented as follows:

a) Confidence levels at the minimum reliability:

$$\max f_{\mathrm{minr}} = \frac{\sum_{i=1}^{12}\sum_{j=1}^{3}\left(NB_{ij} - CP_{ij}\right) A_{ij}\left[a_j\left(SW_{ij} + GW_{ij} + P_{e,i}\right) + b_j\right] - \sum_{i=1}^{12}\sum_{j=1}^{3} A_{ij}\left[CS_i SW_{ij}/\eta_s + CG_i GW_{ij}/\eta_g\right]}{\sum_{i=1}^{12}\sum_{j=1}^{3}\left(SW_{ij}/\eta_s + GW_{ij}/\eta_g\right)} \tag{3a}$$

Constraints

(1) Surface water availability constraint

$$\sum_{i=1}^{12}\sum_{j=1}^{3}\left(1 + \lambda_s^{L}\alpha\right) A_{ij} SW_{ij} \leq Q_s^{R}\alpha\eta_s\beta_s \tag{3b}$$

(2) Groundwater availability constraint

$$\sum_{i=1}^{12}\sum_{j=1}^{3}\left(1 + \lambda_g^{L}\alpha\right) A_{ij} GW_{ij} \leq Q_g^{R}\alpha\eta_g\beta_g \tag{3c}$$

where the submodel means that constraints are satisfied at the minimum reliability, therefore, left-hand side parameters of constraints (3b) and (3c) should achieve their minimum values and the right-hand side parameters achieve their maximum ones.

(3) Fairness constraint

$$\frac{\sum_{i_1}^{I}\sum_{i_2}^{I} \left| \left(SW_{i_1 j} + GW_{i_1 j} + P_{e,i_1}\right) - \left(SW_{i_2 j} + GW_{i_2 j} + P_{e,i_2}\right) \right|}{2n\sum_{i=1}^{I}\left(SW_{ij} + GW_{ij} + P_{e,i}\right)} \leq G_0, \forall j \text{ and } i_1, i_2 \in I \tag{3d}$$

(4) Irrigation water demand constraint

$$ET_{\min,ij} \leq SW_{ij} + GW_{ij} + P_{e,i} \leq ET_{\max,ij}, \forall i, j \tag{3e}$$

(5) Nonnegative constraint

$$SW_{ij} \geq 0, GW_{ij} \geq 0, \forall i, j \tag{3f}$$

Thus, the objective function value f_{minr}^{opt}, and decision variables ($SW_{ij,\mathrm{minr}}^{opt}$ and $GW_{ij,\mathrm{minr}}^{opt}$) can be obtained.

b) Confidence levels at the maximum reliability:

$$\max f_{\mathrm{maxr}} = \frac{\sum_{i=1}^{12}\sum_{j=1}^{3}\left(NB_{ij} - CP_{ij}\right) A_{ij}\left[a_j\left(SW_{ij} + GW_{ij} + P_{e,i}\right) + b_j\right] - \sum_{i=1}^{12}\sum_{j=1}^{3} A_{ij}\left[CS_i SW_{ij}/\eta_s + CG_i GW_{ij}/\eta_g\right]}{\sum_{i=1}^{12}\sum_{j=1}^{3}\left(SW_{ij}/\eta_s + GW_{ij}/\eta_g\right)} \tag{4a}$$

Constraints

(1) Surface water availability constraint

$$\sum_{i=1}^{12}\sum_{j=1}^{3}\left(1 + \lambda_s^{R}(1-\alpha)\right) A_{ij} SW_{ij} \leq Q_s^{L}(1-\alpha)\eta_s\beta_s \tag{4b}$$

(2) Groundwater availability constraint

$$\sum_{i=1}^{12}\sum_{j=1}^{3}\left(1 + \lambda_g^{R}(1-\alpha)\right) A_{ij} GW_{ij} \leq Q_g^{L}(1-\alpha)\eta_g\beta_g \tag{4c}$$

where the submodel means that constraints are satisfied at the maximum reliability, therefore, left-hand side parameters of constraints (4b) and (4c) should become their maximum values and the right-hand side parameters become their minimum ones. In this case, the constraints become more restricted, which will lead to an increased strictness for the constraints and thus a narrow decision space.

(3) Fairness constraint

$$\frac{\sum_{i_1}^{I}\sum_{i_2}^{I} \left| \left(SW_{i_1 j} + GW_{i_1 j} + P_{e,i_1}\right) - \left(SW_{i_2 j} + GW_{i_2 j} + P_{e,i_2}\right) \right|}{2n\sum_{i=1}^{I}\left(SW_{ij} + GW_{ij} + P_{e,i}\right)} \leq G_0, \forall j \text{ and } i_1, i_2 \in I \tag{4d}$$

(4) Irrigation water demand constraint

$$ET_{\min,ij} \leq SW_{ij} + GW_{ij} + P_{e,i} \leq ET_{\max,ij}, \forall i, j \tag{4e}$$

(5) Nonnegative constraint

$$SW_{ij} \geq 0, GW_{ij} \geq 0, \forall i, j \tag{4f}$$

Thus, the objective function value f_{maxr}^{opt} decision variables $SW_{ij,\text{maxr}}^{opt}$ and $GW_{ij,\text{maxr}}^{opt}$ can be obtained. Based on the above analysis, the objective function values and decision variables at given confidence levels can be obtained by solving models (a) confidence levels at the minimum reliability and (b) confidence levels at the maximum reliability. Finally, the optimal solutions can be generated under different water levels (e.g. low level, medium level and high level), i.e. f_{opt}, SW_{ij}^{opt} and GW_{ij}^{opt} at each confidence level including two reliability scenarios.

3. Case study

3.1. Study area

The Heihe River Basin (HRB) is the second largest inland river basin in northwest China. It is divided into upper, middle and lower reaches by the controlled hydrological stations of Yingluoxia and Zhengyixia. The upstream area is the mountainous area where is the runoff production area and midstream area is a main water consumer due to its considerably large area of irrigated oasis. As for the downstream area, it's mainly desert facing serious ecological problems (e.g. degradation of the environment, desertification of land). The elevations of the HRB range from 1300 to 2500 m. Its climate is characterized by arid because this area is situated in the inner of Asia-Europe continent (Zhao et al., 2005). The mean annual temperature is 7.57 °C and mean annual precipitation is 195 mm as well as mean annual potential evapotranspiration is 1710 mm. The shallow, porous and highly permeable soil in the midstream area are characterized as greyish-brown desert soil, sandy loam and sandy soil (Zhao et al., 2010). The midstream area produces the main crops of wheat, maize and economic crops such as cotton, fruits and vegetables. The economic crop is also called as cash crop. Compared with grain crops like maize and wheat, the economic crops can bring high economic benefits to increase farmer's income. Because no multiple cropping exists and thus the value of the multiple cropping index is nearly 100%. The growing period of these crops ranges from March to October.

There are in total 12 irrigation districts (IDs) belonging to three administrative regions in midstream area that are irrigated directly by the main stream, which is selected as the study area (97° 37′–102° 06′E, 37° 44′–42° 40′N). Namely, Daman, Shangsan, Xijun and Yingke IDs of Ganzhou district; Pingchuan, Banqiao, Yanuan, Liaoquan and Shahe IDs of Linze county; Youlian, Luocheng, and Liuba IDs of Gaotai county. The study area (see Fig. 3) is a major irrigation area and grain production base in China that currently faces severe water shortages. With little precipitation and high evapotranspiration, irrigation thereby becomes an important measure for promoting grain production. Moreover, in midstream area, water consumption accounts for over 80% of the total available water resources from the Heihe River, and nearly 90% of the water consumption is used for irrigation (Li et al., 2016a). Crops are mainly irrigated with canal water, and traditional irrigation patterns such as flood and furrow methods are commonly used. Pumping groundwater is mainly for supplementing agricultural irrigation when the supply of canal water is inadequate or seasonal variations (Jiang et al., 2015).

The increasing water demands and shrinking water supplies have exacerbated the competition among water users. Therefore, how to optimize the limited irrigation water allocation, and thus to improve agricultural water productivity have become a growing urgent issue for the development of agriculture. This agricultural water productivity (nowadays called economic water productivity) is defined as the production or and system benefits per unit of irrigation water, which plays an increasingly important role in

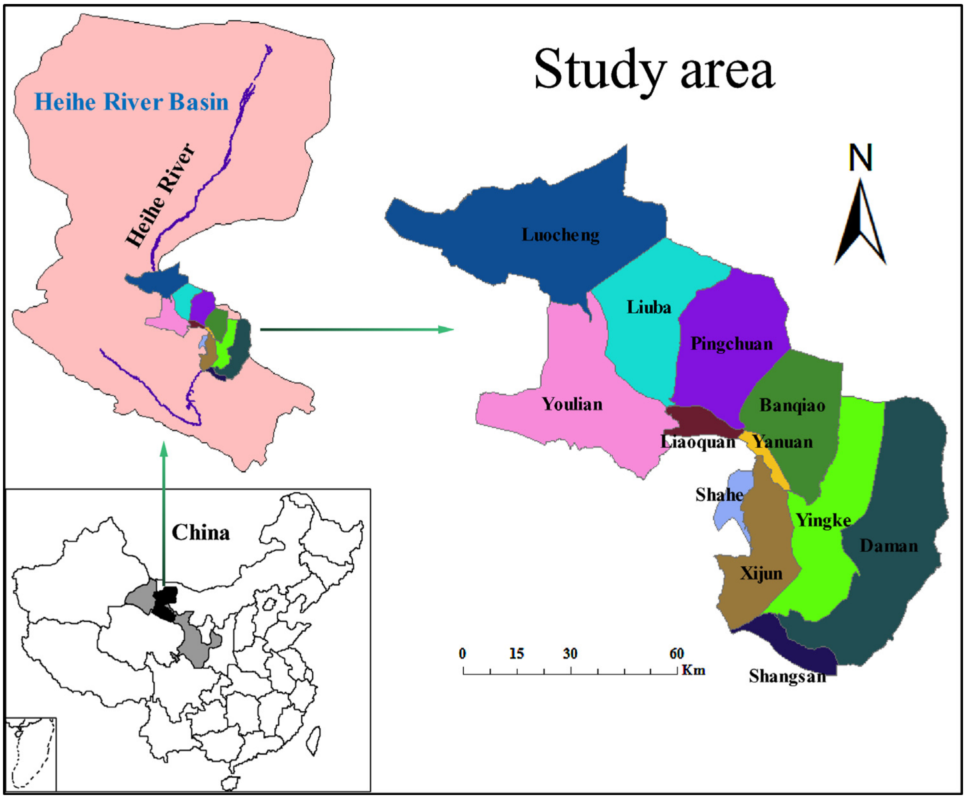

Fig. 3. Geographical position of the study area and distribution of each ID.

irrigation water management problem. The problem can be more compounded by many uncertainties existing in the agricultural systems, such as socio-economic parameters (i.e. market conditions, water use costs) and hydrological elements. Thus, managers attempt to obtain optimal irrigation water allocation solutions to three different crops in these IDs.

In the study system, the problems under consideration include: (1) how to maximize economic water productivity under fuzzy uncertainty; (2) how to reflect the variations of results with double-sided violation of system constraints and (3) how to identify the optimal irrigation water allocation solutions for crops to different IDs under different water levels. Therefore, to address above concerns, a fuzzy linear fractional programming (FLFP) approach with double-sided fuzziness is developed for optimal irrigation water allocation.

3.2. *Data collection and processing*

To deal with such a problem, a series of input parameters need to be acquired not only about crops data among different IDs, but also the information about the precipitation, irrigation water demand and available water resources. A detailed description of the different input parameters can be presented as follows.

3.2.1. *Crop water production function*

The crop water production function (CWPF) is used to express the relation between obtained crop yield and the total amount of actual evapotranspiration (Brumbelow and Georgakakos, 2007), which will directly influence irrigation planning and irrigation water allocation solutions. Two types of CWPF, including the relation between crop yields and actual evapotranspiration in the whole growth period and in each growth stage, are commonly used. In this study, CWPFs for the whole growth stage will be adopted, which basically includes linear and nonlinear functions. Generally, the linear and nonlinear empirical models of CWPFs can be established by the experimental data for the whole growth stage (Chen and Kang, 1995). A linear form is more suitable to the low-yield area where it is lack of irrigation water and has a lower management level (Kang and Cai, 1996; Li et al., 2016a,b). Because the study area is located in an arid region, thus a linear CWPF model is a better choice, namely, $Y = a + bET$ (where Y is crop yield, kg/ha; ET is actual evapotranspiration, m^3/ha; a, b are empirical coefficients fitted by least square method). Different crop water production functions are fitted based on evapotranspiration data and associated yield data obtained from field experiments. The field experiments were conducted in Yingke irrigation district during April to September in 2012–2013 (Jiang et al., 2015). Field experiments are a laborious and time-consuming task that did not conducted within each irrigation district, resulting in a lack of data for all irrigation districts. We believe that soil types and other characteristics have an impact on the water production function, which we did not consider in the study because water is the most dominant limiting factor. Moreover, Yingke irrigation district located in the Ganzhou district is a typical irrigation district in the middle reaches of Heihe River Basin. Therefore, the data is representative. It is thus assumed that crop water production function of all these 12 IDs is the same due to their similar hydrological and meteorological conditions. The expression of linear crop water production functions for the three crops in this study is shown in Table 1.

Table 1
Crop water production function (Y, kg/ha; ET, m^3/ha).

Crops	Water production function
Wheat	$Y = 0.6061ET + 3688.7$
Maize	$Y = 0.7243ET + 6746.6$
Economic crops	$Y = 0.4875ET + 1308.5$

3.2.2. *Water availability*

Crops in these IDs are irrigated by surface water and groundwater. In order to reflect the uncertain characteristics of parameters, both surface water and groundwater availability are considered as a fuzzy number on the right-hand side constraints (Table 2). The triangular fuzzy number is used to describe the fuzzy characteristics, which requires to obtain the middle and the lower values of water availability. In terms of surface water availability, water supply for the midstream area is delivered by the main stream of Heihe River. Thus water supply for all the 12 IDs equals the difference that the runoff from the upstream subtracts the water release to the downstream area. The category of three inflow levels is based on the frequency analysis method (Li et al., 2016a), namely, low level ($P > 75\%$), medium level ($25\% \leq P \leq 75\%$) and high level ($P < 25\%$). Because the standard deviation of the each inflow level is unknown, then t distribution is used to estimate the range of parameters. This study adopts the 95% confidence level lower bounds to express the lower values of fuzzy numbers, and the mean of inflow are to express the middle values. Therefore, for each inflow level, a triangular fuzzy value with lower and middle values denotes the surface water availability for irrigation.

The available groundwater is obtained from historical statistical data. It is also divided into three water levels in the form of triangular fuzzy number. The middle and the lower values of available groundwater are calculated based on the mean and the minimum values of the statistical data. Once the surface water is insufficient for irrigation due to seasonal variations, then pumping groundwater is to make up for this deficiency. Therefore, when the surface water is at low level, more groundwater will be extracted. The effective precipitation among different water levels are obtained from the typical hydrological years (Table 3). Because of the low precipitation in arid area, the precipitation can be fully utilized, so effective precipitation can be obtained directly from the measured data of meteorological stations.

3.2.3. *Irrigation water demand*

In this study, irrigation water demand for these 12 IDs is calculated according to $ET_c = K_c \times ET_0$ (Allen et al., 1998), where ET_c is the actual crop evapotranspiration (mm/day), K_c is the crop coefficient, and ET_0 is the reference crop evapotranspiration (mm/day). Daily reference evapotranspiration is obtained by the FAO 56 Penman-Monteith method based on the meteorological data from Zhangye (No.52562), Linze (No. 52557) and Gaotai

Table 2
Surface water and groundwater availabilities under different water levels (10^6 m^3).

Water level	Surface water availability			Groundwater availability		
	Lower value	Middle value	Upper value	Lower value	Middle value	Upper value
Low	457	543	628	305	317	329
Medium	719	752	784	254	270	286
High	855	950	1045	205	216	227

Table 3
Maximum evapotranspiration $ET_{\max}$ and effective precipitation (m^3/ha).

IDs	Wheat			Maize			Economic crops			Effective precipitation		
	High	Medium	Low	High	Medium	Low	High	Medium	Low	High	Medium	Low
Daman	5958.10	5788.50	5498.40	6149.00	6007.10	5811.40	5593.90	5457.50	5235.30	1253.00	1221.00	1149.00
Shangan	5958.10	5788.50	5498.40	6149.00	6007.10	5811.40	5593.90	5457.50	5235.30	1253.00	1221.00	1149.00
Xijun	5958.10	5788.50	5498.40	6149.00	6007.10	5811.40	5593.90	5457.50	5235.30	1253.00	1221.00	1149.00
Yingke	5958.10	5788.50	5498.40	6149.00	6007.10	5811.40	5593.90	5457.50	5235.30	1253.00	1221.00	1149.00
Pingchuan	6480.76	6291.66	5983.20	6562.76	6413.96	6206.08	6162.16	6014.84	5776.38	1127.70	1098.90	1034.10
Banqiao	6480.76	6291.66	5983.20	6562.76	6413.96	6206.08	6162.16	6014.84	5776.38	1127.70	1098.90	1034.10
Yanuan	6480.76	6291.66	5983.20	6562.76	6413.96	6206.08	6162.16	6014.84	5776.38	1127.70	1098.90	1034.10
Liaoquan	6480.76	6291.66	5983.20	6562.76	6413.96	6206.08	6162.16	6014.84	5776.38	1127.70	1098.90	1034.10
Shahe	6480.76	6291.66	5983.20	6562.76	6413.96	6206.08	6162.16	6014.84	5776.38	1127.70	1098.90	1034.10
Youlian	6829.20	6627.10	6306.40	6838.60	6685.20	6469.20	6541.00	6386.40	6137.10	1014.93	989.01	930.69
Luocheng	6829.20	6627.10	6306.40	6838.60	6685.20	6469.20	6541.00	6386.40	6137.10	1014.93	989.01	930.69
Liuba	6829.20	6627.10	6306.40	6838.60	6685.20	6469.20	6541.00	6386.40	6137.10	1014.93	989.01	930.69

Table 4
K_c values for different crops.

Month	K_c values for different crops		
	Wheat	Maize	Economic crops
March	–	–	[0.34, 0.35]
April	0.22	[0.22, 0.23]	[0.31, 0.35]
May	[0.69, 1.02]	[0.23, 0.33]	[0.63, 0.71]
June	[1.16, 1.35]	[0.56, 1.03]	[0.95, 1.01]
July	[0.74, 1.11]	1.20	[0.99, 1.20]
August	–	[1.09, 1.20]	[0.64, 0.77]
September	–	[0.54, 0.82]	[0.48, 0.62]

(No. 52546) meteorological stations located in midstream area during 1967–2009 (Yin et al., 2012). The total crop evapotranspiration of the whole growth stage can be calculated by the sum of daily value of each growth stage under typical hydrological year. The K_c values vary during different growth stages, which can determine the upper and lower bounds of irrigation water demand (Li et al., 2016b). Therefore, irrigation water demand and K_c values of the crops at different growth stages can be seen in Tables 3 and 4.

3.2.4. Other related parameters

Table 5 presents some basic data of different crops in these IDs, including crop planting area, market price and water use cost per unit irrigation water. Because the rate of canal lining is low, thus nearly 30% of the water is lost during water conveyance (Zhang and Guo, 2017b). Therefore, the irrigation water use coefficients of surface water and groundwater are 0.52, 0.60, respectively. The rate of water loss during water conveyance is considered as triangular fuzzy sets in the left-hand side constraints. In this problem, the rates of water loss during surface water and groundwater conveyance are (0.20, 0.25, 0.30) and (0.10, 0.15, 0.20), respectively.

4. Result analysis

Because the developed model incorporates the DFCCP method into LFP optimization framework, thus this model reflects the characteristics and advantages of the two methods simultaneously. Firstly, the objective function of model is defined as the maximum economic benefits per unit of irrigation water, which also represents the agricultural water use efficiency or economic water productivity. Moreover, the fundamental concept of the DFCCP method is to generate optimal results that double-sided fuzzy constraints can be satisfied at a certain confidence level in optimization model. Therefore, how to choose the confidence level is particularly important. To investigate the results of violating the constraints and generate a range of decision solutions, the results at four α-cut confidence levels (e.g. $\alpha = 0.2$, 0.4, 0.6 and 0.8) are analyzed and compared. Different α-cut levels reflect the subjective attitudes or preferences of decision-makers, and thus affect the corresponding changing trend of the results under different satisfaction degrees of fuzzy uncertainty.

Table 6 presents the obtained results of irrigation water allocation from FLFP model under medium water level at α-cut levels from 0.2 to 0.8. For the same α-cut confidence level under the minimum and maximum reliabilities, the overall results for different crops in Ganzhou district and Gaotai county almost are the same. For example, at $\alpha = 0.2$ under the minimum and maximum reliabilities (see second column of Table 6), the obtained results for

Table 5
Basic data of different crops in the study irrigation districts.

IDs	Crop planting area (10^3 ha)			Average crop price (Yuan/kg)			Crop production costs (Yuan/kg)			Surface water cost (Yuan/m^3)	Groundwater cost (Yuan/m^3)
	Wheat	Maize	Economic crops	Wheat	Maize	Economic crops	Wheat	Maize	Economic crops		
Daman	0.17	11.57	2.46	2.04	2.42	5.02	0.45	0.45	0.60	0.18	0.40
Shangsan	0.32	11.30	0.42	2.04	2.42	5.02	0.45	0.45	0.60	0.13	0.40
Xijun	0.63	17.67	0.29	2.04	2.42	5.02	0.45	0.45	0.60	0.20	0.40
Yingke	1.11	8.50	2.41	2.04	2.42	5.02	0.45	0.45	0.60	0.16	0.40
Pingchuan	0.57	2.89	0.42	2.10	2.38	4.84	0.45	0.45	0.60	0.21	0.50
Bnaqiao	0.14	3.68	0.19	2.10	2.38	4.84	0.45	0.45	0.60	0.21	0.50
Yanuan	0.09	1.33	0.27	2.10	2.38	4.84	0.45	0.45	0.60	0.21	0.50
Liaoquan	0.29	1.51	0.50	2.10	2.38	4.84	0.45	0.45	0.60	0.21	0.50
Shahe	0.19	0.95	0.16	2.10	2.38	4.84	0.45	0.45	0.60	0.21	0.50
Youlian	0.51	4.89	0.94	2.27	2.52	5.16	0.45	0.45	0.60	0.33	0.60
Luocheng	0.66	3.53	2.42	2.27	2.52	5.16	0.45	0.45	0.60	0.18	0.60
Liuba	0.79	3.28	1.00	2.27	2.52	5.16	0.45	0.45	0.60	0.32	0.60

Table 6

Optimal solutions of FLFP model under medium water level (surface water; groundwater, m^3/ha).

IDs	$\alpha = 0.2$			$\alpha = 0.4$			$\alpha = 0.6$			$\alpha = 0.8$		
	Wheat	Maize	Economic crops	Wheat	Maize	Economic crops	Wheat	Maize	Economic crops	Wheat	Maize	Economic crops
Minimum reliability scenario												
Daman	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9
Shangsan	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9
Xijun	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9
Yingke	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9
Pingchuan	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8
Banqiao	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8
Yanuan	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8
Liaoquan	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8
Shahe	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8	0; 1732.3	0; 1787.4	0; 1607.8
Youlian	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9
Luocheng	0; 1993.2	1145.1; 874.2	0; 1884.9	0; 1993.2	1931.7; 87.7	0; 1884.9	1993.2; 0	2019.3; 0	453.3; 1431.6	796.3; 1196.9	2019.3; 0	1884.9; 0
Liuba	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9
Maximum reliability scenario												
Daman	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9
Shangsan	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9
Xijun	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9
Yingke	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9
Pingchuan	383.3; 1349.1	0; 1787.4	458.7; 1149.1	708.9; 1023.5	0; 1787.4	693.0; 914.7	742.5; 989.8	0; 1787.4	713.8; 894.0	758.6; 973.8	768.9; 1018.5	727.4; 880.4
Banqiao	754.2; 978.2	0; 1787.4	641.6; 966.2	829.4; 903.0	0; 1787.4	748.1; 859.7	836.0; 896.4	1267.5; 519.8	757.4; 850.3	840.5; 891.8	1300.2; 487.1	763.9; 843.9
Yanuan	789.7; 942.6	502.9; 1284.5	582.5; 1025.2	840.8; 891.5	1155.1; 632.3	729.6; 878.2	845.3; 887.1	698.6; 1088.7	742.8; 865.0	848.4; 884.0	733.3; 1054.1	751.4; 856.3
Liaoquan	636.4; 1095.9	1407.3; 380.1	378.9; 1228.9	794.1; 938.3	1787.4; 0	664.3; 943.5	808.2; 924.1	681.7; 1105.7	693.6; 914.2	817.5; 914.9	720.2; 1067.1	707.4; 900.4
Shahe	711.0; 1021.4	153.2; 1634.1	671.4; 936.4	815.8; 916.6	692.6; 1094.8	758.1; 849.7	825.2; 907.2	747.6; 1039.8	765.7; 842.1	831.3; 901.0	719.7; 1067.7	770.9; 836.8
Youlian	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9
Luocheng	1993.2; 0	2019.3; 0	1884.9; 0	1993.2; 0	2019.3; 0	1884.9; 0	1993.2; 0	2019.3; 0	1884.9; 0	1993.2; 0	2019.3; 0	1884.9; 0
Liuba	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9

three crops among the majority of IDs show that crops are irrigated by groundwater completely, and no surface water is used for irrigation. Under such circumstance, the allowable groundwater is firstly utilized for irrigation due to its higher irrigation water use coefficient and lower rate of water loss during water conveyance (i.e. $\eta_g > \eta_s$ and $\lambda_g < \lambda_s$). Luocheng is an exception for the optimal irrigation water allocation of three crops. That is, under the minimum reliability, the result is 0; 1993.2 m^3/ha (surface water; groundwater) for wheat, 1145.1; 874.2 m^3/ha for maize and 0; 1884.9 m^3/ha for economic crops. While under the maximum reliability, the result is 1993.2; 0 m^3/ha for wheat, 2019.3; 0 m^3/ha for maize and 1884.9; 0 m^3/ha for economic crops respectively. Similarly, the results can be examined at α = 0.4, 0.6, 0.8 under both the minimum and maximum reliabilities. However, IDs in Linze county have different results. Under the minimum reliability scenario, all the crops are irrigated by groundwater totally but the varying results can be obtained under the maximum reliability scenario. Furthermore, the sum of irrigated surface water and groundwater is also equal to their minimum crop water requirements. This is because if the linear CWPF projects a positive vertical axis intercept, the empirical value of crop water productivity will decline as ET increases (Wichelns, 2014). The intercept will thereby help determine whether water productivity will increase or decrease at higher levels of ET for a linear CWPF. Thus, it is evident that those crops suffer from water deficiency in order to achieve the maximum economic benefits with the minimum irrigation amount. Additionally, the results also indicate that surface water resources are not fully utilized. More importantly, this also shows that it is urgent need to take water-saving measures such as canal lining to reduce the water loss and thus improve the irrigation efficiency of surface water. On the other hand, such a great quantity of surface water is not fully utilized, indicating that there is still not achieve its full potential for the development of water-saving agriculture in the region.

Furthermore, at α = 0.2 under the minimum reliability, the allocated surface water is less than or equals to that under the maximum reliability. In contrast, the allocated groundwater presents the opposite result. Taking wheat as an example, the allocated surface water in IDs of Linze county is zero under the minimum reliability while the results under the maximum reliability are 383.3 m^3/ha (Pingchuan), 754.2 m^3/ha (Banqiao), 789.7 m^3/ha (Yanuan), 636.4 m^3/ha (Liaoquan) and 711.0 m^3/ha (Shahe). The reason is that, according to the DFCCP algorithm, the surface water and groundwater availabilities under the minimum reliability conditions are significantly higher than the maximum reliability. More groundwater availabilities are used for irrigation due to the structure of the developed optimization model. Therefore, the overall optimal solution is to give priority to the use of groundwater to meet the actual crop water requirements.

The results in Table 6 also shows that a higher confidence level corresponds to a higher total irrigation water allocation (i.e. the equation: $\sum_{i=1}^{I}\sum_{j=1}^{J}A_{ij}\left(SW_{ij}/\eta_s+GW_{ij}/\eta_g\right)$). For example, when α-cut confidence level is raised from o.2 to 0.8, the total of allocated irrigation water is increased from 237.09 × 10^6 m^3 to 239.19 × 10^6 m^3 under the minimum reliability. The total of allocated irrigation water is increased from 240.52 × 10^6 m^3 to 242.44 × 10^6 m^3 under the maximum reliability. Based on fuzzy arithmetic that can be used to generalize crisp mathematical operators to fuzzy sets (Zadeh, 1978), an increased α-cut confidence level leads to an increased possibility and decreased admissible risk of violation of constraints, which means an increased strictness for the double-sided fuzzy constraints. Thus less water availabilities (right-hand side fuzzy parameters of constraints) and larger rate of water loss during water conveyance (left-hand side fuzzy parameters of constraints) will be generated. Therefore, in this case, the allocated groundwater availabilities are limited and more surface water availabilities should be used for irrigation, leading to an increased total irrigation water use.

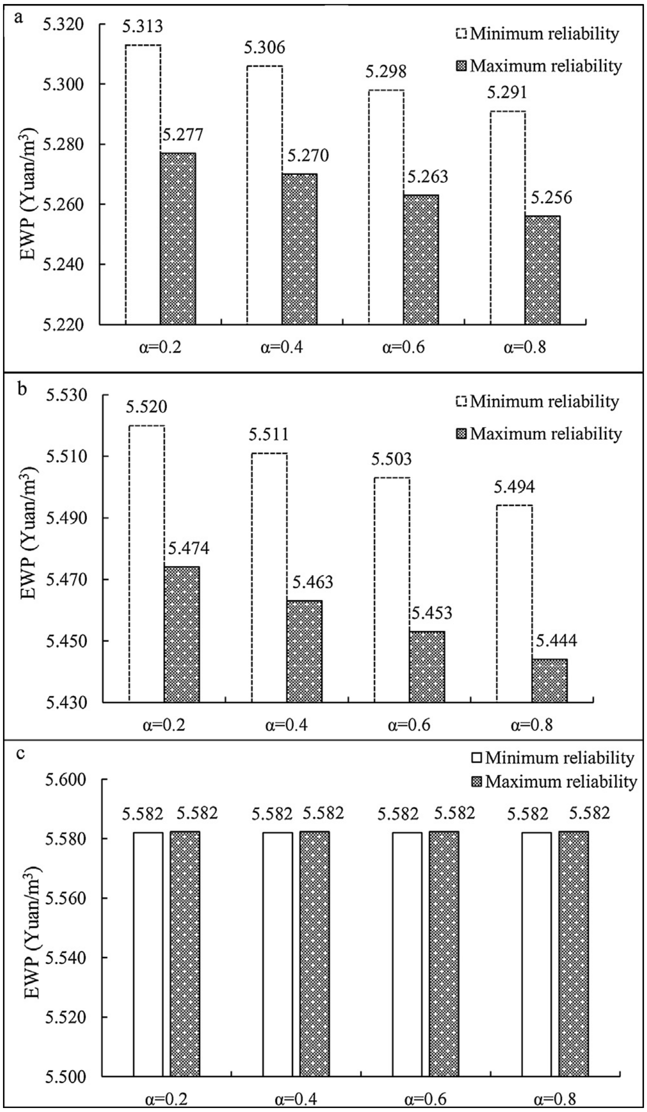

Fig. 4. a, b and c. Economic water productivity (EWP) of FLFP model under high (a), medium (b) and low (c) water levels.

Fig. 4a–c present the variations of the maximum economic water productivity between total economic benefits and total irrigation water use at different confidence levels under the minimum and maximum reliabilities scenarios. Taking the medium water level as example shown in Fig. 4b, the results of economic water productivity from the developed model are decreased with an increased confidence level. For example, when the confidence level is raised, the economic water productivity is decreased from 5.520 Yuan/m^3 to 5.494 Yuan/m^3 under the minimum reliability. Certainly, similar changing trends of the results can be obtained under high and low water levels. This is mainly because, with the confidence level increasing, the double-sided constraints become more strictly established and thus lead to a narrower decision space. This will conversely result in increased total irrigation water use and water use costs, and thus reduce system economic benefits and economic water productivity. Moreover, the economic water productivity is 5.474 Yuan/m^3 to 5.444 Yuan/m^3

Table 7
Optimal solutions of linear DFCCP model under medium water level (surface water; groundwater, m^3/ha).

IDs	$\alpha = 0.2$			$\alpha = 0.4$			$\alpha = 0.6$			$\alpha = 0.8$		
	Wheat	Maize	Economic crops	Wheat	Maize	Economic crops	Wheat	Maize	Economic crops	Wheat	Maize	Economic crops
Minimum reliability scenario												
Daman	0; 4567.5	1526.1; 3260.0	0; 4236.5	4567.5; 0	1698.1; 3088.0	0; 4236.5	0; 1383.8	1780.9; 3005.2	0; 4236.5	1383.8; 0	1991.7; 2794.4	0; 4236.5
Shangsan	4567.5; 0	4786.1; 0	4236.5; 0	4567.5; 0	4786.1; 0	4236.5; 0	1950.4; 0	4786.1; 0	4236.5; 0	1383.8; 0	4786.1; 0	4236.5; 0
Xijun	0; 4567.5	0; 4786.1	0; 4236.5	0; 4567.5	0; 4786.1	0; 4236.5	0; 1383.8	0; 4786.1	0; 4236.5	0; 1383.8	0; 4786.1	0; 4236.5
Yingke	4567.5; 0	4786.1; 0	4236.5; 0	4567.5; 0	4786.1; 0	4236.5; 0	1383.8; 0	4786.1; 0	4236.5; 0	1383.8; 0	4786.1; 0	4236.5; 0
Pingchuan	5192.8; 0	5315.1; 0	4915.9; 0	2920.8; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0
Banqiao	5192.8; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0
Yanuan	5192.8; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0
Liaoquan	5192.8; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0
Shahe	5192.8; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0
Youlian	4151.3; 0	5696.2; 0	5397.4; 0	1993.2; 0	5696.2; 0	5397.4; 0	1993.2; 0	5696.2; 0	5397.4; 0	1993.2; 0	4697.1; 0	5397.4; 0
Luocheng	5638.1; 0	5696.2; 0	5397.4; 0	5638.1; 0	5696.2; 0	5397.4; 0	5638.1; 0	5696.2; 0	5397.4; 0	1993.2; 0	5696.2; 0	5397.4; 0
Liuba	5638.1; 0	5696.2; 0	5397.4; 0	1993.2; 0	5696.2; 0	5397.4; 0	1993.2; 0	5696.2; 0	5397.4; 0	1993.2; 0	5696.2; 0	5397.4; 0
Maximum reliability scenario												
Daman	0; 1383.8	2462.7; 2323.4	0; 4236.5	1383.8; 0	2661.7; 2124.4	0; 4236.5	0; 1383.8	2898.0; 1888.2	0; 4236.5	1383.8; 0	3089.6; 1696.5	0; 4236.5
Shangsan	1383.8; 0	4786.1; 0	4236.5; 0	1383.8; 0	4786.1; 0	4236.5; 0	1383.8; 0	4786.1; 0	4236.5; 0	1383.8; 0	4786.1; 0	4236.5; 0
Xijun	0; 1383.8	0; 4786.1	0; 4236.5	0; 1383.8	0; 4786.1	0; 4236.5	0; 1383.8	0; 4786.1	0; 4236.5	0; 1383.8	0; 4786.1	0; 4236.5
Yingke	1383.8; 0	4786.1; 0	4236.5; 0	1383.8; 0	4786.1; 0	4236.5; 0	1383.8; 0	4786.1; 0	4236.5; 0	1383.8; 0	4786.1; 0	4236.5; 0
Pingchuan	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0
Banqiao	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5270.0; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0
Yanuan	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	1787.4; 0	4915.9; 0
Liaoquan	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	3756.3; 0	4915.9; 0
Shahe	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	5315.1; 0	4915.9; 0	1732.3; 0	1787.4; 0	4915.9; 0	1732.3; 0	1787.4; 0	4915.9; 0
Youlian	1993.2; 0	2019.3; 0	5397.4; 0	1993.2; 0	2019.3; 0	5397.4; 0	1993.2; 0	2019.3; 0	5397.4; 0	1993.2; 0	2019.3; 0	5397.4; 0
Luocheng	1993.2; 0	5696.2; 0	5397.4; 0	1993.2; 0	5696.2; 0	5397.4; 0	1993.2; 0	5696.2; 0	5397.4; 0	1993.2; 0	5696.2; 0	5397.4; 0
Liuba	1993.2; 0	5245.9; 0	5397.4; 0	1993.2; 0	3077.4; 0	5397.4; 0	1993.2; 0	2019.3; 0	5397.4; 0	1993.2; 0	2019.3; 0	5397.4; 0

under the maximum reliability scenario. Obviously, the maximum reliability is slightly lower than that under the minimum reliability scenario. Compared with the minimum reliability scenario, this confidence level of constraints satisfaction is more reliable because the risk level is lower. Therefore, in this case, the decision maker will be more conservative in the irrigation water allocation management.

5. Discussion

To better summarize the characteristics and advantages of the FLFP model, a linear DFCCP model is introduced into the study for comparison. This linear model is an economy-oriented optimization model, its objective function is to maximize the system economic benefits with the same parameter settings of fuzzy uncertainty and a set of constraints. Therefore, optimal solutions of irrigation water allocation can be obtained by solving linear DFCCP model (Table 7). Figs. 5–7 show the comparison of the economic water productivity and total irrigation water use of two models under high, medium and low water levels. In fact, under different water levels, the FLFP model leads to lower system economic benefits than the linear DFCCP model at different confidence levels. However, in terms of economic water productivity, the FLFP model is much greater than the linear model. Taking medium water level as an example (Fig. 6a), when the confidence level raised from 0.2 to 0.8, the economic water productivity resulting from FLFP model is decreased from 5.520 Yuan/m^3 to 5.494 Yuan/m^3, while that from linear model is increased from 2.046 Yuan/m^3 to 2.141 Yuan/m^3. Such a notable difference between the two models can be explained by further comparisons of the total irrigation water use amount. In other words, the linear model leads to more system economic benefits, but it also consumes a large amount of irrigation water resources, resulting in less economic water productivity.

As shown in Figs. 5–7, total irrigation water use of FLFP model is greatly lower than that of linear DFCCP model. In detail (see Fig. 6a), as the confidence level raised, total irrigation water use resulting from FLFP model will fluctuate within a range of 237.09×10^6 m^3 and 239.19×10^6 m^3 while that from linear model is decreased from 807.60×10^6 m^3 to 765.85×10^6 m^3. This is because the FLFP model focuses on the maximum economic water productivity, that is, maximum net system benefits with minimum irrigation water allocation from the irrigated area. But the linear DFCCP model puts more emphasis on the total system benefits from the overall irrigation districts. Fig. 8 illustrates the system benefits of the FLFP and linear DFCCP models under the minimum and maximum reliabilities and high water level. Apparently, the system benefits of the linear DFCCP model are roughly greater than FLFP model. Thus, the total system benefits and farmer's economic income of FLFP model are much less due to yield reduction, but these saved

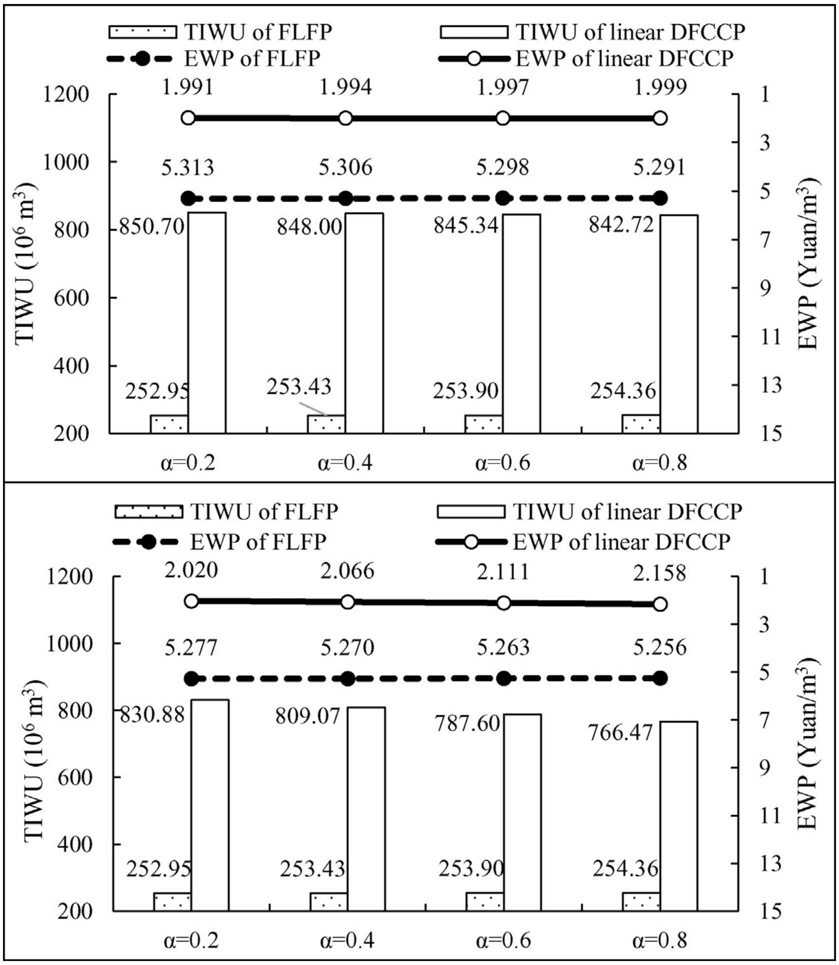

Fig. 5. a, b. Economic water productivity (EWP) and total irrigation water use (TIWU) of two models under high water level (i.e. the minimum (a) and maximum (b) reliability scenarios).

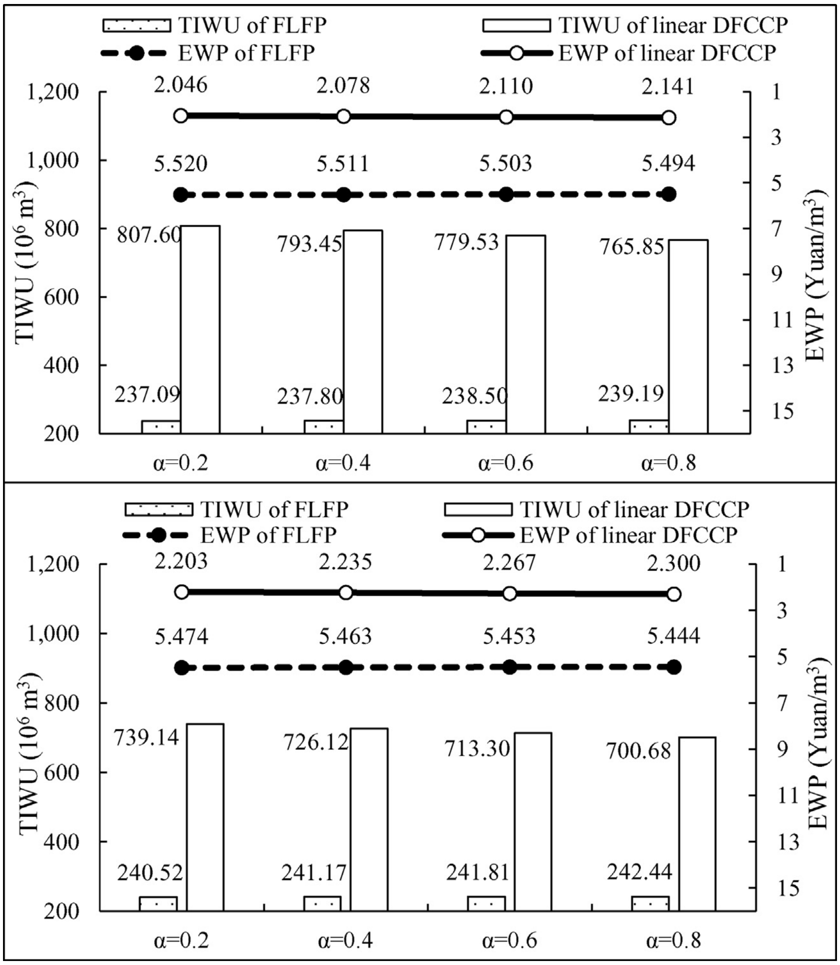

Fig. 6. a, b. Economic water productivity (EWP) and total irrigation water use (TIWU) of two models under medium water level (i.e. the minimum (a) and maximum (b) reliability scenarios).

water can be used for other purpose to compensate for this. For instance, water saved by FLFP model can be used to discharge to the downstream area for ecological restoration (Cheng et al., 2014), or transfer agricultural water to the second industry and tertiary industry to bring an incremental benefit of water resources through agricultural water savings reallocation (Zhang and Guo, 2016). Therefore, the results of the FLFP model show that the potential in the study area is significant in terms of economic water productivity and water savings. On the other hand, in practical problems, the objective of the maximum economic water productivity may lead to allocate less irrigation water to achieve a relative higher agricultural water productivity, which will lead to water deficit irrigation conditions of crops and thus reduce the expected agricultural crop yields. However, this situation can be resolved through market trade and national grain subsidy policy to guarantee food security and give economic compensation for farmers respectively.

Furthermore, when the confidence level increases, the economic water productivity of FLFP model decreases while the economic water productivity of linear model increases. Fig. 5a, b present the variation trends between the minimum and maximum reliabilities scenarios. The economic water productivity under the minimum reliability is slightly higher than that under the maximum reliability of the FLFP model. However, the linear model shows an opposite result. Therefore, compared with linear model, the FLFP model is an efficiency-oriented optimization model that can effectively deal with ratio problem and reflect economic water productivity. More practically, it can reflect the tradeoffs and interrelationships through a ratio of economic water productivity by taking into account multiple input factors, which is helpful in the sustainable development of agriculture.

In order to highlight the characteristics of FLFP model, a linear fractional programming (LFP) model is introduced for comparison purpose. Without consideration of double-sided fuzzy chance-constrained constraints, its deterministic parameters can be derived from setting the confidence level $\alpha = 1.0$. Therefore, the solutions of LFP model are special cases in the solutions generated from FLFP method. Accordingly, the solution of the LFP model can be obtained (Table 8). In this case, the decision alternative will be restricted to a single solution. Meanwhile, the flexibility and effectiveness of decision making will be undermined without considering the fuzzy uncertainty of parameters. Therefore, the FLFP model can support the in-depth analysis of the relationship between economic water productivity and different confidence levels, and can provide more effective decision alternatives.

To sum up, the above analysis shows that the FLFP model is derived from combining with LFP and DFCCP model, which can be used to deal with the problem of irrigation water allocation under uncertainty. The FLFP model has the following advantages: (1) it can deal with two objectives directly rather than subjective determination of weight coefficients; (2) it effectively reflects

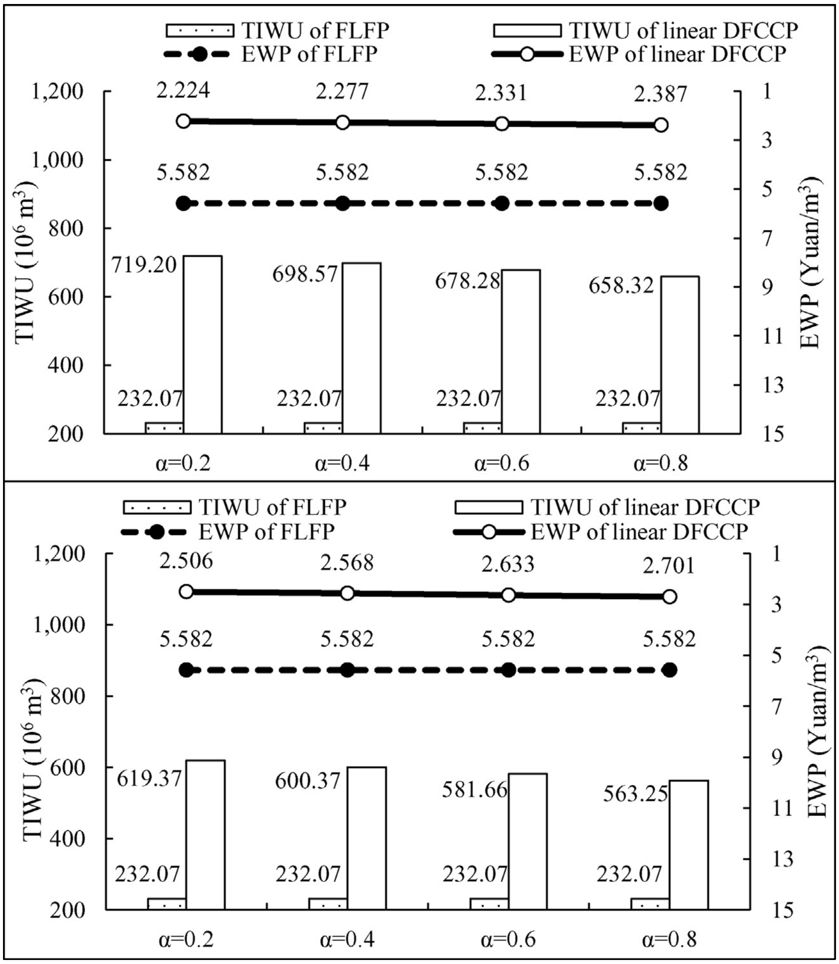

Fig. 7. a, b. Economic water productivity (EWP) and total irrigation water use (TIWU) of two models under low level (i.e. the minimum (a) and maximum (b) reliability scenarios).

economic water productivity and fuzzy parameters on double-sided constraints; (3) it introduces the concept of confidence levels of fuzzy constraints-satisfaction under both minimum and maximum reliabilities to generate more flexible solutions and (4) it facilitates in-depth analysis of interrelationships among economic water productivity, system benefits and varying confidence levels. Therefore, the results of the FLFP model can provide decision support when applied in the irrigation water allocation under complex uncertainty. It is an attempt to demonstrate that these solutions are feasible and flexible when considering economic water productivity, so as to support the irrigation water management in the study area. The results can also help managers to identify desired irrigation water allocations for supporting agricultural sustainability.

Table 8
Optimal solutions of FLFP model under medium water level at α = 1.0 (surface water; groundwater, m^3/ha).

IDs	Minimum reliability			Maximum reliability		
	Wheat	Maize	Economic crops	Wheat	Maize	Economic crops
Daman	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9
Shangsan	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9
Xijun	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9
Yingke	0; 1383.8	0; 1482.2	0; 1234.9	0; 1383.8	0; 1482.2	0; 1234.9
Pingchuan	134.6; 1597.8	0; 1787.4	251.5; 1356.2	820.5; 911.9	1054.4; 733.0	771.7; 836.0
Banqiao	693.0; 1039.4	0; 1787.4	551.9; 1055.9	855.2; 877.1	1557.8; 229.5	784.6; 823.1
Yanuan	751.4; 980.9	0; 1787.4	454.5; 1153.2	858.4; 874.0	875.3; 912.1	779.8; 828.0
Liaoquan	497.9; 1234.4	0; 1787.4	128.4; 1479.4	847.9; 884.4	880.3; 907.0	761.4; 846.4
Shahe	621.8; 1110.5	882.6; 904.7	600.4; 1007.4	851.6; 880.7	822.0; 965.4	787.9; 819.9
Youlian	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9
Luocheng	1993.2; 0	2019.3; 0	1884.9; 0	1993.2; 0	2019.3; 0	1884.9; 0
Liuba	0; 1993.2	0; 2019.3	0; 1884.9	0; 1993.2	0; 2019.3	0; 1884.9

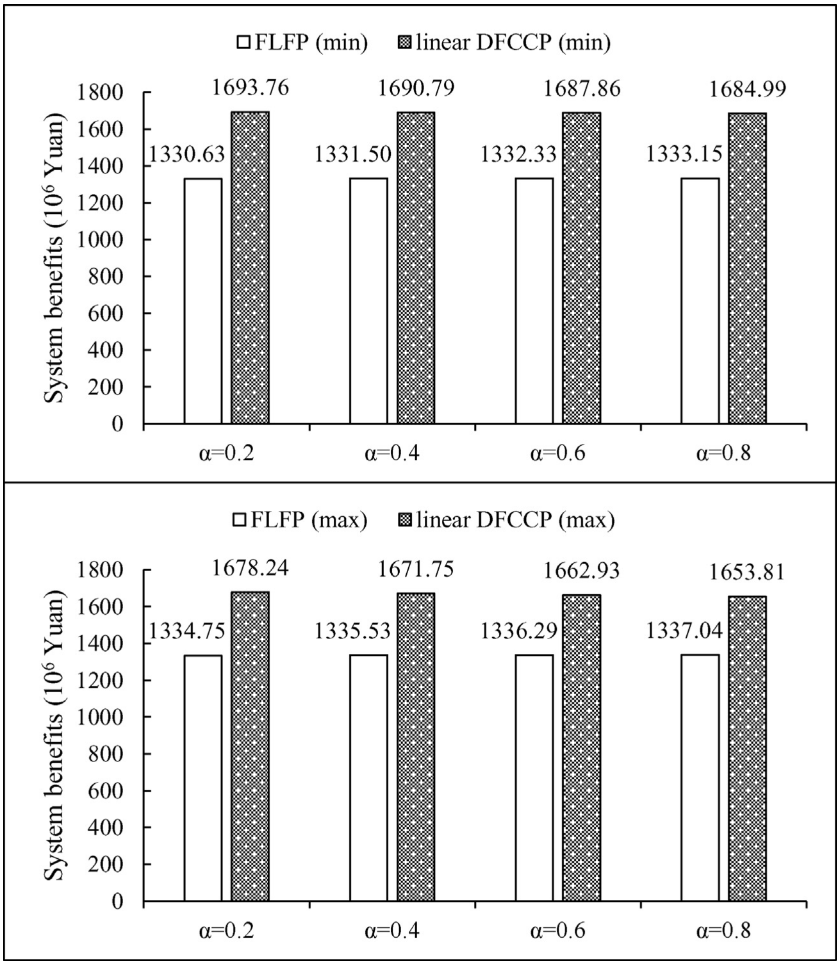

Fig. 8. a, b. System benefits of two models under high water level (i.e. the minimum (a) and maximum (b) reliability scenarios).

6. Conclusions

A fuzzy linear fractional programing (FLFP) approach with double-sided fuzziness is developed for optimal irrigation water allocation under uncertainty. In the developed model, the double-sided fuzzy chance-constrained programming (DFCCP) is incorporated into a linear fractional programming (LFP) optimization framework. Therefore, the FLFP model can deal with ratio optimization problems with double-sided fuzzy chance constraints. The concept of confidence level of fuzzy constraints-satisfaction under both minimum and maximum reliabilities is introduced to generate more flexible solutions. Thus, the final solutions can be generated by giving different confidence levels for constraint. The developed FLFP model is then applied to a case study for irrigation water allocation to demonstrate its applicability.

The obtained results are useful for supporting irrigation water management. The results of FLFP model show that: (1) the sum of irrigated surface water and groundwater is equal to the minimum crop water requirements in FLFP model due to its objective function; (2) the allocated irrigation water in IDs of Linze county is sensitive to the variations of confidence levels; (3) a higher confidence level corresponds to a higher total irrigation water allocation and a lower economic water productivity and (4) the economic water productivity under maximum reliability is slightly lower than that under the minimum reliability scenario; (5) the results of economic water productivity under three water level are: high level < medium level < lower level while the total system benefits show an opposite result.

This study attempts to develop a modelling framework for ratio problems involving fuzzy uncertainties to deal with irrigation water allocation problem. The results suggest that it is also applicable to other resources and environmental management problems. However, the FLFP model still needs to be improved. Confidence level is critical in the decision-making process of fuzzy constraints-satisfaction but there are no effective tools to choose a proper one. This difficulty should deserve further investigations. Besides, this approach can be further enhanced through incorporating the highly complex information within the optimization framework to reflect the spatial variations of crop, soil and water, such as process-based simulation model and more filed experiments. As a potentially crucial problem, water quality and saline problems will be paid attention to in our future scientific research.

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