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ORIGINAL PAPER

The interval copula-measure Me based multi-objective multi-stage stochastic chance-constrained programming for seasonal water resources allocation under uncertainty

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Abstract
A copula-measure Me based interval multi-objective multi-stage stochastic chance-constrained programming (CMIMOMSP) model is proposed for water consumption optimization. It can conduct water allocation amid multiple users and multiple stages, and deal with the uncertainties presented as interval numbers, random fuzzy interval numbers, and stochastic variables. It improves upon multi-stage stochastic chance-constrained programming by introducing the multi-objective programming, and it can tradeoff the relationships amid economic benefit, full usage of water resources, and economic loss. It enhances the accuracy of copula function and conditional distribution function through proposing the interval functions. Besides, it can deal with the impact of the decision attitudes of managers on water allocation by formulating the function equation between water demand and the optimistic-pessimistic factor. The CMIMOMSP model is applied to a case study of the Heihe River Basin to verify its application. The results indicate that: (1) the optimistic-pessimistic factors have different degrees of positive influences on water allocation for industrial, domestic and ecological sectors; (2) the joint violated probability and optimistic-pessimistic factor have various range of impacts on agricultural water allocation; (3) tthe objective function values have different variation tendencies with the rise of joint violated probabilities and optimistic-pessimistic factors. Its robustness is enhanced by comparing it with the three single-objective programming models. The CMIMOMSP model can provide various water allocation schemes for managers with different risk attitudes in semi-arid and arid districts.

Keywords Interval copula function · Interval conditional distribution · Multi-objective programming · Multi-stage stochastic programming · Measure Me · Uncertainty

1 Introduction

Water resources play an important role in social and economic development, especially for arid and semi-arid regions. In recent years, the contradictions between water supplies and water demands become serious, and water resources competitions amid multiple water users aggravate with variations of climate change and human activities (Gu et al. 2013; Zeng et al. 2014; Zhang et al. 2017, 2019). Besides, seasonal uneven distributions of water resources increase the difficulties of water management. Therefore, it is essential to optimize water resources amid multiple water users at multiple stages to alleviate the water resources contradictions and improve water-use efficiency.

The interval multi-stage stochastic programming approach (IMSP) is a useful tool to address the above problems (Zhou et al. 2013; Liu et al. 2016). It can conduct water allocation amid multiple associated stages and deal with uncertainties presented as interval numbers and stochastic variables, but it cannot address the stochastic effects in the constraints. Therefore, the interval multi-stage stochastic chance-constrained programming

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(IMSCCP) approach is developed by integrating the chance-constrained programming (CCP) with the IMSP approach (Li et al. 2006, 2009, 2014; Li and Guo 2015; Suo et al. 2017; Li and Huang 2008; Safaei 2014; Zhang et al. 2019). For example, Wang et al. 2019 established the IMSCCP model for water resources management. It builds connections of runoff at adjacent two stages by the surplus water resources at previous stage and reflect the responses of system's outputs on different water availability violated probabilities. Nevertheless, the IMSCCP exists three problems in previous studies: (1) it cannot conduct multi-variable analysis, and thus the dependency of available water resources at adjacent periods is neglected; (2) it cannot measure the runoff level at the current period under the impact of known runoff level in the previous period; (3) it is incapable of dealing with relationships between individual violated probability and joint violated probability;. The above problems can be addressed by copula functions that have been widely used in water resources management fields. For example, Fan et al. analyzed the dependency of multi variables (e.g. flood peak, flood volume and flood duration) by developing copula functions for water resources management (Fan et al. 2016). Kong et al. (2018) measured the dependency of storage volume of water in multiple reservoirs by formulating the copula functions to manage water resources. Therefore, the copula function and conditional distribution functions are incorporated into the IMSCCP approach Chen et al. 2017; Yu et al. 2018; Guo et al. 2010; Zhang et al. 2019; Yang et al. 2016). However, the copula function exists uncertainties caused by variations of average values of runoff and estimation methods. To characterize the uncertainties of copula function, the interval copula function and interval conditional distribution function are built. Therefore, the interval copula-based interval multi-stage stochastic chance-constrained programming (CIMSP) is developed in this paper. Nevertheless, there are no researches about building and integrating the interval copula function and interval conditional distribution function with the IMSCCP model to manage water resources.

The CIMSP can optimize the water allocation at multiple stages with consideration of the dependency relationship of runoff, and uncertainties of copula functions, and interval numbers and stochastic variables, but it is incapable of quantifying the decision attitudes of managers. In reality, managers with different optimistic-pessimistic opinions have various attitudes towards one water allocation scheme. In general, optimistic managers usually set higher water demands, while pessimistic managers make lower water demands. In other words, water demands can reflect the attitudes of managers to some extent. And thus the relationship fitting between water demand and optimistic-pessimistic factors can provide a basis for the optimization of water resources for managers with different attitudes. There are many methods to quantify the attitudes for decision-makers, including possibility measure, necessary measure, and credibility measure. Among these methods, a single approach has respective emphasis, but cannot cover all situations of attitudes of managers. For example, the possibility measure is suitable for managers with optimism while the necessary measure is applicable for managers with pessimism. Compared with the above methods, the measure Me is an improved method to handle all kinds of views by introducing optimistic-pessimistic factors into the possibility and necessary measures (Xu and Yao 2011; Xu and Zhou 2013; Tu et al. 2015). Therefore, the copula-measure Me based interval multi-stage stochastic chance programming (CMIMSP) model is developed through integrating the measure Me with the CIMSP model in this paper. Nevertheless, the related studies have rarely been reported.

In general, the optimal water allocation obtained from the CMIMSP model usually exceeds the water allocation target to reach the higher economic benefit and avoid water shortage, which inevitably causes the surplus water resources. The tradeoffs between water shortage and water surplus, and between economic benefit and potential economic loss are related to the overall economic-social benefits meanwhile the above three objectives are contradictory. To reach higher comprehensive benefits of multiple objectives, the copula–measure Me based interval multi-objective multi-stage stochastic chance-constrained programming (CMIMOMSP) is developed by integrating the multi-objective programming with the CMIMSP approach.

In this paper, a copula-measure Me based interval multi-objective multi-stage stochastic chance-constrained programming (CMIMOMSP) model is developed for water resources management. It integrates the interval copula function, measure Me, multi-objective programming, chance-constrained programming with the interval multi-stage stochastic programming approaches. It can allocate water resources amid multiple water users under multiple seasons and tradeoff the relationships between multiple objectives. Besides, it can deal with the dependency relationship of runoff at two adjacent seasons, at the same time the uncertainties of copula functions. In addition, it can cope with the parameters presented as interval numbers, random fuzzy interval numbers, and stochastics variables with the known probability distribution. Moreover, it can reflect the optimistic-pessimistic attitudes of managers. It supports policy analysis by formulating the different water availability scenarios and optimistic-pessimistic scenarios. The developed model is applied to a case of the middle reaches of the Heihe River Basin to optimize water allocation amid agricultural, industrial, domestic, and

ecological sectors at four seasons to verify its application. The results will be useful for the managers to allocate the water resources to multiple water users reasonably, and contribute to the sustainable development of water resources.

2 Methodology

Notation
i: index of subareas (i = 1: Ganzhou district; i = 2: Linze county; i = 3 : Gaotai county)
j: index of water users (j = 1: agricultural sector; j = 2: industrial sector; j = 3: domestic sector; j = 4: ecological sector)
t : index of periods (t = 1: spring; t = 2: summer; t = 3: autumn; t = 4: winter)
h :index of flow level (h = 1: high flow level; h = 2 Medium flow level; h = 3: low flow level)
NB: net benefit coefficient (Yuan/m^3);
WT: water allocation target (10^4m^3)
Ph: occurrence probabilities of stochastic variables
C: penalty coefficient when the water allocation target is not satisfied (Yuan/m^3)
θ :auxiliary variable
$E^{\pm Me}[E^r[W^{\pm}_{ijt,\max}]]$: water demands of water users
WA: water allocation, decision variables (10^4m^3)
Q: available water resources (10^4m^3)
ε: surplus water resources (10^4m^3)
p: joint violated probabilities
D: water shortage (10^4m^3)

2.1 The random fuzzy interval number

The random fuzzy interval numbers are used to characterize the uncertainty of the average value of the stochastic variable. The average value is supposed to be fuzzy interval numbers where the minimum possible value, possible value, and the maximum possible value of fuzzy numbers are interval numbers. Taking the triangular fuzzy number as an example, its expression is $\overset{\sim}{\overline{W}}{}^{\pm}_{ij} \sim N(\tilde{u}{}^{\pm}_{ij}, \sigma^2_{ij})$, where$\tilde{u}{}^{\pm}_{ij} = (m^{\pm}_{ij}, s^{\pm}_{ij}, n^{\pm}_{ij})$; Using the expected value operator and measure Me to transform the fuzzy interval numbers into the interval expected value$E^{Me\pm}[E^r(\overset{\sim}{\overline{W}}{}^{\pm}_{ij})]$ as follows, and the introduction of the measure Me is shown in the "Appendix":

$E^{Me\pm}[E^r(\overset{\sim}{\overline{W}}{}^{\pm}_{ij})] = [E^{Me-}[E^r(\overset{\sim}{\overline{W}}{}^{\pm}_{ij})] = \frac{1}{2}(m^-_{ij} + s^-_{ij}) + \frac{\lambda}{2}(n^-_{ij} - m^+_{ij}), \quad E^{Me+}[E^r(\overset{\sim}{\overline{W}}{}^{\pm}_{ij})] = \frac{1}{2}(m^+_{ij} + s^+_{ij}) + \frac{\lambda}{2}(n^+_{ij} - m^-_{ij})]$The diagram of random fuzzy interval number can be characterized as Fig. 1.

The random fuzzy interval variable is converted into the expected interval numbers as a function of positive-negative factor λ through the above transformation. The λ ranges from 0 to 1, and 0.5 represents that the decision-makers have compromised attitudes. The zero represents that decision-maker is pessimistic to the stochastic fuzzy event while the one donates that decision-makers have optimistic attitudes toward to the stochastic fuzzy event.

2.2 The interval copula function and interval conditional function

To deal with the dependency of runoff at adjacent two seasons and corresponding uncertainties of dependent relationships, at the same time fit the conditional cumulative distribution function (CDF) of runoff under each flow level, the interval copula function and interval conditional CDF are developed. The runoff corresponding to the cumulative probability of less than 20% is regarded as the low flow level, and 80% as medium flow level and 100% as high flow level.

(1) The concept of copula

The copula function is a multivariate probability distribution with its marginal distribution uniform, which can be expressed as follows:

$$F(x_1, x_2, ..., x_n) = C(\mathrm{F}_{X_1}(x_1), \mathrm{F}_{X_2}(x_2), ..., \mathrm{F}_{X_n}(x_n)) \tag{1}$$

where $\mathrm{F}_{X_1}(x_1), \mathrm{F}_{X_2}(x_2), ..., \mathrm{F}_{X_n}(x_n)$ are marginal distributions of random vector $(X_1, X_2, ..., X_n)$. If these marginal distributions are continuous, then a single copula function C exists, which can be written as follows (Fan et al. 2016; Salvadori et al. 2007):

$$C(u_1, u_2, ..., u_n) = \mathrm{F}(\mathrm{F}^{-1}_{X_1}(u_1), \mathrm{F}^{-1}_{X_2}(u_2), ..., \mathrm{F}^{-1}_{Xn}(u_n) \tag{2}$$

The steps of determining the copula function are as follows:

(1) Analyze the dependency of variables

The Kendall and Spearman rank correction coefficient are applied to measure the dependency of variables and corresponding significant levels, are shown as follows:

$$\tau = \frac{2}{n(n-1)} \sum_{1 \le i \le j} sign[(x_i - x_j) - (y_i - y_j)] \tag{3 - 1}$$

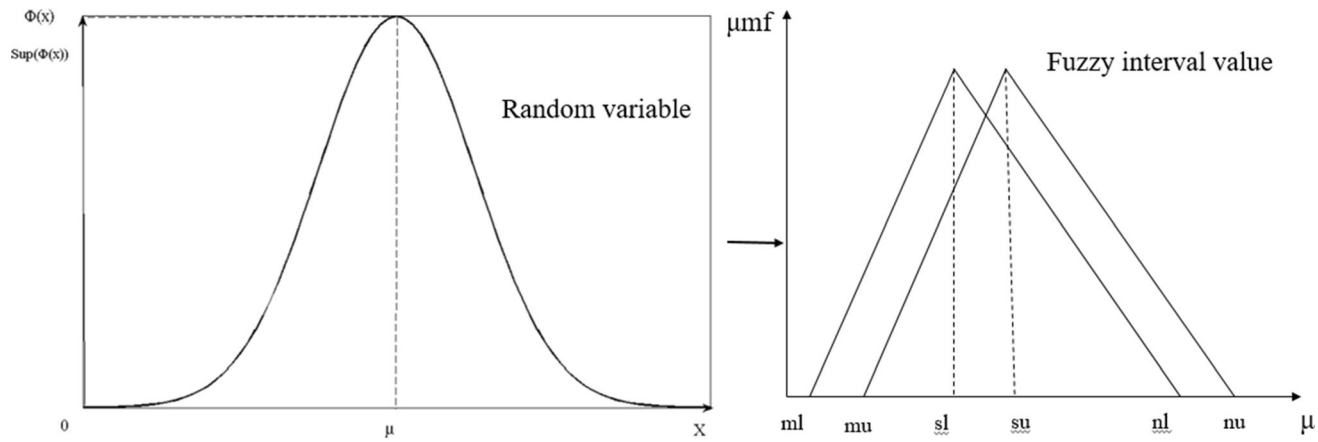


Fig. 1 The random fuzzy interval value

$$\rho_n = \frac{\sum_{i=1}^{n}(R_i - R)(S_i - S)}{\sqrt{\sum_{i=1}^{n}(R_i - R)^2 \sum_{i=1}^{n}(S_i - S)^2}} \quad (3-3)$$

where ρ is the rank correction coefficient, R_i and S_i are the ranks of variables X and Y, separately; R and S are the average value of Ri and Si, respectively.

(2) Fit the marginal distribution and select the appropriate marginal distribution

There are many kinds of marginal distribution functions, including normal distribution, P-III distribution, and Gumbel distribution. The goodness of distribution functions of variables can be measured by criteria of RMSE, MAE and PCAA, and the equations are expressed as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x(i) - y(i))^2} \quad (4-1)$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|x(i) - y(i)| \quad (4-2)$$

$$PCRR = \frac{\sum_{i=1}^{n}(x(i) - x_m)(y(i) - y_m)}{\sqrt{\sum_{i=1}^{n}(x(i) - x_m)^2 \sum_{i=1}^{n}(y(i) - y_m)^2}} \quad (4-3)$$

where the x, and y donate the observed value and simulated value; x_m and y_m are the average value of observed data and simulated data; n is the numbers of samples.

The smaller *RMSE, MEA*, and higher *PCRR* corresponds to better fitting effects of distribution functions.

(3) Fit the copula function and choose the appropriate copula function

The usually used copula functions are Clayton, Frank, Ali–Mikhail–Haq (AHM) and Gumel-Hougaard (GH). The related index method which expresses the relationship between the estimated parameter of the 2-copula function and Kendall rank correction coefficient is used to determine the parameter of the 2-copula function, shown as follows (Table 1):

The goodness assessment of copula functions is used to select the appropriate copula function. The assessed criterions include the Akaike information criterion (AIC) information method and ordinary least squares (OLS) method (Wang et al. 2019; Thevaraja and Rahman 2019).

The smaller AIC and OLS indicate better fitting effects of copula functions.

(2) Conditional distribution function

The conditional distribution can be obtained if an appropriate copula function is selected. The conditional cumulative distribution function of U_2 given $U_1 \le u_1$ can be expressed as follows:

$$C_{U_2|U_1 \le u_1} = P(\mathrm{U}_2 \le u_2 | \mathrm{U}_1 \le u_1) = \frac{C(u_1, u_2)}{u_1} \quad (5)$$

(3) The interval copula function and interval conditional CDF

Table 1 The relationship between parameter of 2-copula function and τ of Kendall method

Copula function	The relationship between θ and τ
Gumbel–Hougaard	$\tau = 1 - 1/\theta$
Clayton	$\tau = \theta/(2+\theta)$
Ali–Mikhail–Haq	$\tau = (1 - \frac{2}{3\theta}) - \frac{2}{3}(1 - \frac{1}{\theta})^2 \ln(1-\theta)$
Frank	$\tau = 1 - \frac{4}{\theta}[\frac{1}{\theta}\int_0^\theta \frac{t}{\exp(t)-1}dt - 1]$

To address uncertainties of average values of marginal distributions of random variables caused by the estimation method and limited water availability, the interval copula function and interval conditional CDF are built based on the deterministic copula function. The expressions are shown as follows:

$$C^{\pm}(u_1, u_2, ..., u_n) = F^{\pm}(F_{X_1}^{-1}(u_1), F_{X_2}^{-1}(u_2), ..., F_{Xn}^{-1}(u_n)) \quad (6-1)$$

$$C^{\pm}{}_{U_2|\mathrm{U}_1 \le u_1} = P(\mathrm{U}_2 \le u_2 | \mathrm{U}_1 \le u_1) = \frac{C^{\pm}(u_1, u_2)}{u_1} \quad (6-2)$$

The determination of interval copula function and interval conditional CDF is shown as follows:

1. The average values of marginal distribution are expressed as interval numbers based on the 95% confidence interval.
2. Fitting the interval marginal distribution, interval copula distribution function and interval conditional distribution function.

2.3 Copula-measure Me based interval multistage stochastic chance-constrained programming (CMIMSP)

The CMIMSP can allocate water resources to multiple water users at various successive stages with consideration of dependency of runoff at adjacent reasons, uncertain parameters, water availability constraints joint violated probabilities, and optimistic-pessimistic factors. The multilayered scenario tree will be used for dynamic analysis in a planning horizon, which is shown in Fig. 2.

The expressions of the CMIMSP model are shown as follows:

Objective function: maximizing the economic benefit

$$Maxf^{\pm} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} NB_{ijt}^{\pm} WT_{ijt}^{\pm} - \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{h=1}^{H} ph_{th} P_{ijt}^{\pm} D_{ijth}^{\pm} \quad (7-a)$$

Subject to:

Water availability constraints

$$\Pr\left\{\sum_{i=1}^{I}\sum_{i=1}^{J} WA_{ijth}^{\pm} \le Q_{th}^{\pm} + \varepsilon_{(t-1)h}^{\pm}\right\} \ge 1 - p \quad \forall t, h \quad (7-b)$$

$$C(1-p_1, 1-p_2, ...1-p_t) = 1 - p \quad (7-c)$$

$$\varepsilon_{(t-1)h}^{\pm} = Q_{(t-1)h}^{\pm} - \sum_{i=1}^{I}\sum_{i=1}^{J} WA_{ijth}^{\pm} + \varepsilon_{(t-2)h}^{\pm} \quad (7-d)$$

$$WA_{ijth}^{\pm} = WT_{ijt}^{\pm} - D_{ijth}^{\pm} \quad (7-e)$$

Water demand constraints

$$WA_{ijth}^{\pm} \le E^{Me\pm}[E^{r}(\tilde{\overline{W}}_{ij}^{\pm})] \quad \forall i, j, t, h \quad (7-f)$$

Non-negative constraints

$$WA_{ijth}^{\pm} \ge 0 \quad \forall i, j, t, h \quad (7-g)$$

2.4 Copula-measure Me based interval multi-objective multi-stage stochastic chance-constrained programming (CMIMOMSP)

The CMIMSP approach cannot deal with the relationship between water shortage and water surplus, at the same time the relationships amid multiple objectives. Therefore, the CMIMOMSP model is established to overcome the disadvantages of the CMIMSP model, where the expression are as follows:

Objective 1: maximize net economic benefit.

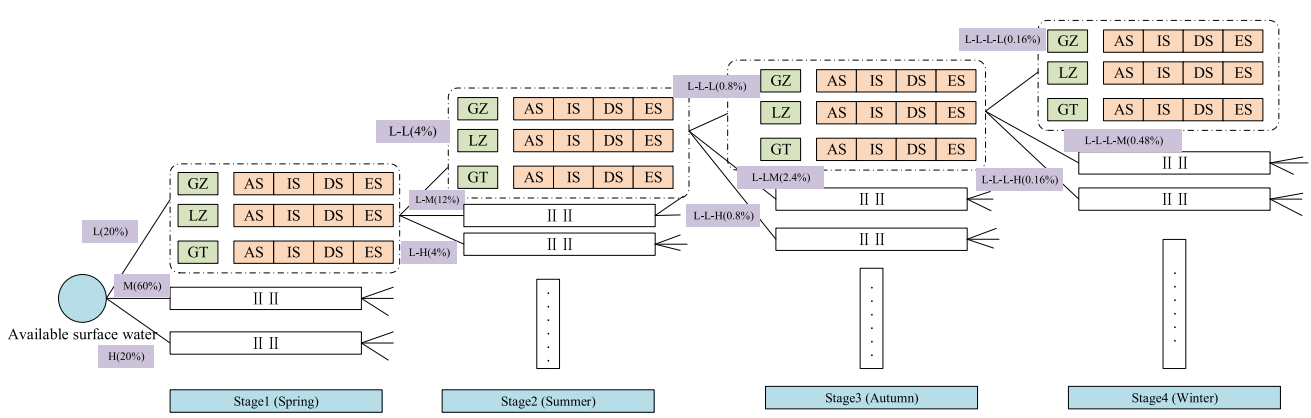


Fig. 2 Diagram of the IMSP model. *Notes*: The GZ, LZ and GT represent Ganzhou district, Linze county and Gaotai county, respectively; The AS, IS, DS, and ES donate the agricultural,industrial, domestic and ecological sectors, separately

The economic benefit is calculated by the regular benefit that is gained by multiplying the water allocation benefit with water allocation subtracting the economic penalty caused by water shortage.

$$\text{Max} f_1^{\pm} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} NB_{ijt}^{\pm} WT_{ijt}^{\pm} - \sum_{h=1}^{H}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} ph_{th} C_{ijt}^{\pm} D_{ijth}^{\pm} \quad (8-\text{a})$$

Objective 2: minimize the deviation between water allocation and target water allocation.

The deviation between water allocation and water allocation target under all flow levels should be diminished as possible to avoid water surplus and water shortage.

$$\text{Min} f_2^{\pm} = \sum_{h=1}^{H}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} ph_{th} |WA_{ijth}^{\pm} - WT_{ijt}^{\pm}| \quad (8-\text{b})$$

The transformation of Eq. (8-b) is based on the method proposed by Xu et al. (2009), and after transformation the linear equations are as follows:

$$\text{Min} f_2^{\pm} = \sum_{h=1}^{H}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} ph_{th} (WA_{ijth}^{\pm} - WT_{ijt}^{\pm} + 2\theta_{ijth}^{\pm}) \quad (8-\text{b}-1)$$

$$WA_{ijth}^{\pm} - WT_{ijt}^{\pm} + \theta_{ijth}^{\pm} \geq 0 \quad \forall i,j,t,h \quad (8-\text{b}-2)$$

Objective 3: minimize the potential economic loss.

The potential economic loss is caused by the difference between water allocation and maximum water demand, without taking advantage of market potentiality, and it supposes that the potential economic loss will be zero if the water allocation satisfies the maximum water demand.

$$\text{Min} f_3^{\pm} = \sum_{h=1}^{H} ph_h \frac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} NB_{ijt}^{\pm}(E^{\pm Me}[E^r[WT_{ijt,\max}^{\pm}]] - WA_{ijth}^{\pm})}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} E^{\pm Me}[E^r[WT_{ijt,\max}^{\pm}]]} \quad (8-\text{c})$$

Subject to:

(1) Water availability constraints

The relationship between joint and individual violated probabilities of water availability constraints are determined by the interval copula function, and the available water resources under each stage and flow level are obtained from the inverse functions of the interval conditional CDF.

$$\sum_{i=1}^{I}\sum_{j=1}^{4} WA_{ijth}^{\pm} \leq (Q_{th}^{p_l})^{\pm} + \varepsilon_{(t-1)h}^{\pm} \quad \forall h,t,l \quad (8-\text{d})$$

$$\varepsilon_{(t-1)h}^{\pm} = (Q_{(t-1)h}^{p_l})^{\pm} - \sum_{i=1}^{I}\sum_{j=1}^{J} WA_{ijth}^{\pm} + \varepsilon_{(t-2)h}^{\pm} \quad \forall h,t,l \quad (8-\text{e})$$

$$C(1-p_1, 1-p_2, \ldots, 1-p_L) = 1-p \quad (8-\text{f})$$

(2) Water demand constraints

The minimum and maximum water demands are expressed as the functions of optimistic-pessimistic factors. Set multiple groups of water demands by giving the different optimistic-pessimistic factors to measure the influences of attitudes of decision-makers on the system's outputs.

$$E^{\pm Me}[E^r[W_{ijth,\min}^{\pm}]] \leq WA_{ijth}^{\pm} \leq E^{\pm Me}[E^r[W_{ijth,\max}^{\pm}]] \quad \forall i,j,t,h \quad (8-\text{g})$$

(4) Auxiliary constraints

$$WA_{ijth}^{\pm} - WT_{ijt}^{\pm} + \theta_{ijth}^{\pm} \geq 0 \quad \forall i,j,t,h \quad (8-\text{h})$$

(5) Non-negative constraints

$$WA_{ijth}^{\pm} = W_{ijt}^{\pm} - D_{ijth}^{\pm} \geq 0 \quad \forall i,j,t,h \quad (8-\text{i})$$

2.5 Modified minimum deviation Method for solving CMIMOMSP model

The framework of the CMIMOMSP model is shown in Fig. 3.

The detailed solution process for the CMIMOMSP model can be obtained as follows:

Step 1: Acquire relevant parameters of the model, including interval number, random fuzzy interval number, and stochastic variables, the interval copula function.

Step 2: Formulate a CMIMOMSP model.

Step 3: Calculate the best and worst value of each objective and the interval weight of each objective based on the interval analytic hierarchy process (IAHP) method.

Step 4: Transform the CMIMOMSP model into a linear format by an improved method based on the modified minimum deviation method proposed by Zhang et al. (2019).The interval AHP method is used to determine the interval weight of each objective (Li 2016). The corresponding objective of the model is as follows:

$$\text{MinF}^{\pm} = \omega_1^{\pm} \frac{f_1^{\max} - f_1^{\pm}}{f_1^{\max} - f_1^{\min}} + \omega_2^{\pm} \frac{f_2^{\pm} - f_2^{\min}}{f_2^{\max} - f_2^{\min}} + \omega_3^{\pm} \frac{f_3^{\pm} - f_3^{\min}}{f_3^{\max} - f_3^{\min}} \quad (9)$$

where $f_s^{\max}$,$f_s^{\min}$ are the maximum value and minimum value of objective function f_s, respectively; w is the weight of each objective.

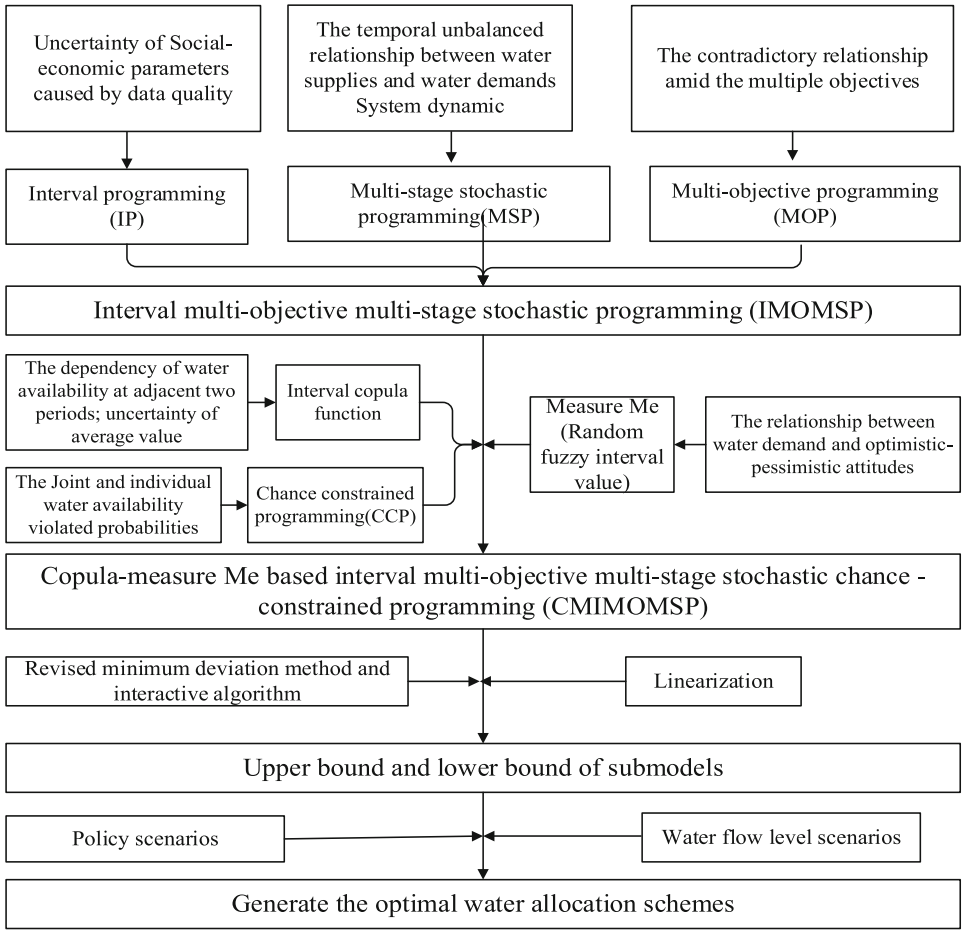


Fig. 3 The diagram of the CMIMOMSP model

Step 5: Solve the F^{-} sub-model and obtain corresponding solution alternatives.

Step 6: Solve the F^{+} sub-model and obtain corresponding solution alternatives.

Step 7: Get the solution of upper and lower sub-models.

Step 8: Formulate a series of policy scenarios composed of multiple groups of joint violated probabilities and optimistic-pessimistic levels.

Step 9: Obtain the optimal solution under different policy scenarios.

Step 10: End.

3 Case study

3.1 Study area

The study area is located in the middle reaches of the Heihe River basin (HRB), Gansu province, northwest China, shown in Fig. 4. The central administrative regions include the Ganzhou district, Linze county, and Gaotai county, and the primary water users are the agricultural sector, industrial sector, domestic sector, and the ecological sector. The main water resources are surface water from the HRB and groundwater. The seasonal uneven water resources distributions and increasing water resources requirements aggravate competitions amid water users and subareas. Therefore, the optimization of water resources amid multiple water users and various stages is essential and vital.

The IMSP model cannot deal with the dependency relationship of seasonal runoff and uncertainty of dependent relationship, and the optimistic-pessimistic attitudes of managers. Besides, it is unable to address the multiple conflicting objectives. In reality, the runoff at the current season is affected by the runoff in the previous season, presenting the dependency relationship. Besides, decision-makers have different attitudes towards one water allocation scheme. Moreover, the relationships between economic benefit, the relationship between water shortage and water surplus, and potential economic loss are conflicting objectives. To address the above problems, the improved model based on the IMSP with considerations of above factors is necessary.

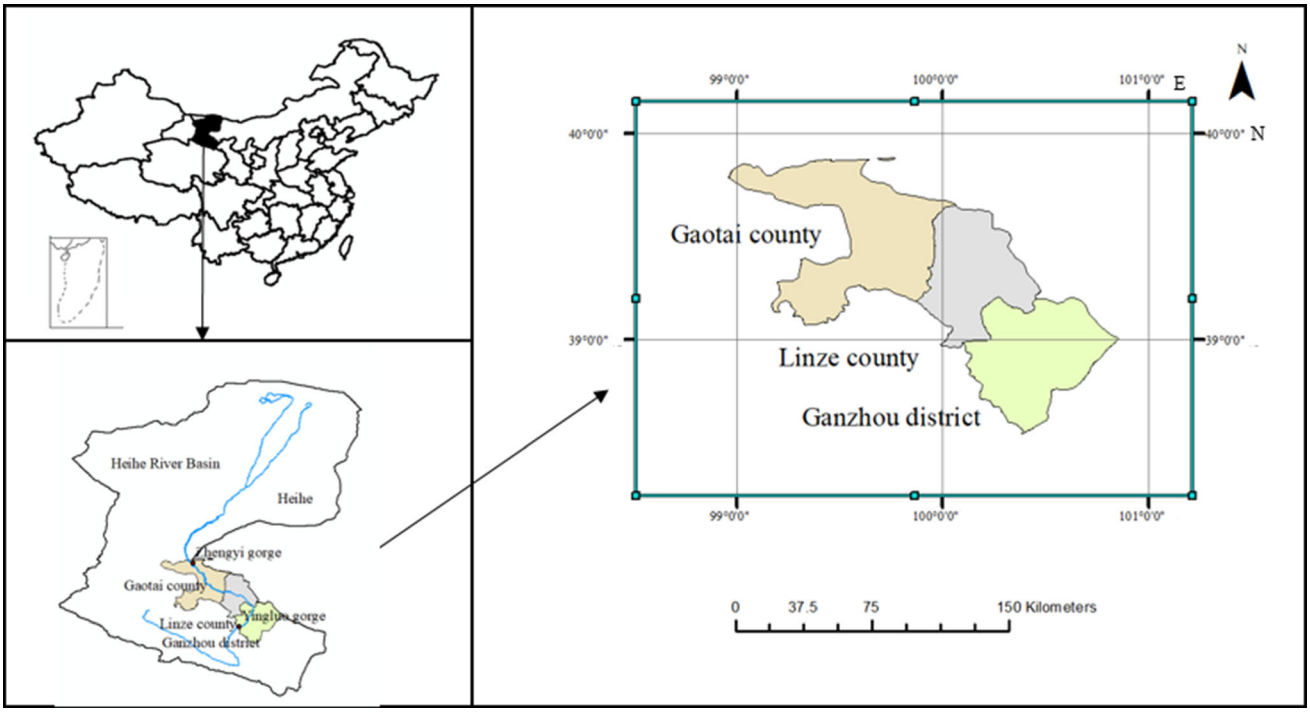


Fig. 4 Study area

3.2 Data collection

The runoff at adjacent seasons has a significant dependency relationship tested by the Kendall and Spearman rank correction coefficient methods. Therefore, the runoff at adjacent seasons are used to build the 2-copula function, and the seasonal runoff data from the year 1962 to the year 2013 is used to determine their respective marginal cumulative distribution functions (MCDF). The MCDF of seasonal runoff is fitted as normal distribution because the goodness of normal distribution is better than the P-III distribution tested by the RMSE, MAE and PACC method. The spring runoff follows normal distribution as N (2.33, 0.47), summer runoff follows N (8.67, 1.85), autumn runoff follows N (4.16, 1.30), and winter runoff follows N (1.57, 0.22), respectively. The Gumbel–Houggard copula function is applied to fit the copula function of seasonal runoff. The estimated parameters of the Gumbel–Houggard copula function are 1.30, 1.17 and 1.38 for spring-summer copula, summer-autumn copula and autumn-winter copula, respectively. The interval copula function is determined through the relationship between the estimated parameter and the related coefficient of the Kendall test (Feng et al. 2017). And three interval copula CDF are built, including the spring-summer copula CDF, summer-autumn copula CDF, autumn-winter copula CDF, respectively. Besides two representative scenarios of constraints violation levels (p, p1, p2, p3, p4) being (0.1, 0.1, 0.0369, 0.075, 0.0475), and (0.15, 0.1, 0.1646, 0.0465, 0.2232) are selected, where p represents the joint constraint-violation level; p1, p2, p3, and p4 denote the individual constraint-violation levels corresponding to the water availability at spring, summer, autumn and winter seasons, separately, which is displayed on Table 2. Another two representative scenarios of optimistic-pessimistic factor (λ) being 0 and 1 are selected to generate different water demands, separately, shown in Table 3. Therefore, four policy scenarios are formulated, where first scenario (S1) is composed of A and C: lower joint constraint violation level and pessimistic attitudes; second scenario (S2) is formed by B and C: higher joint constraint-violation level and pessimistic attitudes; Third scenario (S3) is developed by A and D: lower joint constraint violation level and optimistic attitudes; Fourth scenario (S4) is established by B and D: higher joint constraint-violation level and optimistic attitudes, shown in Table 4. The water availability of four seasons under different flow levels are shown in Table 5. Except for the runoff data, the input data of the model includes water allocation target, fuzzy minimum and maximum water

Table 2 The scenarios of joint violated probabilities and individual violated probabilities

Scenarios	p	P1	P2	P3	P4
A	0.1	0.08	0.0369	0.075	0.0475
B	0.15	0.1	0.1646	0.0465	0.2232

Table 3 The scenarios of different optimistic-pessimistic factors

Scenarios	C	D
Optimistic-pessimistic factor	0	1

Table 4 The policy scenarios composed of different joint violated probabilities and optimistic-pessimistic factors

Policy scenarios	S1	S2	S3	S4
Combination	A + C	B + C	A + D	B + D

Table 6 The annual net benefit coefficient and penalty coefficient of water users

Sectors	AS	IS	DS	ES
Annual net benefit when water demand is satisfied (Yuan/m^3)				
GZ	[0.30, 0.33]	[0.53, 0.58]	[0.55, 0.61]	[0.31, 0.34]
LZ	[0.21, 0.23]	[0.53, 0.58]	[0.55, 0.61]	[0.30, 0.33]
GT	[0.32, 0.35]	[0.53, 0.58]	[0.55, 0.61]	[0.33, 0.36]
Annual penalty when water is not delivered (Yuan/m^3)				
GZ	[0.39, 0.43]	[0.69, 0.76]	[0.72, 0.79]	[0.40, 0.44]
LZ	[0.28, 0.30]	[0.69, 0.76]	[0.72, 0.79]	[0.39, 0.43]
GT	[0.42, 0.46]	[0.69, 0.76]	[0.72, 0.79]	[0.43, 0.47]

demand, net benefit coefficient, penalty coefficient and the weights of objectives. The water allocation target is equal to the maximum water demand. The calculation of water demand refers to Wang et al. (2020). And the minimum, average and maximum water demands are regarded as the minimum possible value, possible value and maximum possible values of fuzzy water demands. The minimum water demand is calculated by multiplying the water demand as 0.6 to promise the basic water demands of industrials. And net benefit coefficient and penalty coefficient refer to Li et al. (2015), shown in Table 6. The weights of objectives are obtained by the interval analytical hierarchy process (IAHP) method. The interval numbers are all determined by 95% confidence level. The data is attained from Zhangye statistical yearbook, Annual report of water conservancy, and corresponding references, and local survey.

4 Result analysis and discussion

The results of optimal water allocation, water allocation target, and objective values under four policy scenarios are obtained from solving the CMIMOMSP model.

Figure 5 shows the seasonal interval copula CDF and Fig. 6 displays the seasonal interval conditional CDF under each flow level. It indicates that the values of conditional CDF for one water availability will increase as the values of other water availability enlarges. This illustrates positive correlation structures between the water availability under each flow level at two adjacent reasons. This also discloses that the conditional CDF is better than the independent CDF.

4.1 Optimal water allocation targets under four scenarios

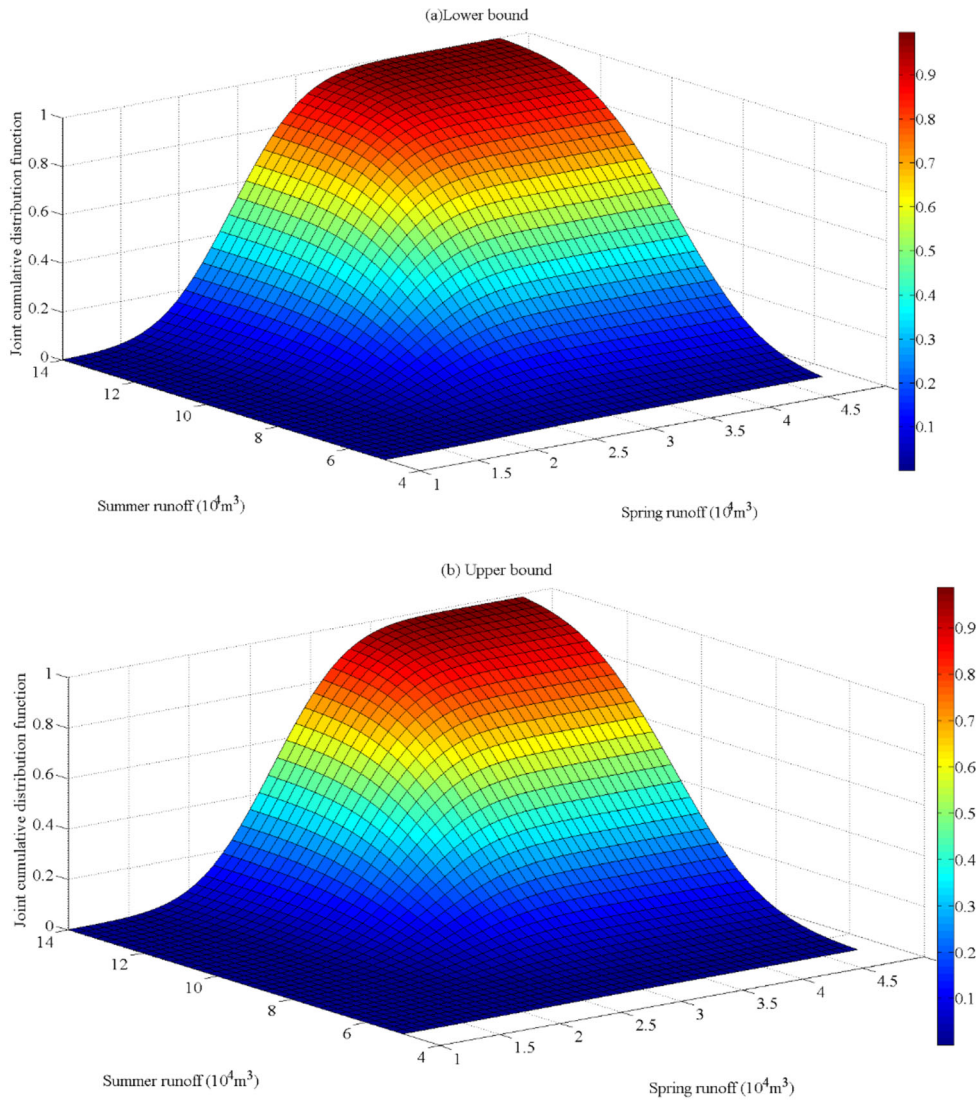
The effect of joint violated probabilities (p) and optimistic-pessimistic factors (λ) on the water allocation target can be figured out through the controlling variable method. The influence of joint violated probabilities on water allocation target with the pessimistic attitude can be obtained by comparing S1 and S2, and the effect of joint violated probabilities with the optimistic attitude can be attained by evaluating the S3 and S4; and the effect of attitude with lower water availability violated level can be gained through judging the S1 and S3; the effect of attitude with higher water availability violated level can be attained by assessing the S2 and S4.

Figure 7a represents the optimal water allocation target of industrial, domestic, and ecological sectors in GZ under four policy scenarios when the flow level is medium.

Table 5 The seasonal water availabilities under different flow levels and corresponding occurrence probabilities

Flow levels	Probability	Water availability (10^4 m^3)			
		Spring	Summer	Autumn	Winter
High	0.2	[28,470, 29,051]	[73918,75427]	[49,217, 50,222]	[24,058 24,549]
Medium	0.6	[25,240, 25,755]	[65,394, 66,729]	[38,926, 39,720]	[22,378, 22,835]
Low	0.2	[23,179 23,653]	[56,659, 57,816]	[32,689, 33,356]	[22,197, 22,650]

Fig. 5 The interval copula functions of runoff at adjacent two seasons



(a) The interval copula function of runoff for spring-summer seasons

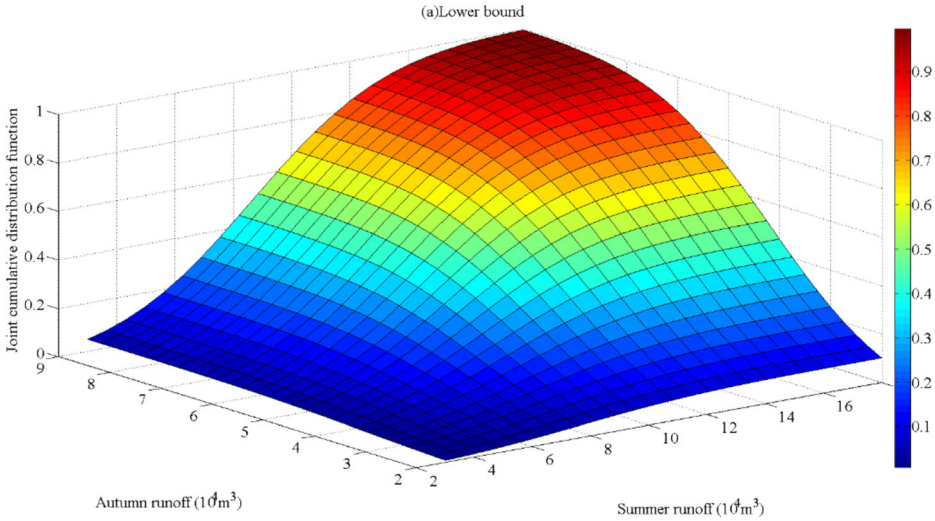
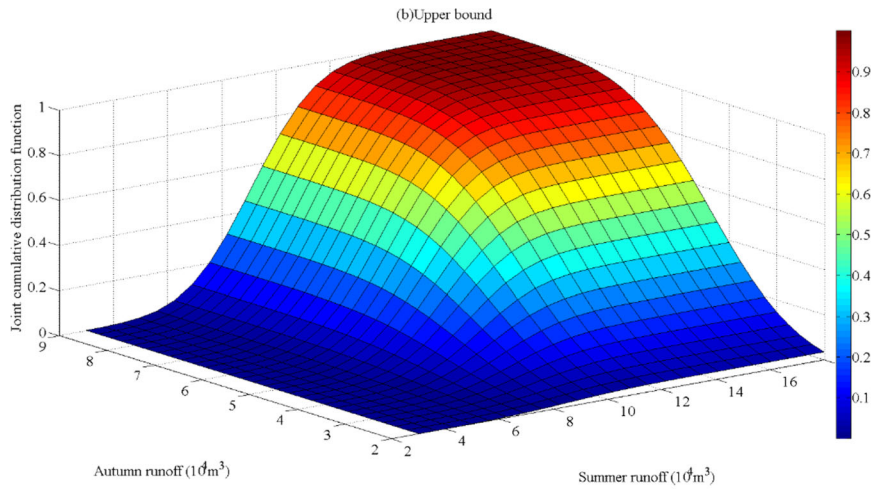
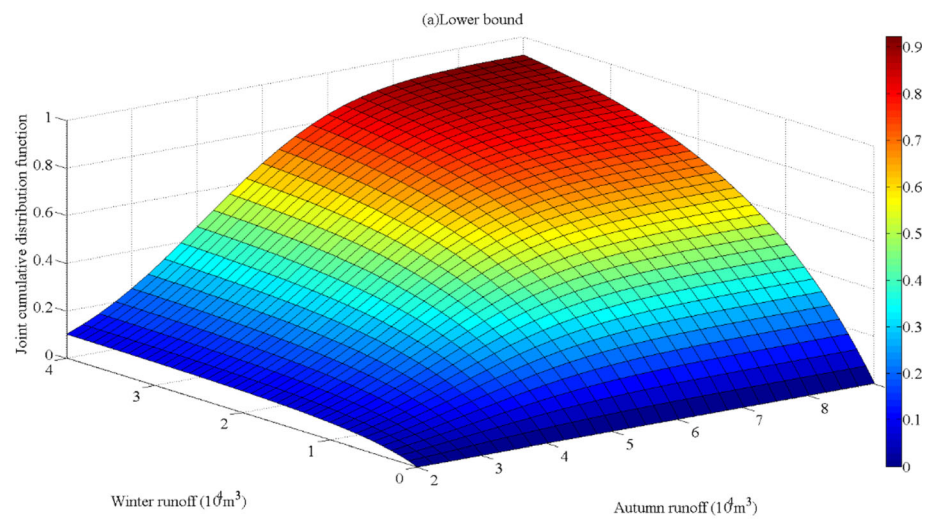
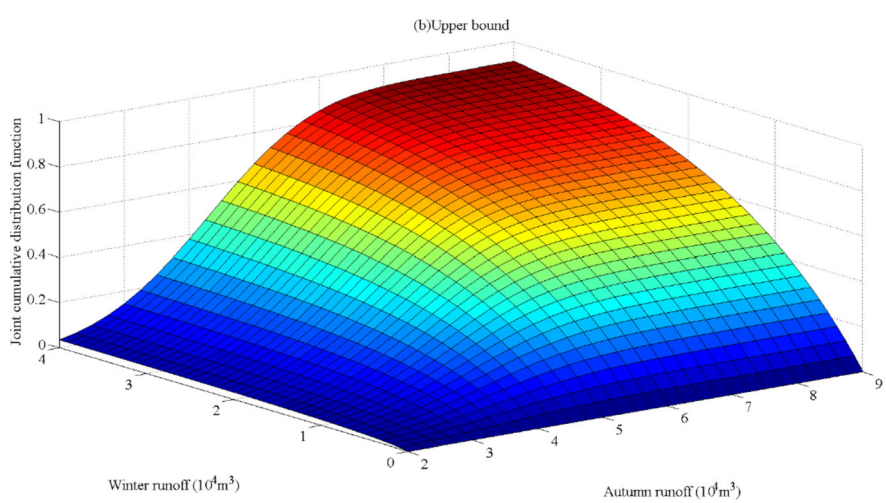


Fig. 5 continued



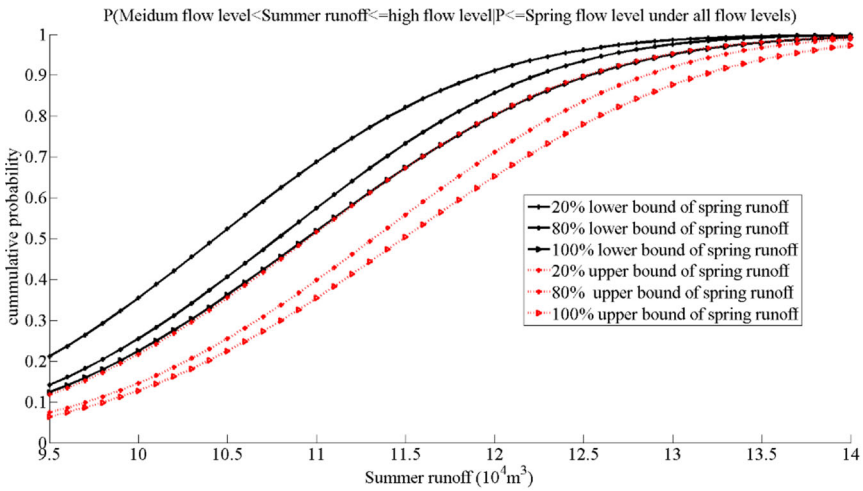
(b) The interval copula function of runoff for summer-autumn seasons



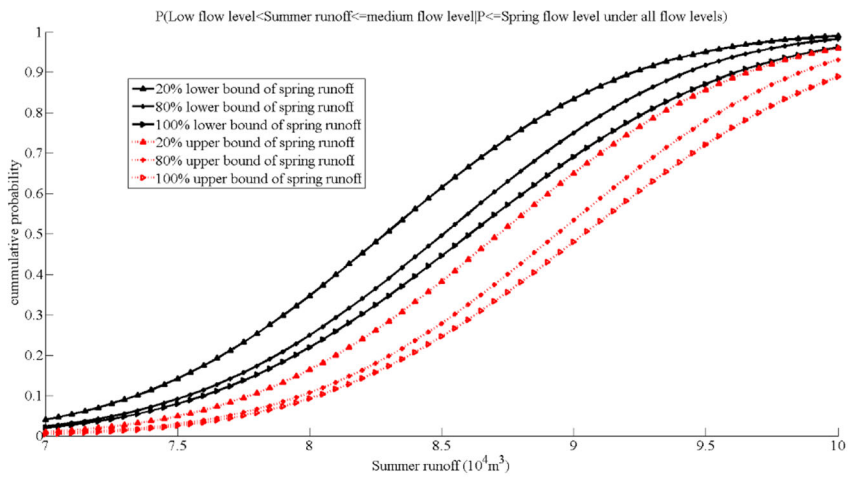


(c) The interval copula function of runoff for autumn -winter seasons

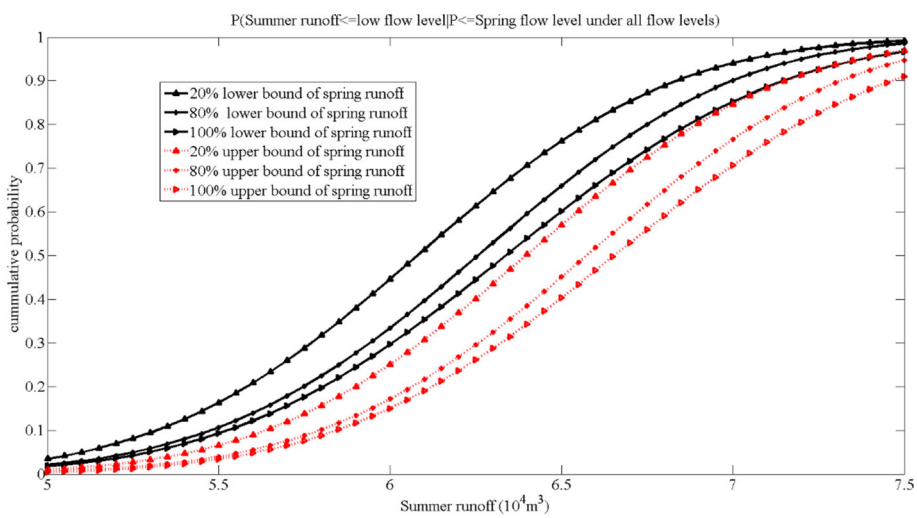
Fig. 6 The conditional cumulative distribution functions of seasonal runoff



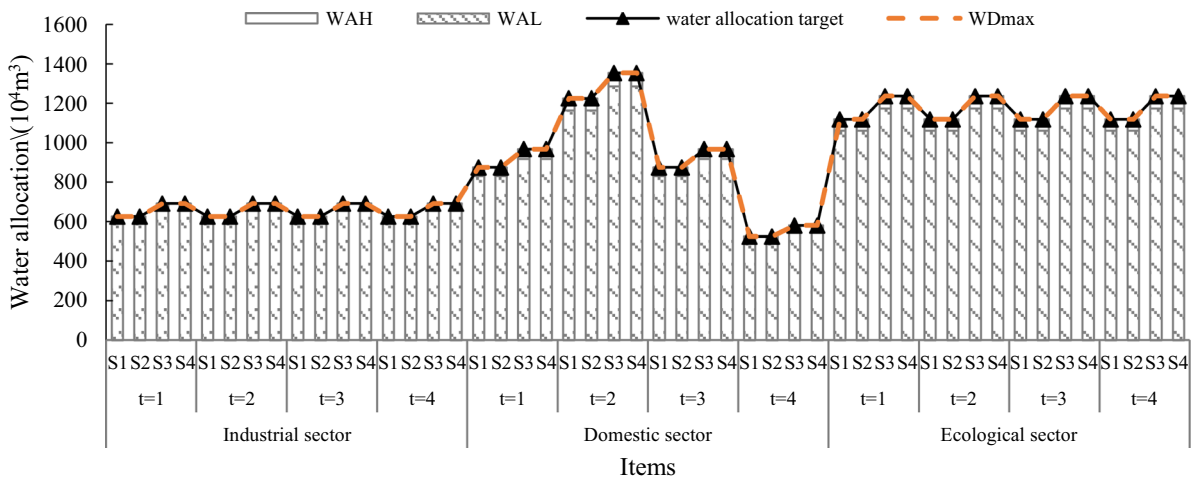
(a) The conditional cumulative distribution functions of runoff under high flow level



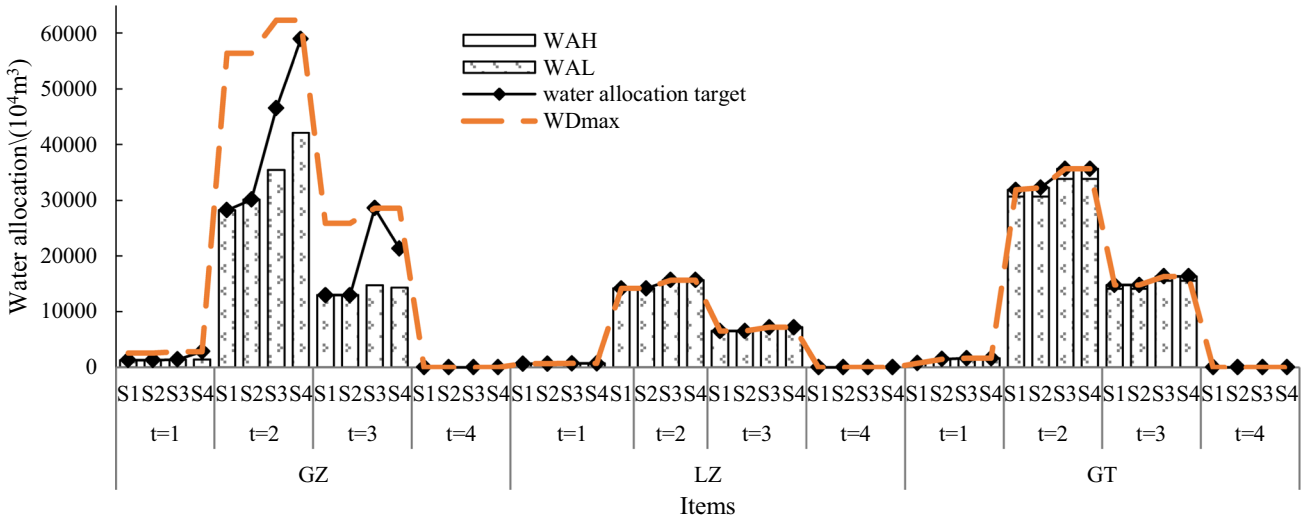
(b) The conditional cumulative distribution functions of runoff under medium flow level



(c) The conditional cumulative distribution functions of runoff under low flow level



(a) The optimal water allocation of industrial, domestic and ecological sectors



(b) The optimal water allocation of agricultural sector under four policy scenarios

Fig. 7 The optimal water allocation under four policy scenarios

Figure 7b donates the optimal water allocation target of the agricultural sector in three subareas. Figure 7a indicates that the water allocation targets of industrial, domestic, and ecological sectors reach the upper bound of water allocation under four policy scenarios. The water allocation target keeps unchanged with the variation of p and increases with the rise of λ. For example, the water allocation target of the industrial sector keeps the same as p values are 0.1 and 0.15 while increases by 65.84×10^4 m^3 as λ values are 0 and 1. It indicates that joint violated probabilities have no significant impact on the water allocation target and the optimistic-pessimistic factor has a positive effect on the water allocation target. Therefore, the decision-makers could select the combination of joint violated probabilities and optimistic-pessimistic factors based on their attitudes to risk events

Figure 7b reveals that the water allocation target of the agricultural sector in LZ and GT arrives at the upper bound of water allocation while that in GZ is between the lower bound and upper bound of water allocation target. The water allocation differences in the agricultural sector amid three subareas are caused by the water allocation benefit, where the upper bound of water allocation target is first satisfied for the sector with higher water allocation benefit under the limited water availability. This is also suitable for differences in water allocation target among the agricultural, industrial, ecological, and agricultural sectors. The benefit coefficient of the domestic sector is the biggest,

followed by the industrial sector, the ecological sector, and the agricultural sector. The impacts of joint violated probabilities and optimistic-pessimistic factors on agricultural water allocation target are conducted. Taking GZ as an example, in spring, the water allocation target stays unchanged with q when λ is 0, while it increases when λ takes 1.0. The water allocation target increases with λ when p take 0.1 and 0.15. It indicates that the optimistic-pessimistic factor has a different degree of positive impact on water allocation target, and joint violated probability has a significant positive effect on the water allocation target when λ is 1.0. In summer, the water allocation target is sensitive to both p and λ, and the sensitivity degrees are different. For example, the water allocation targets change 1901.52×10^4 m^3 for a combination of S1 and S2, 12415.41×10^4 m^3 for a combination of S3 and S4, 18325.11×10^4 m^3 for a combination of S1 and S3 and $28{,}830 \times 10^4$ m^3 for a combination of S2 and S4. It shows that the condition that λ changes from 0 to 1 and p is 0.15 has the biggest influence on water allocation target and the condition that p varies from 0.1 to 0.15 and λ value 0 has the smallest impact on water allocation target. In autumn, the water allocation target keeps unchanged with p when λ is 0 and decreases when λ is 1.0. This is because lower λ constrains the effects of p, and the water availability decreases with the increase of p when λ is 1.0 under the impact of dependency of seasonal runoff. Besides, the water allocation target increases with the λ in autumn. In summary, the annual water allocation target increases with the p and λ. The higher water allocation target will bring out the higher economic benefit at the same time the bigger penalty loss, the higher potential economic loss, and higher water waste. The manager should make a decision on which joint probabilities and optimistic-pessimistic factors are selected based on their attitudes and preferences. If managers require to improve economic benefit and lessen the potential economic loss, the water allocation target with a higher joint violated probability and higher optimistic-pessimistic factor could be selected. If managers want to take advantage of water resources, the water allocation target with a lower joint violated probability and lower optimistic-pessimistic factors could be chosen.

4.2 Optimal water allocation under four scenarios

The water allocation is expressed as interval numbers to quantify the uncertainty in the water resources management system. Figure 7 shows that the upper bound of water allocation of four water-use sectors in LZ and GT reaches the respective water allocation targets. The water allocations of industrial, domestic and ecological sectors under S1 equal to the water allocations under S2 while S3 is the same with S4, which indicates that the joint violated probabilities have no effect on water allocation on the above sectors, while the positive-pessimistic factor has a positive effect on water allocation of above sectors. The reason for the differences in water allocation under four policy scenarios is that the water demand limits the water allocation target, and the higher λ corresponds to bigger water demand and further leads to higher water allocation target and water allocation. Figure 8 shows the water allocation of the agricultural sector in GZ under four policy scenarios and three flow levels. It discloses that the water allocation all reaches the water allocation target under a high flow level while it does not arrive at water allocation target under medium and low flow levels. This is because the available water resources decrease with the reductions of flow levels. It illustrates that the water allocation exists differences amid four policy scenarios. For example, taking medium flow level as an example, in spring, the water allocation satisfies water allocation target under S1, S2, and S3, while it does not meet the water allocation target under S4. This is because compared with the other three scenarios, the water allocation target under S4 is higher, and the insufficient available water resources could not meet its water allocation target under medium flow level. It also indicates that the joint violated probability has no effect on water allocation, while the positive-pessimistic factor has a positive effect on water allocation in spring under three flow levels. In summer, the water allocation meets the water allocation target under S1 and S2 while it does not satisfy the water allocation target under S3 and S4. The reason is similar to the spring, where the water allocation targets under S3 and S4 are higher than the S1 and S2. It also discloses that the joint violated probabilities and positive-pessimistic factors have a different degree of significant influence on water allocation. In autumn, the relationship between water allocation and water allocation target is the same with spring, presenting that the water allocations under a combination of S1 and S2, and a combination of S3 and S4 are the same, and do not reach the water allocation targets.

Notes: The WAHH, WAMH, and WALH are upper bound of water allocation under high, Medium and low flow levels, respectively; the WAHL, WAML and WALL refer to the lower bound of water allocation under high, Medium and low flow levels, separately; the WT and Wdmax donate the water allocation target and the maximum water demand, respectively.

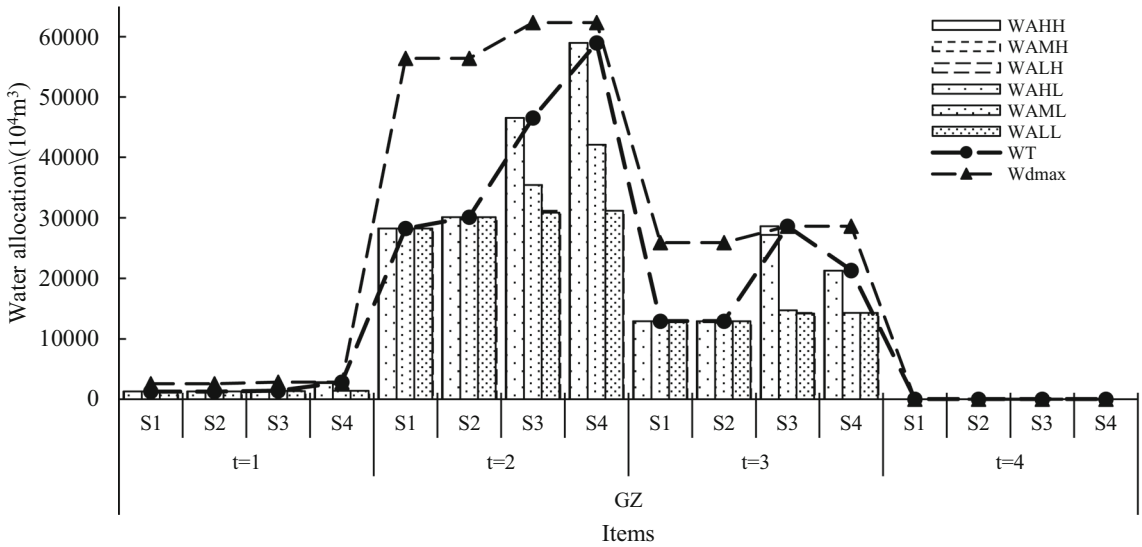


Fig. 8 The water allocation of agricultural sector in GZ under four policy scenarios

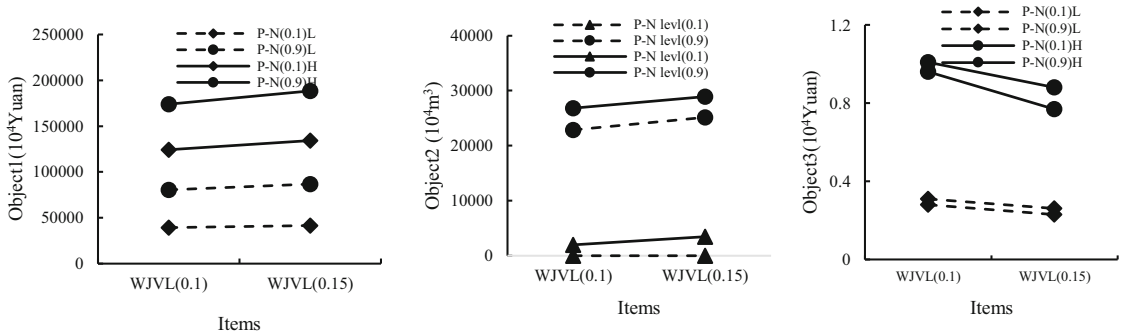
4.3 Comparative analysis of objectives

(a) Comparative analysis of economic benefits

The economic benefits under four policy analysis scenarios are shown in Fig. 9a. The economic benefit presented as interval numbers to reflect the uncertainties in the process of decision making. It indicates that the economic benefits increase with the rise of p and λ. For example, the economic benefit enlarges by [2280.75, 3301.32] × 10^4 Yuan when λ is 0, and p changes from 0.1 to 0.15, while increases by [4248.37, 4672.41] × 10^4 Yuan as λ is 1, and p changes from 0.1 to 0.15. It illustrates that the increased ranges of economic benefits with higher joint violated probability are larger than the lower joint violated probability. The economic benefits increase by [1990.71, 26,832] × 10^4 Yuan, and [3491.53, 28892.64] × 10^4 Yuan, respectively as λ varies from 0 to 1.0 and p equals 0.1 and 0.15. It discloses that higher water demand with an optimistic attitude and higher joint violated probabilities have bigger economic benefits.

(b) Comparative analysis of the full usage of water resources

The deviation between water allocation and water allocation target increases with the p and λ. The deviation between the water allocation target and water allocation is presented as the water shortage because the water allocation does not exceed the water allocation target. And the water shortage will enlarge with the rise of p and λ. The water shortages are [22880.82, 24841.29] × 10^4 m^3 and [25168.90, 25401.11] × 10^4 m^3, separately for p of 0.1 and 0.15 when λ changes from 0 to 1. It demonstrates that the λ has a more significant influence on water shortage when p is higher. This is because the water allocation target enlarges with the rise of λ, and there is no sufficient water supply satisfying the water allocation target under medium and low flow levels. And it also indicates that the higher joint violated probabilities result in a bigger water



(a) Economicbenefit **(b)** utilization of water resources **(c)** water short age economic loss

Fig. 9 The objectives under four policy scenarios

shortage. The water shortages are [0, 1500.83] × 104 m^3 and [2060.64, 2288.08] × 10^4 m^3 for λ of 0 and 1.0 as p changes from 0.1 to 0.15. It implies that less water demand with a pessimistic attitude is beneficial to reducing the water shortage.

(c) Comparative analysis of potential economic loss

The potential economic loss decreases with variations of p and λ. It means that the difference between maximum water demand and water allocation becomes less and the satisfactory degree of the potential economic benefit enlarges. For example, the potential economic losses decrease by [0.05, 0.13] and [0.05, 0.19] for λ of 0 and 1.0 when p changes from 0.1 to 0.15. The influence of q on potential economic loss under λ of 0 is less than λ of 1. It indicates that the higher joint violated probability under the optimistic attitude has a more important influence on potential economic loss. The potential economic losses reduce by [0.03, 0.05] and [0.031, 0.11] for p of 0.1 and 0.15 when λ varies from 0 to 1.0, which also discloses optimistic-pessimistic factor under higher joint violated levels has a more significant impact on potential economic loss. The potential economic loss decreases with the rise of p because more available water resources will be allocated. Comparing S1 and S3, it indicates that the potential economic loss decreases with the rise of λ, which is because the water allocation target and water allocation enlarge with the λ, and the difference between water allocation and maximum water demand lessens. This is also suitable for S2 and S4, S3, and S4.

In summary, three objectives are contradictory. It shows that the economic benefit increases, the difference between water allocation target and water allocation enlarges and the penalty economic losses decrease with the rise of joint violated probabilities and optimistic-pessimistic factors. The economic benefit, and deviation between water allocation target and water allocation have positive relationships with joint violated probabilities and optimistic-pessimistic factors, while the potential economic loss has an opposite relationship with joint violated probabilities and optimistic-pessimistic factors. At the same time, the higher joint violated probabilities correspond to the higher water shortage risk. Managers could select the parameter combination and corresponding water allocation schemes based on their risk attitudes and optimistic-pessimistic attitudes.

4.4 The comparison with the single-objective programming model

The objective values under the S1 scenario are compared with the three single-objective programming models, shown in Table 7. The objective function of the first model is maximizing the economic benefit, and the second is minimizing the deviation between water allocation target and water allocation, and the third is minimizing the potential economic loss. It indicates that the single-objective programming could obtain their respective best objective, but cannot have consideration of other objectives. Compared with the single-objective programming, the CMIMOMSP model could achieve water resources management considering the tradeoff amid three objectives, and figure out the variations of objectives under different policy scenarios. And the optimal water allocation scheme obtained from the CMIMOMSP model could reach the tradeoff amid economic benefit, full usage of water resources and potential economic loss faced by water managers in water resources management.

4.5 Analysis of copula tail dependence

The upper bound and lower bound of the copula tail dependence of seasonal runoff are shown as Table 8.

It shows that the lower bound of tail dependence seasonal runoff is higher than the upper bound, indicating that the dependency of seasonal runoff at low flow level is higher than the seasonal runoff at a high flow level. Therefore, the managers should pay more attention to the low flow level to deal with the drought condition.

5 Conclusion

In this study, a copula-measure Me based interval multi-objective multi-stage stochastic chance-constrained programming (CMIMOMSP) method is built for water resources optimization. The developed CMIMOMSP model is applied to a case study of the Heihe River Basin to verify its application, and the result provides that the proposed model can be a reliable and effective tool. The innovations of this paper are as follows:

1. It can deal with the uncertainties presented as interval number, random fuzzy interval number, and stochastic variable with known probability distributions.
2. It can tradeoff the relationships amid the multiple objective functions expressed as economic benefits, the relationship between water shortage and water surplus, and potential economic loss.
3. It can quantify the dependency relationship of water availability at adjacent two seasons, and uncertainties of fitted copula function and conditional distribution function.
4. It can Measure the function relationship between water demands and optimistic-pessimistic factors.

Table 7 Comparison between the CMIMOMSP model and single-objective models

Models	Objective1	Objective2	Objective3	CMIMOMSP
EB	[42272, 50550]	[29352, 32287]	[38517, 48004]	[39601, 45240]
UWS	[25956, 29994]	0	[44369, 48673]	[0, 3039]
PEL	[0.2, 0.7]	[2.76,5.68]	[0.18, 0.52]	[0.29, 0.96]

The EB, UWS and PEL are economic benefit, utilization of water resources, and potential economic loss, respectively

Table 8 The interval copula tail dependence

Items	Spring–smmer		Summer–autum		Autum–winter	
	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound
Gumber copula	0.30	0.59	0.19	0.55	0.35	0.61

5. It can conduct policy analysis composed of a series of joint violated probabilities, and optimistic-pessimistic factors.

It is demonstrated that the CMIMOMSP model is more robust and applicable for coping with water consumption optimization problems in arid and semi-arid district.

However, the CMIMOMSP model cannot deal with the problems of water security, water quality and sustainability, and these problems can be addressed by incorporating the water security constraints, water quality constraints, and sustainability constraints into the CMIMOMSP model, which will be studied in the future.

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Appendix

The measure Me can be defined as follows:

$$Me\{\xi \le r\} = Nec\{\xi \le r\} + \lambda(Pos\{\xi \le r\} - Nec\{\xi \le r\}),$$

where $Pos\{\xi \le r\}=\sup_{u \le r} \mu(u)$ and $Nec\{\xi \le r\}$=1- $\sup_{u > r} \mu(u)$; Pos is the possibility measure; Nec is the necessary measure; λ is the optimistic-pessimistic parameter to determine the combined attitude of a manager.

The probability distribution of triangular fuzzy numbers $\widetilde{\xi} = (\xi_1, \xi_2, \xi_3)$ based on the measure Me is show as follows:

$$Me\left\{\widetilde{\xi} \ge r\right\}=\begin{cases} 1 & r \le \xi_1 \\ \lambda + (1-\lambda)\dfrac{\xi_2 - r}{\xi_2 - \xi_1} & \xi_1 \le r < \xi_2 \\ \lambda\dfrac{\xi_3 - r}{\xi_3 - \xi_2} & \xi_2 \le r < \xi_3 \\ 0 & r > \xi_3 \end{cases}$$

where ξ_1, ξ_2, ξ_3 are the minimum possible value, possible value and maximum possible value, respectively.

The expected value of $\widetilde{\xi}$ based on the measure Me.

$$\begin{aligned} E[\xi] &= E^{Nec}[\xi] + \lambda(E^{Pos}[\xi] - E^{Nec}[\xi]) \\ &= \lambda E^{Pos}[\xi] + (1-\lambda)E^{Nec}[\xi] \end{aligned}$$

Finally, the expected value of $\widetilde{\xi}$ is expressed as $E^{Me}[\xi] = \frac{1}{2}(\xi_2 + \xi_1) + \frac{\lambda}{2}(\xi_3 - \xi_1)$

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